Modeling

- Need to usefully represent objects in the world
- Need to provide for easy interaction
  - manual modeling
    - user would like to “fiddle” until it is right (e.g. CAD)
    - user has an idea what an object is like
  - fitting to measurements
    - laser range finder data
- Support rendering/geometric computations
Modeling tools

- Polygon meshes
- Fitting curves to points (from data)
- Fitting curves to points (user interaction)
- Generating shapes with sweeps
- Constructive solid geometry
Polygon Meshes

• Common, straightforward, often built in (e.g. torus mesh)
• Ready to render (many of the representations discussed soon are often be reduced to polygon meshes for rendering)
• Problems
  – Awkward to provide user editing
  – The number of polygons can be very large
    • Some kind of adaptive process makes sense
    • More polygons at high curvature points
    • More polygons where the object is larger
    • extra care then needs to be taken to avoid temporal aliasing
Explicit curve representation

- Usual representation learned first
- Generally less useful in graphics, but know the term
- Explicit curve is a function of one variable. Examples
  - line, \( y=m\times x + b \)
  - circle (need to glue two together) \( y = \pm \sqrt{r^2 - x^2} \)
- Explicit surface is a function of two variables. Examples
  - plane \( z = m\times x + n\times y + b \)
Implicit representation

- Also less useful for this section, but again, know the term
- An implicit curve is given by the vanishing of some functions
  - circle on the plane, $x^2 + y^2 - r^2 = 0$
  - twisted cubic in space, $x^2 y - z = 0$, $x^2 z - y^2 = 0$, $x^2 x - y = 0$
- An implicit surface is given by the vanishing of some functions
  - sphere in space $x^2 + y^2 + z^2 - r^2 = 0$
  - plane $a x + b y + c z + d = 0$
Parametric representation

• A parametric curve is given as a function of one parameter. Examples:
  – circle as \((\cos t, \sin t)\)
  – twisted cubic as \((t, t^2, t^3)\)

• A parametric surface is given as a function of two parameters. Examples:
  – sphere as \((\cos s \cos t, \sin s \cos t, \sin t)\)

• Advantage - easy to compute normal, easy to render, easy to put patches together, ranges can be easy (e.g. half circle)
• Disadvantage - ray tracing can be hard
Generating Surfaces

- We can construct surfaces from curves in a variety of user intuitive ways
- Extruded surfaces
- Generalized cones
- Surfaces of revolution
- Sweeping (generalized cylinders)
Extruded surfaces

- Geometrical model - Pasta machine
- Take curve and “extrude” surface along vector
- Many human artifacts have this form - rolled steel, etc.

\[ (x(s,t), y(s,t), z(s,t)) = (x_c(s), y_c(s), z_c(s)) + t(v_0, v_1, v_2) \]
Cones

- From every point on a curve, construct a line segment through a single fixed point in space - the vertex
- Curve can be space or plane curve, but shouldn’t pass through the vertex
Surfaces of revolution - 1

- Plane curve + axis
- “spin” plane curve around axis to get surface
- Choice of plane is arbitrary, choice of axis affects surface
- In the example to the right, curve is on x-z plane, axis is z axis. (think of \( x_c(s) \) as a radius)

\[
(x(s,t), y(s,t), z(s,t)) = (x_c(s)\cos(t), x_c(s)\sin(t), z_c(s))
\]
Surfaces of revolution -2

Many artifacts are SOR’s, as they’re easy to make on a lathe.

Controlling is quite easy - concentrate on the cross section.

Axis crossing cross-section leads to ugly geometry.
Sweeps/Generalized Cylinders

Figure 3.8: Banana example. A banana is represented by an affine transformation surface. The cross section is scaled, translated along z from -1 to 1, and rotated around the y axis.

[Synder 92, via CMU course page]
Sweeps/Generalized Cylinders

MetaCreations, via CMU course page