

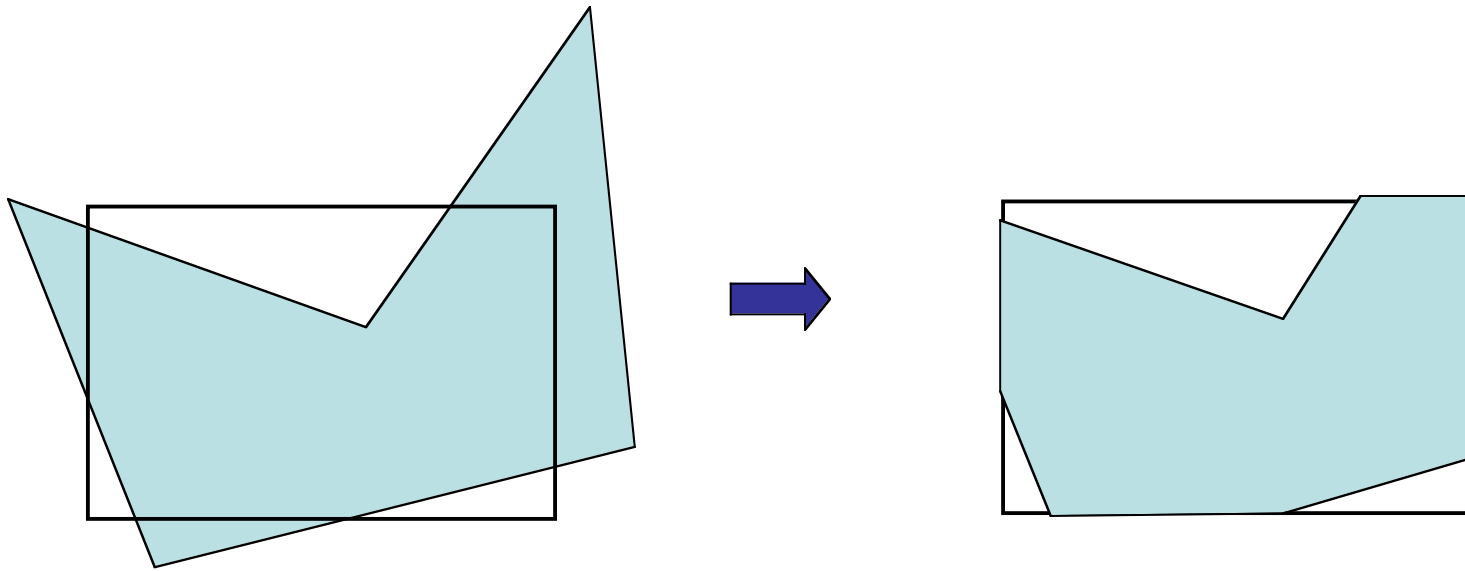
Administrative

Lecture notes should be accessible from anywhere with alternative URL.

Let the TA have a crack at assignment problem--send E-mail to him (mingde @ cs.arizona.edu) and cc me.

Next term: There will be a grad course in graphics which will focus on advance topics--open to undergrads.

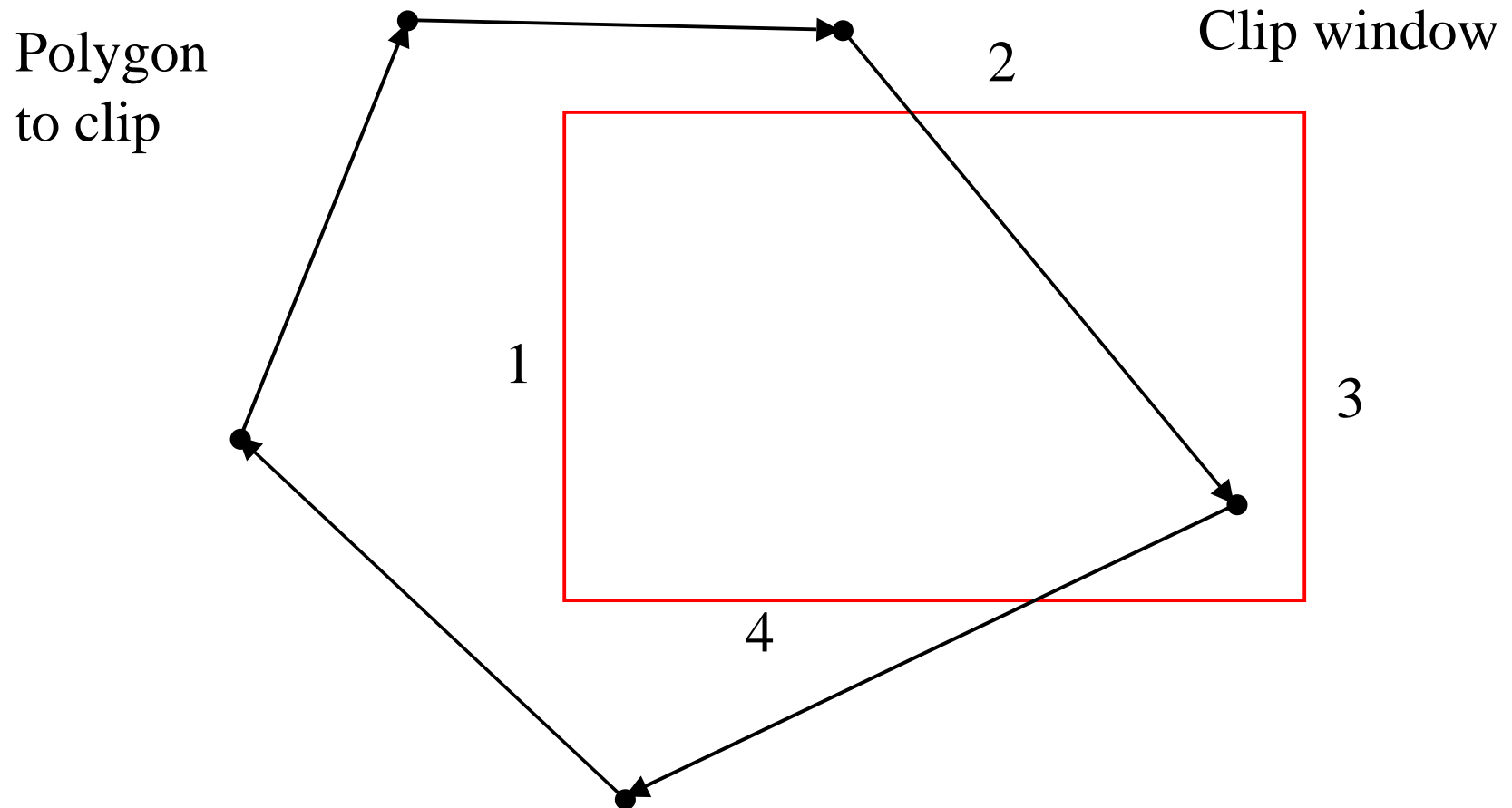
Polygon clip (against convex polygon)



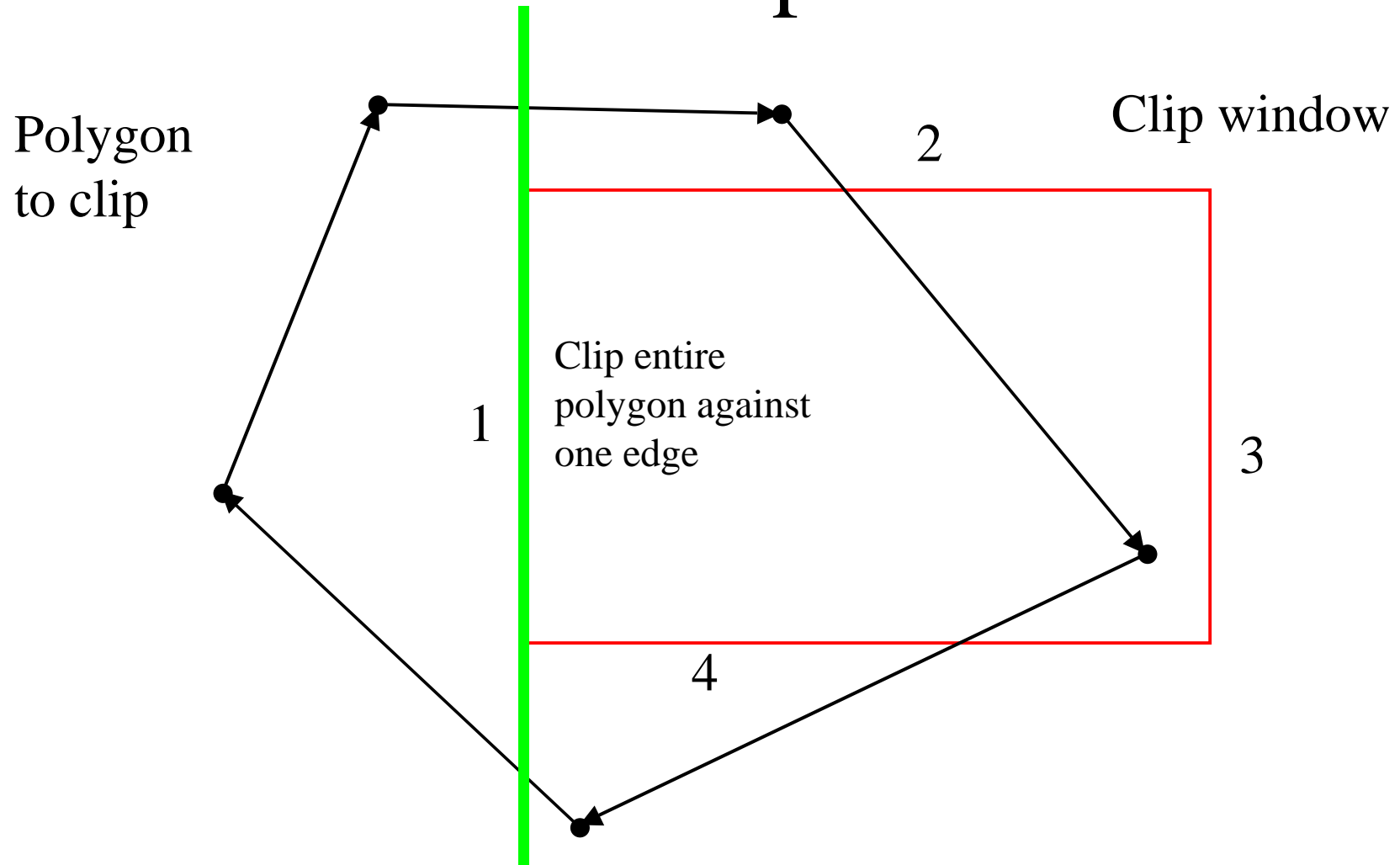
Sutherland-Hodgeman polygon clip

- Recall: polygon is convex if any line joining two points inside the polygon, also lies inside the polygon; implies that a point is inside if it is on the right side of each edge.
- Clipping each edge of a given polygon doesn't make sense - how do we reassemble the pieces? We want to arrange doing so on the fly.
- Clipping the polygon against each edge of the clip window in *sequence* works if the clip window is *convex*.
- (Note similarity to Sutherland-Cohen line clipping)

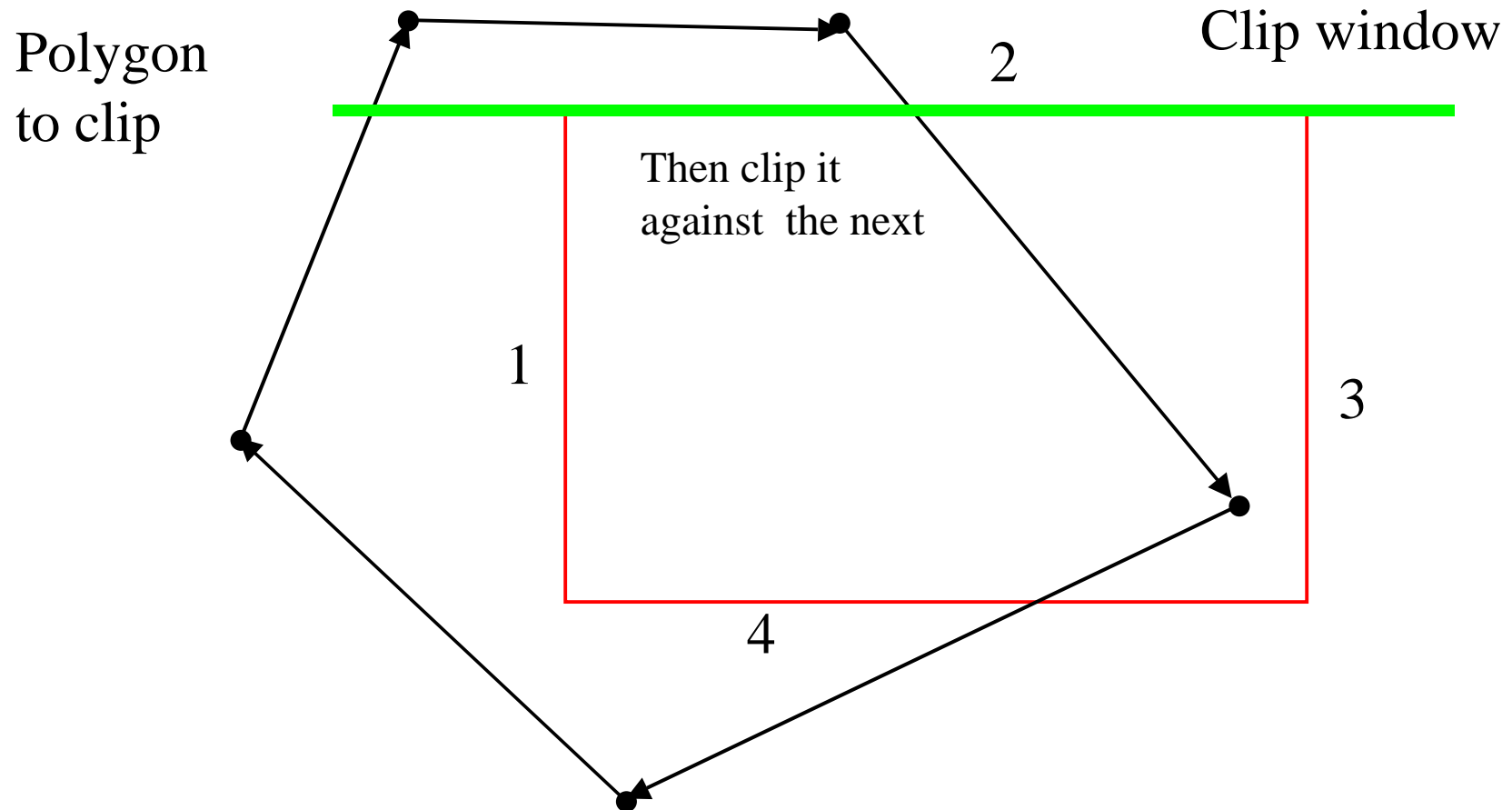
Sutherland-Hodgeman polygon clip



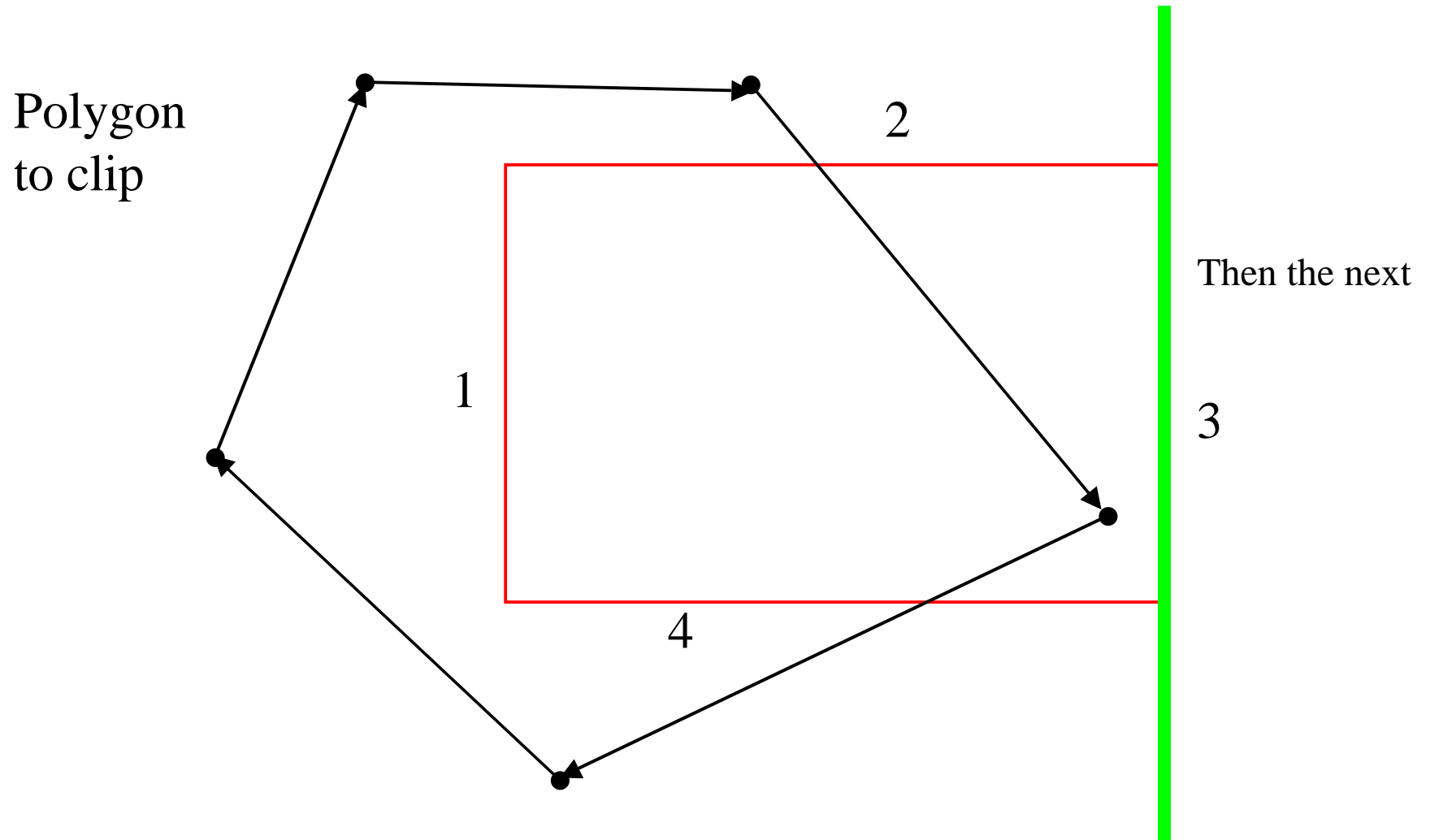
Sutherland-Hodgeman polygon clip



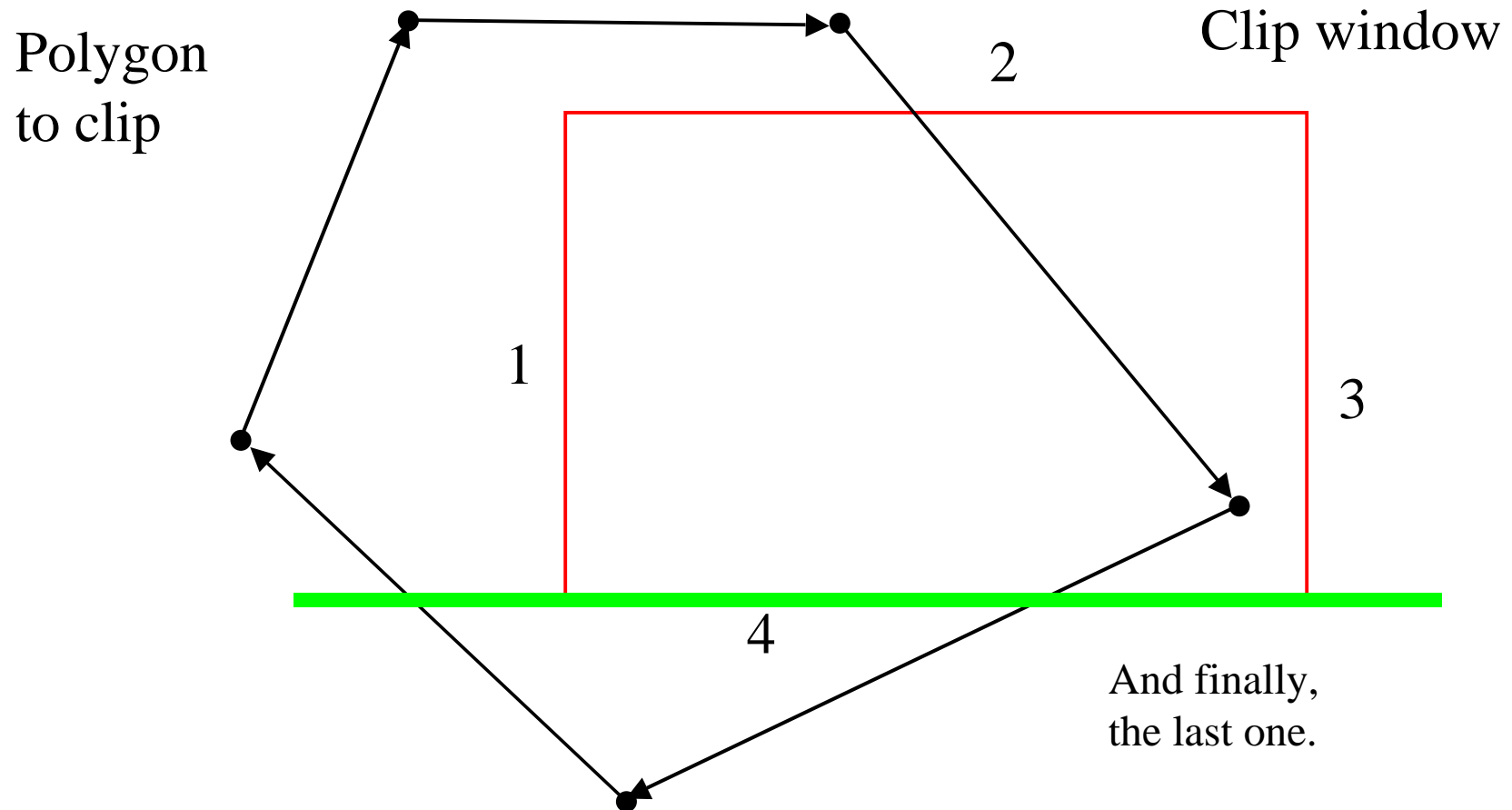
Sutherland-Hodgeman polygon clip



Sutherland-Hodgeman polygon clip

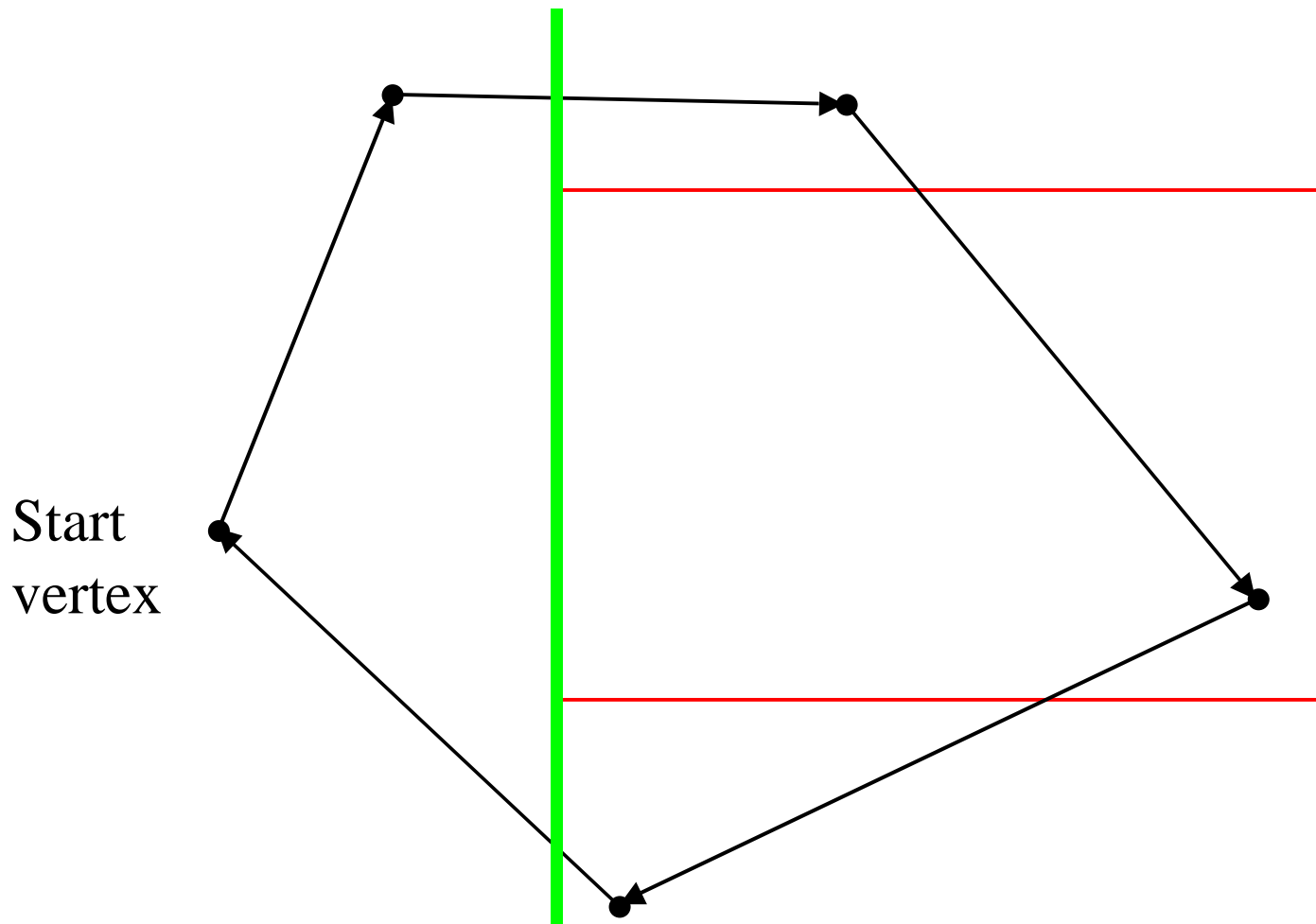


Sutherland-Hodgeman polygon clip



Clipping against current clip edge

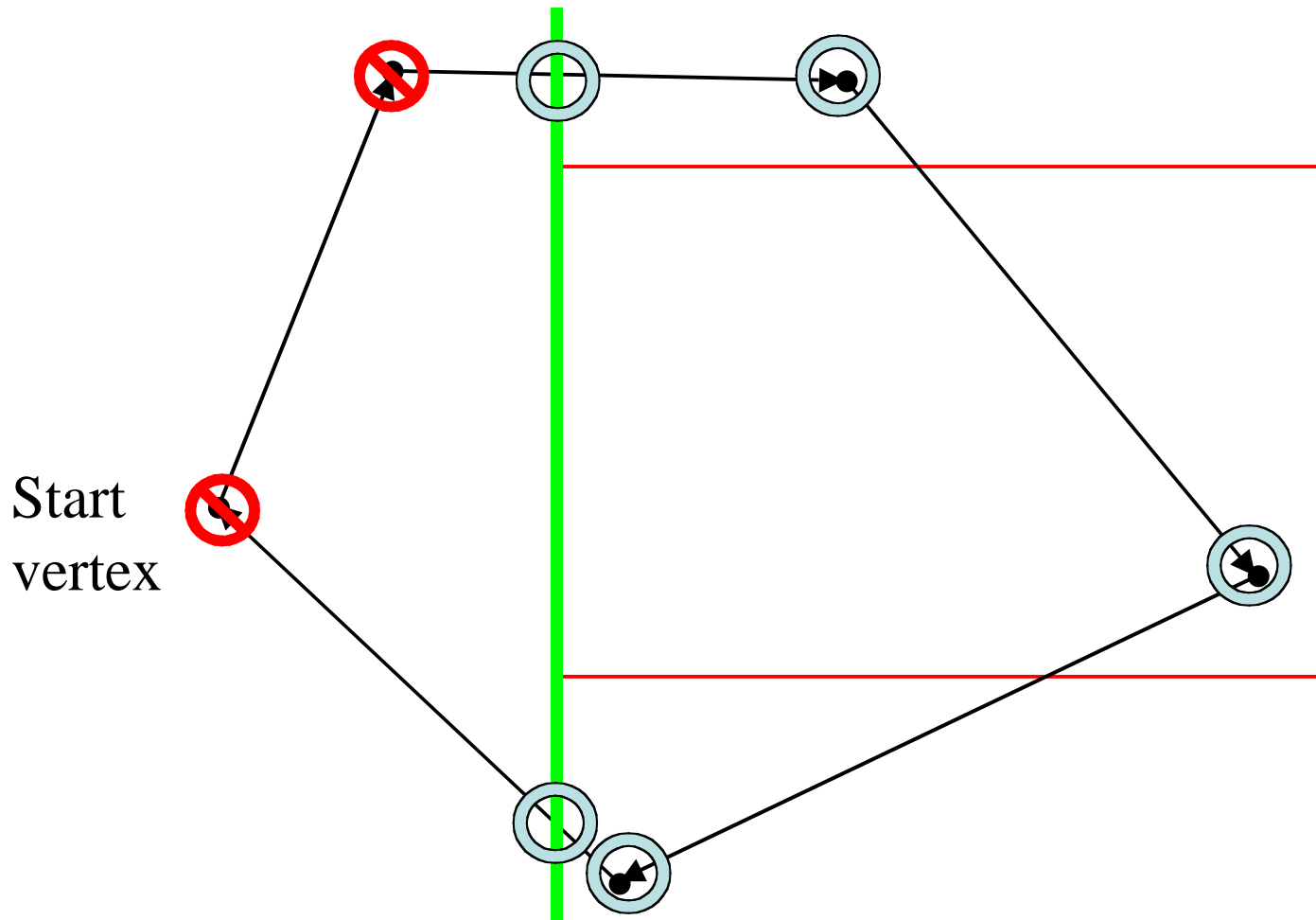
- Polygon is a list of vertices
 - Think of process as rewriting polygon, vertex by vertex
 - Check start vertex
 - in - emit it
 - out - ignore it
 - Walk along vertices and for each edge consider four cases and apply corresponding action.
- Four cases:
 - polygon edge crosses clip edge going from out to in
 - emit crossing, next vertex
 - polygon edge crosses clip edge going from in to out
 - emit crossing
 - polygon edge goes from out to out
 - emit nothing
 - polygon edge goes from in to in
 - emit next vertex



polygon edge crosses clip edge going from out to in \implies emit crossing,
next vertex

polygon edge crosses clip edge going from in to out \implies emit crossing

polygon edge goes from out to out \implies emit nothing

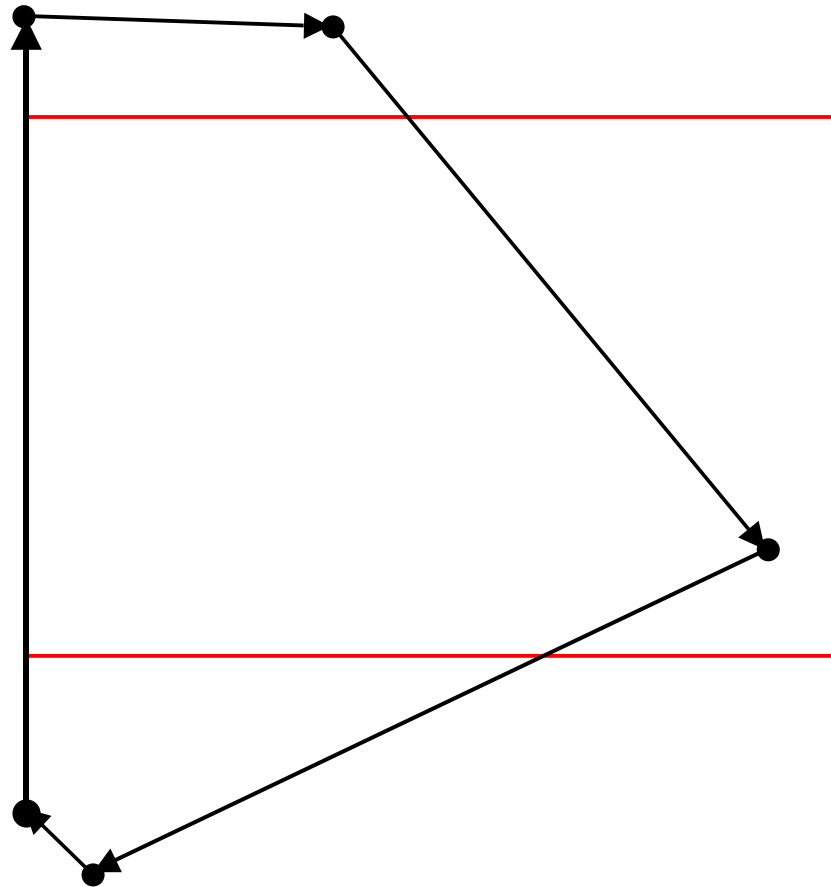


polygon edge crosses clip edge going from out to in \implies emit crossing,
 next vertex

polygon edge crosses clip edge going from in to out \implies emit crossing

polygon edge goes from out to out \implies emit nothing

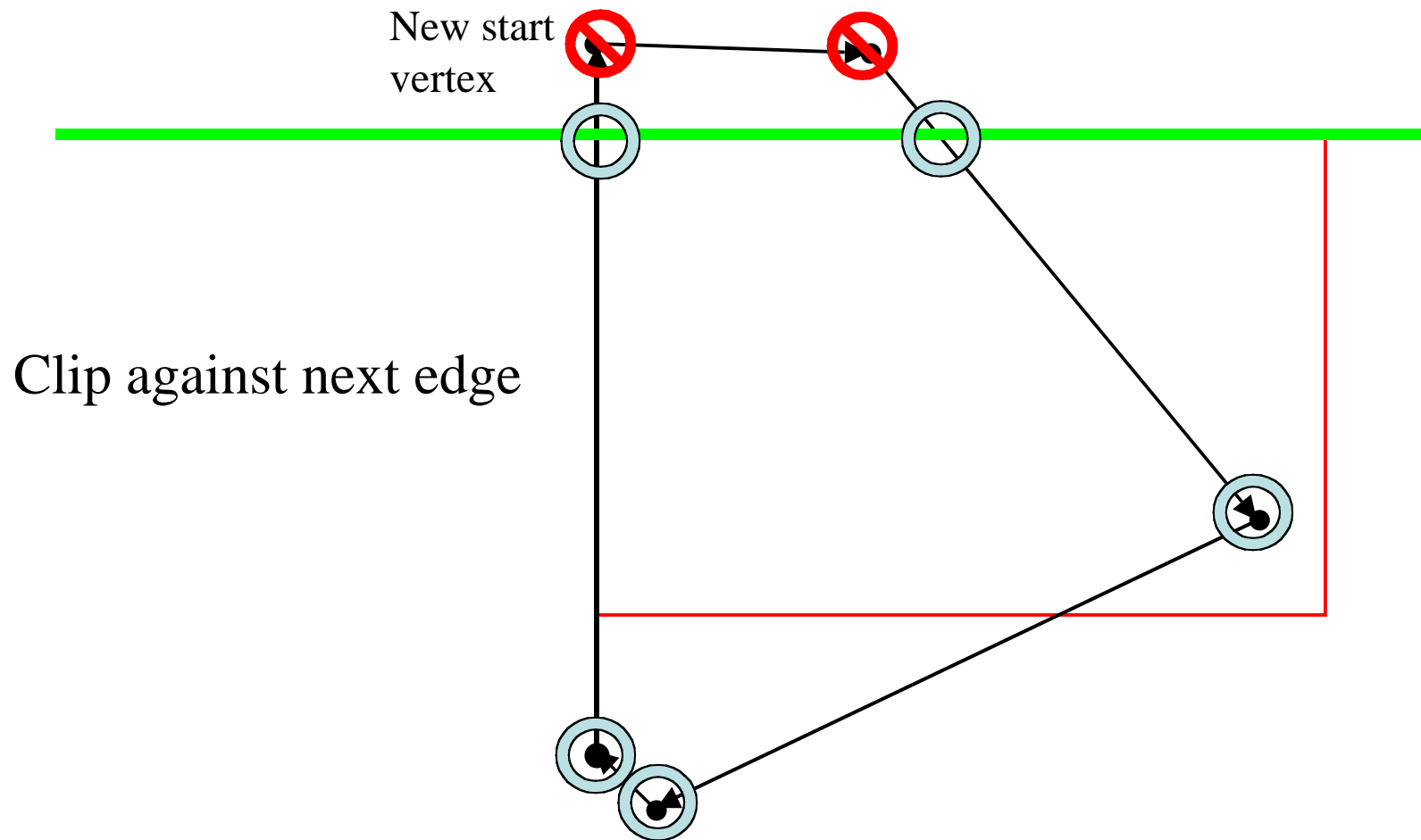
Now have



polygon edge crosses clip edge going from out to in \implies emit crossing,
next vertex

polygon edge crosses clip edge going from in to out \implies emit crossing

polygon edge goes from out to out \implies emit nothing

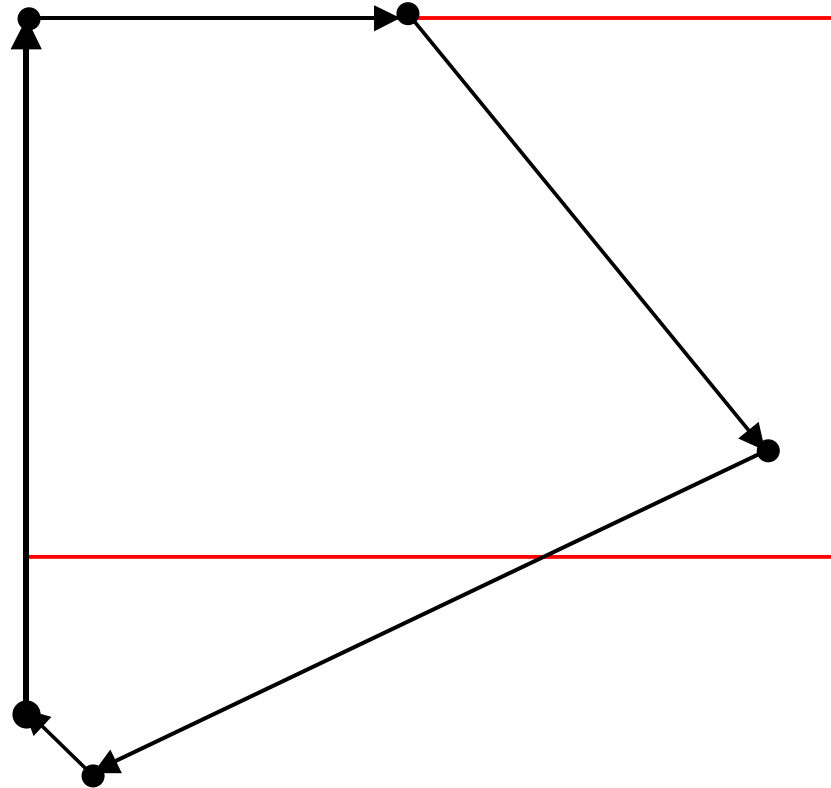


polygon edge crosses clip edge going from out to in \implies emit crossing,
 next vertex

polygon edge crosses clip edge going from in to out \implies emit crossing

polygon edge goes from out to out \implies emit nothing

Now have

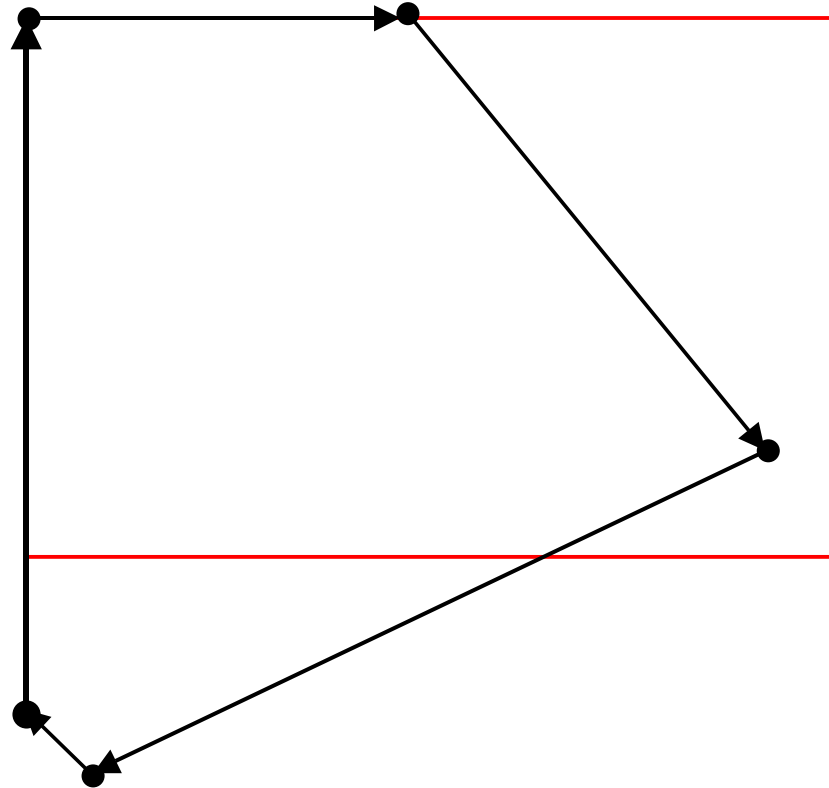


polygon edge crosses clip edge going from out to in \implies emit crossing,
next vertex

polygon edge crosses clip edge going from in to out \implies emit crossing

polygon edge goes from out to out \implies emit nothing

Clipping against
next edge (right)
gives

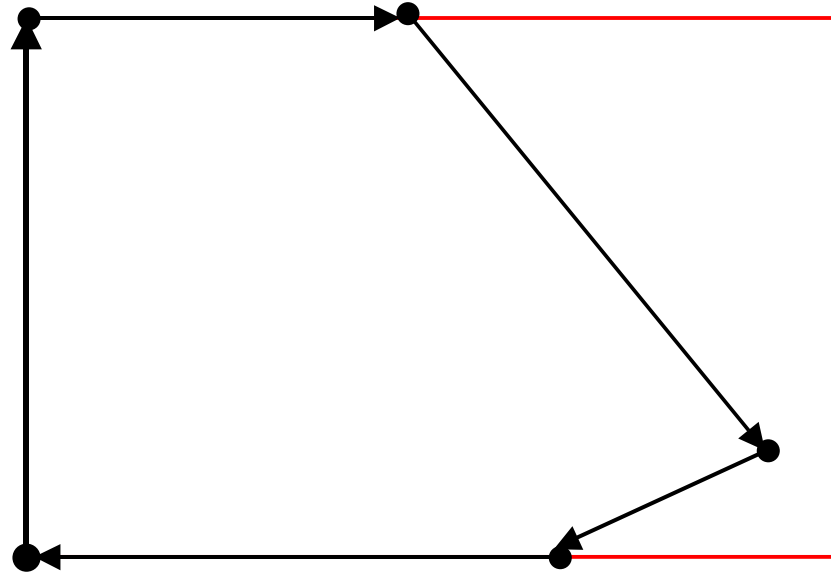


polygon edge crosses clip edge going from out to in \Rightarrow emit crossing,
next vertex

polygon edge crosses clip edge going from in to out \Rightarrow emit crossing

polygon edge goes from out to out \Rightarrow emit nothing

Clipping against
final(bottom)
edge gives



polygon edge crosses clip edge going from out to in ==> emit crossing,
next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

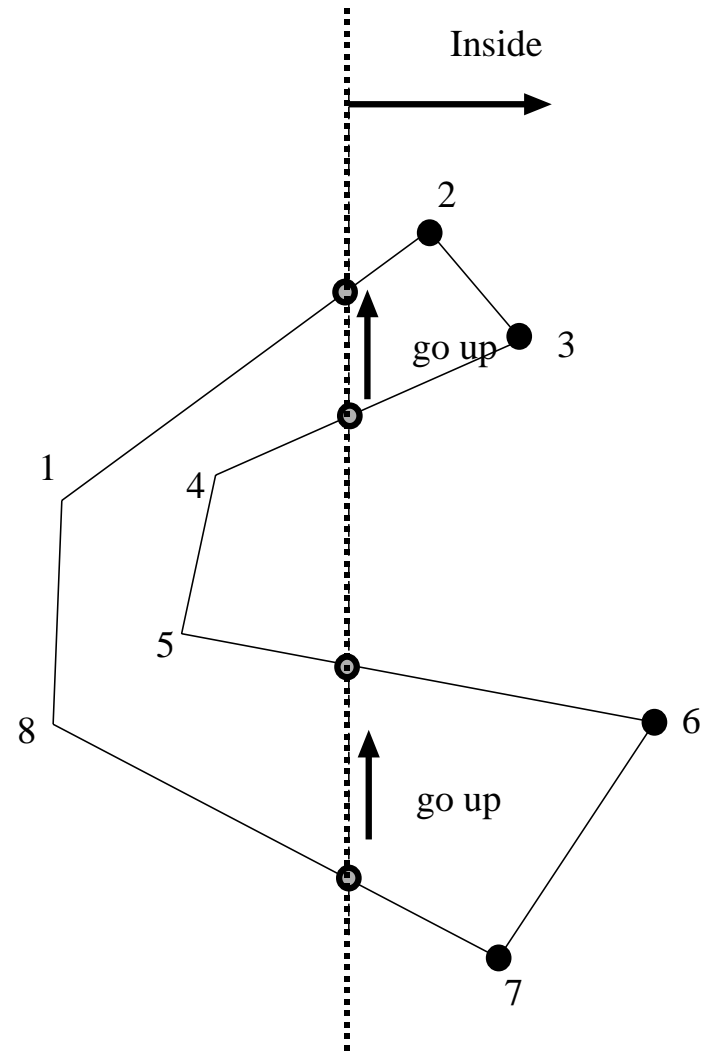
More Polygon clipping

- Notice that we can have a pipeline of clipping processes, one against each edge, each operating on the output of the previous clipper -- substantial advantage.
- Unpleasantness can result from concave polygons; in particular, polygons with empty interior.
- Options:
 - modify algorithm (Weiler Atherton)
 - write polygon fill carefully; but note extra edges, etc.

Weiler Atherton

for clockwise polygon:

- for out-to-in pair, follow usual rule
- for in-to-out pair, follow clip edge
- then jump to next vertex (which is on the outside) and start again
- only get a second piece if polygon is convex
- easiest to start outside



Additional remarks on clipping

- Clipping polygon, line against concave clip region is significantly harder. In both cases, efficient algorithms are known.
- Although everything is in terms of polygons/lines clipped against lines in 2D, all - except Nicholl-Lee-Nicholl - will work in 3D against convex regions.
- This is because the central issue in each algorithm is the inside outside decision.
- Inside-outside decisions can be made for lines in 2D, planes in 3D. e.g testing $x \geq 0$
- Hence, all (except N-L-N) can be used to clip:
 - lines against 3D convex regions (e.g. cubes) (CS, LB)
 - polygons against 3D convex regions (e.g. cubes) (SH, WA)
- NLN could work in 3D, but the number of cases increases too much to be practical.

2D Transformations (§5.2)

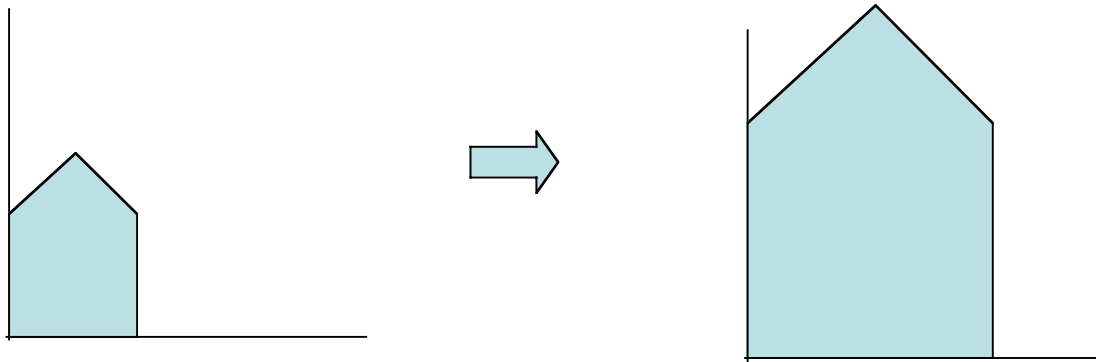
Have a look at §5.1--it reviews math that we will be using throughout the course.

2D Transformations

- Represent transformations by matrices
- To transform a point, represented by a vector, multiply the vector by the appropriate matrix.
- To transform lines, transform endpoints
- To transform polygons, transform vertices

2D Transformations

- Scale (stretch) by a factor of k

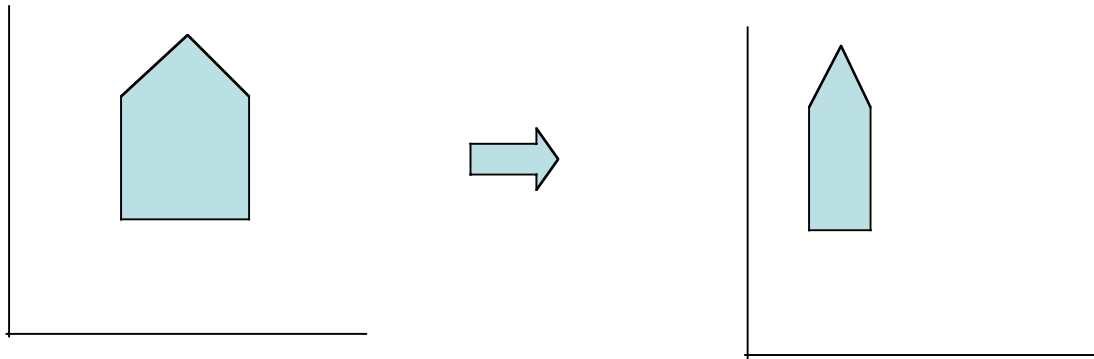


$$M = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

($k = 2$ in the example)

2D Transformations

- Scale by a factor of (S_x, S_y)

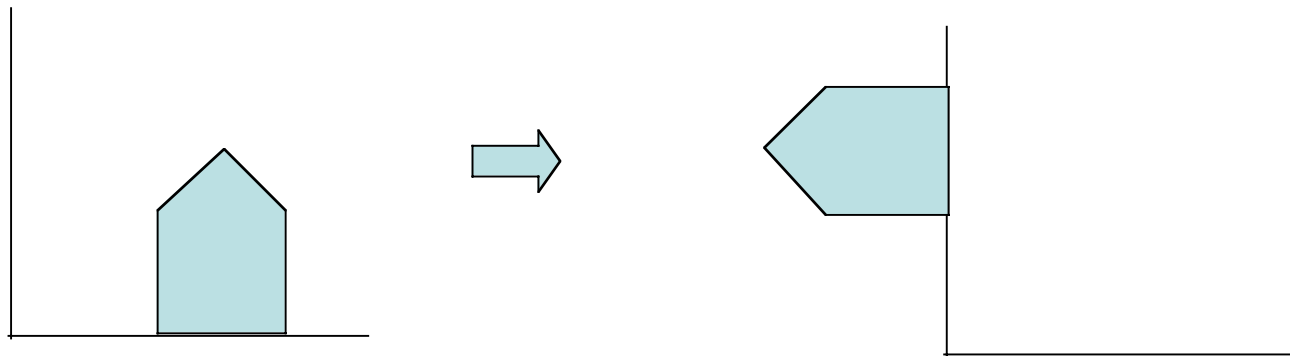


$$M = \begin{vmatrix} S_x & 0 \\ 0 & S_y \end{vmatrix}$$

(Above, $S_x = 1/2$, $S_y = 1$)

2D Transformations

- Rotate around origin by θ (Orthogonal)

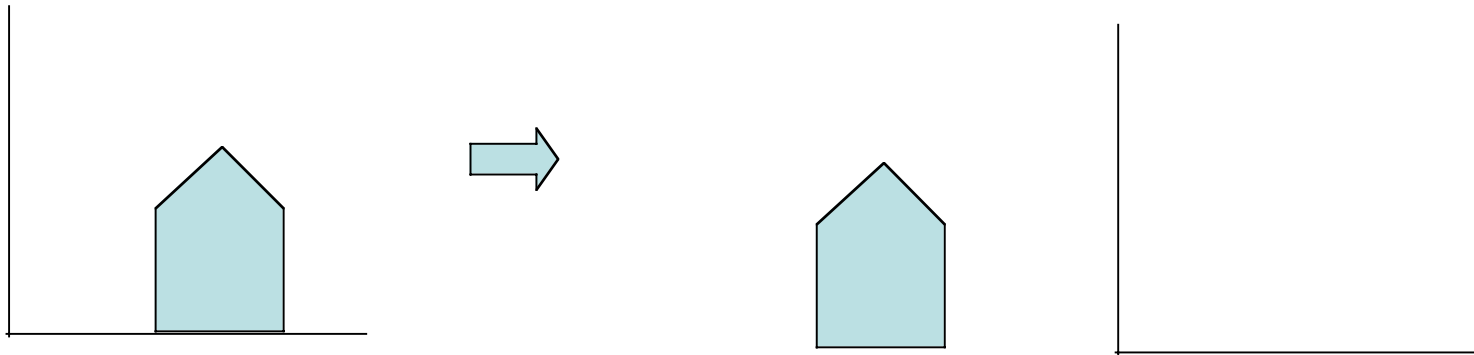


$$M = \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

(Above, $\theta=90^\circ$)

2D Transformations

- Flip over y axis (Orthogonal)

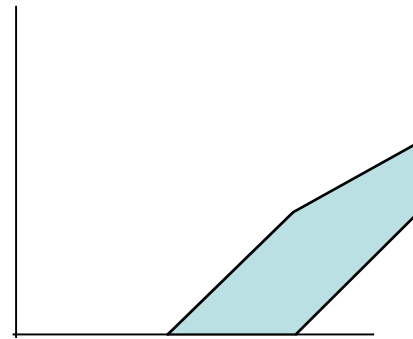
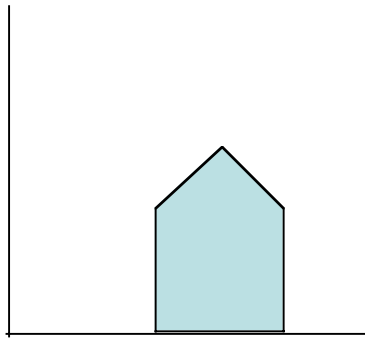


$$M = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

Flip over x axis is ?

2D Transformations

- Shear along x axis

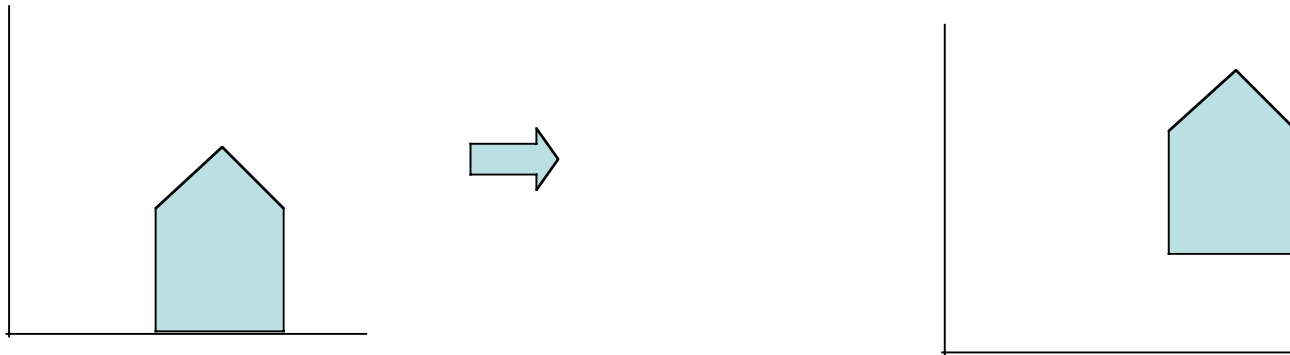


$$M = \begin{vmatrix} 1 & a \\ 0 & 1 \end{vmatrix}$$

Shear along y axis is ?

2D Transformations

- Translation ($\mathbf{P}_{\text{new}} = \mathbf{P} + \mathbf{T}$)



$\mathbf{M} = ?$

Homogenous Coordinates (§5.3)

- Represent 2D points by 3D vectors
- $(x,y) \rightarrow (x,y,1)$
- Now a multitude of 3D points (x,y,W) represent the same 2D point, $(x/W, y/W, 1)$
- Represent 2D transforms with 3 by 3 matrices
- Can now do translations
- Homogenous coordinates have other uses/advantages (later)

2D Translation in H.C.

$$\mathbf{P}_{\text{new}} = \mathbf{P} + \mathbf{T}$$

$$(x', y') = (x, y) + (t_x, t_y)$$

$$\mathbf{M} = \left| \begin{array}{ccc} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{array} \right|$$

2D Scale in H.C.

$$\mathbf{M} = \begin{vmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

2D Rotation in H.C.

$$M = \begin{vmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Composition of Transformations

(§5.4)

- If we use one matrix, M_1 for one transform and another matrix, M_2 for a second transform, then the matrix for the first transform followed by the second transform is simply $M_2 M_1$
- This generalizes to any number of transforms
- Computing the combined matrix first can save lots of computation

Composition Example

- Matrix for rotation about a point, P
- Problem--we only know how to rotate about the origin.
- Solution--translate to origin, rotate, and translate back (see §5.4)