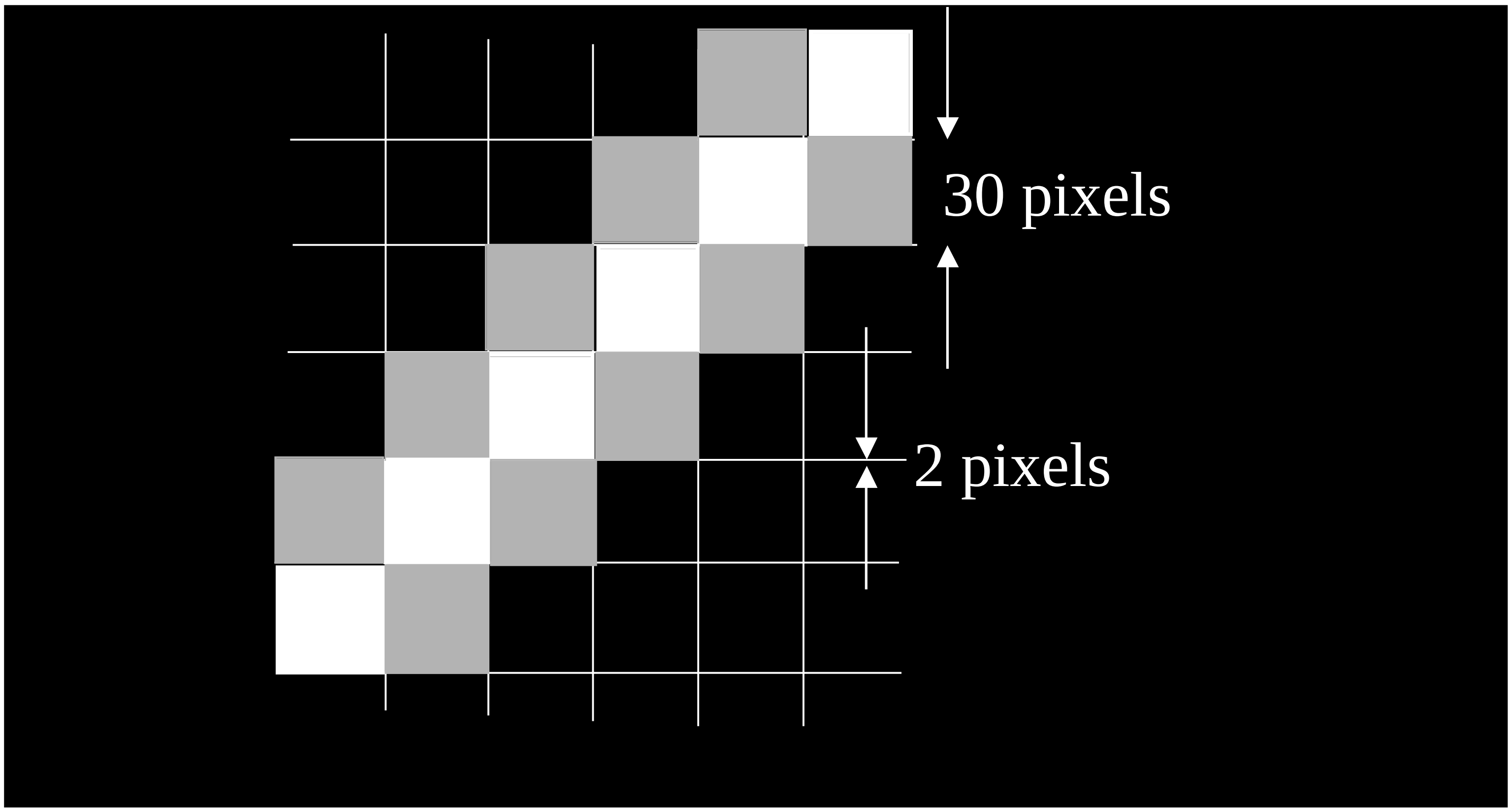
Administrative

Assignment Example

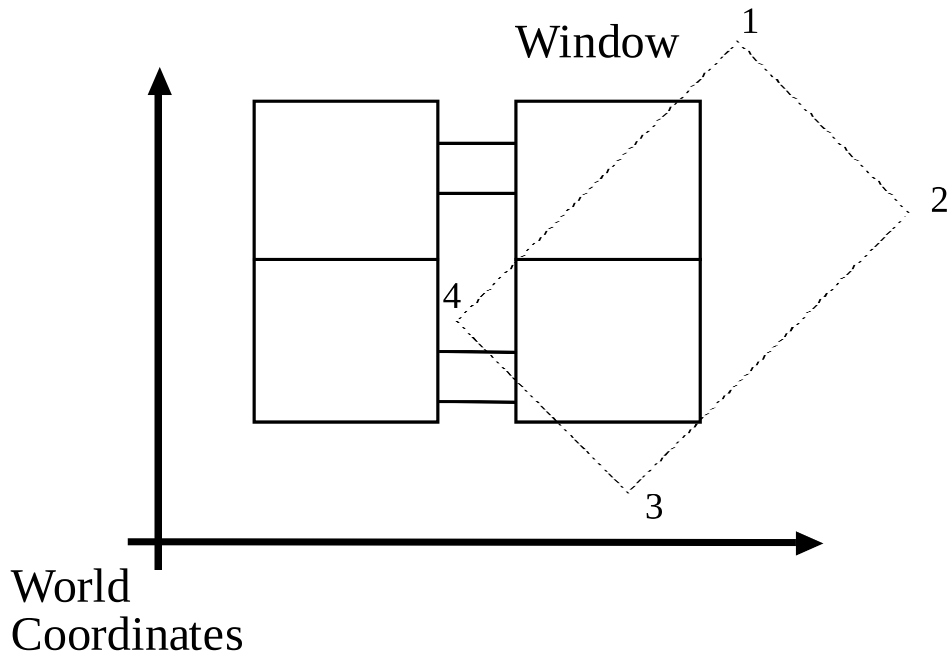


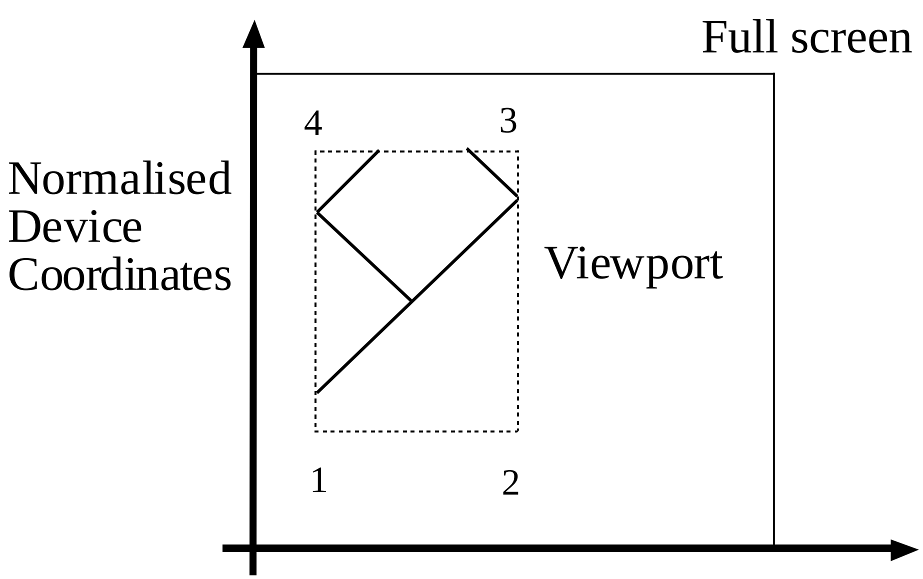
2D transformations (continued)

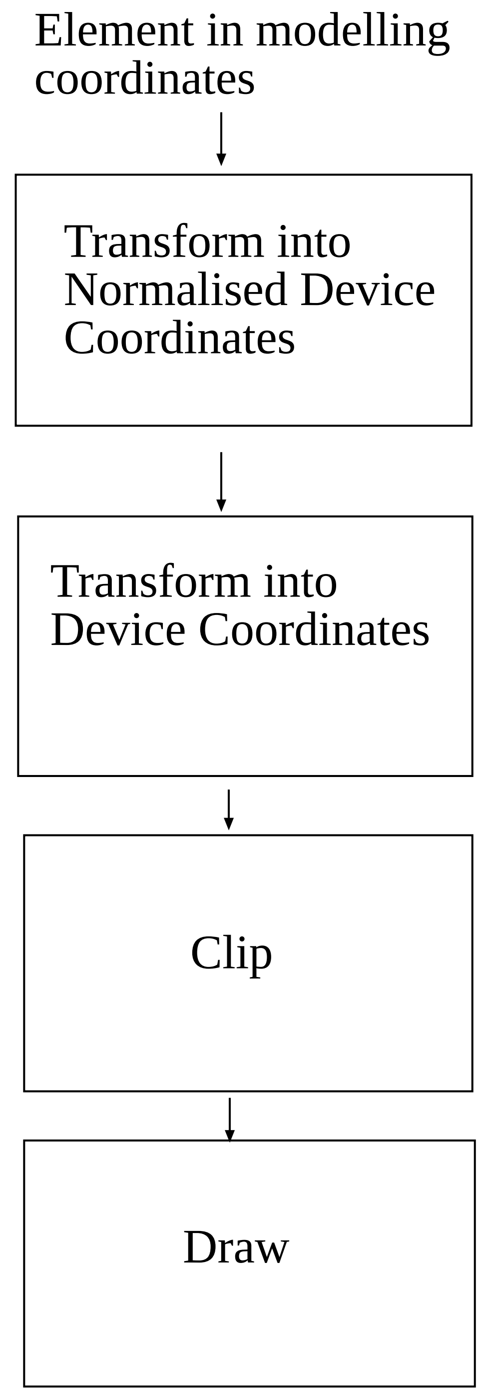
- The transformations discussed so far are invertable (why?). What are the inverses?

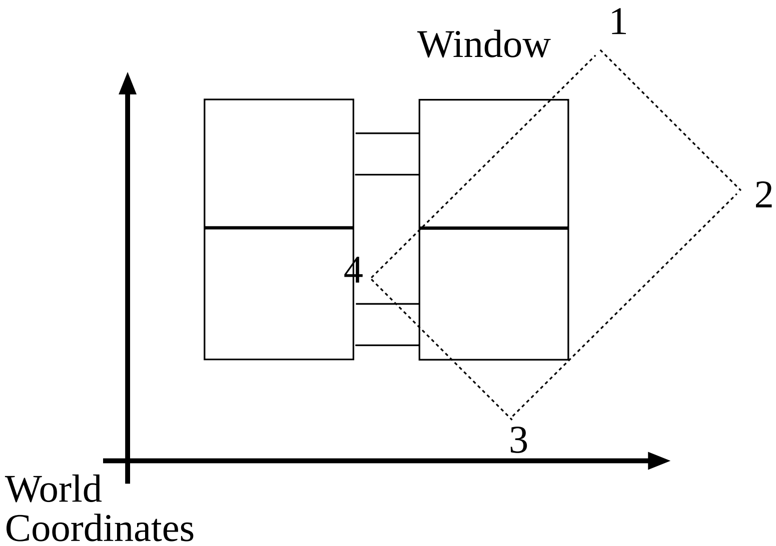
2D viewing (§5.5 and more)

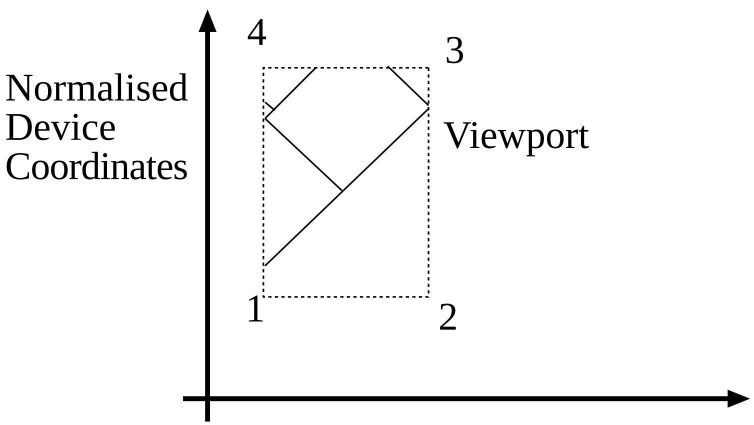
- 3 Significant coordinate systems are usual
  - World coordinates or modeling coordinates - where the model is defined (meters, miles, etc.)
  - Normalized device coordinates; usually (0-1) in each variable.
  - Device coordinates: the actual coordinates of the pixels on the frame-buffer or the printer
- Main issue: constructing transformations between coordinate systems
- Terminology:
  - window = region on drawing that will be displayed (rectangle)
  - viewport = region in NDC's/DC's where this rectangle is displayed (often simply entire screen).


Window
1
2
3
4
World
Coordinates

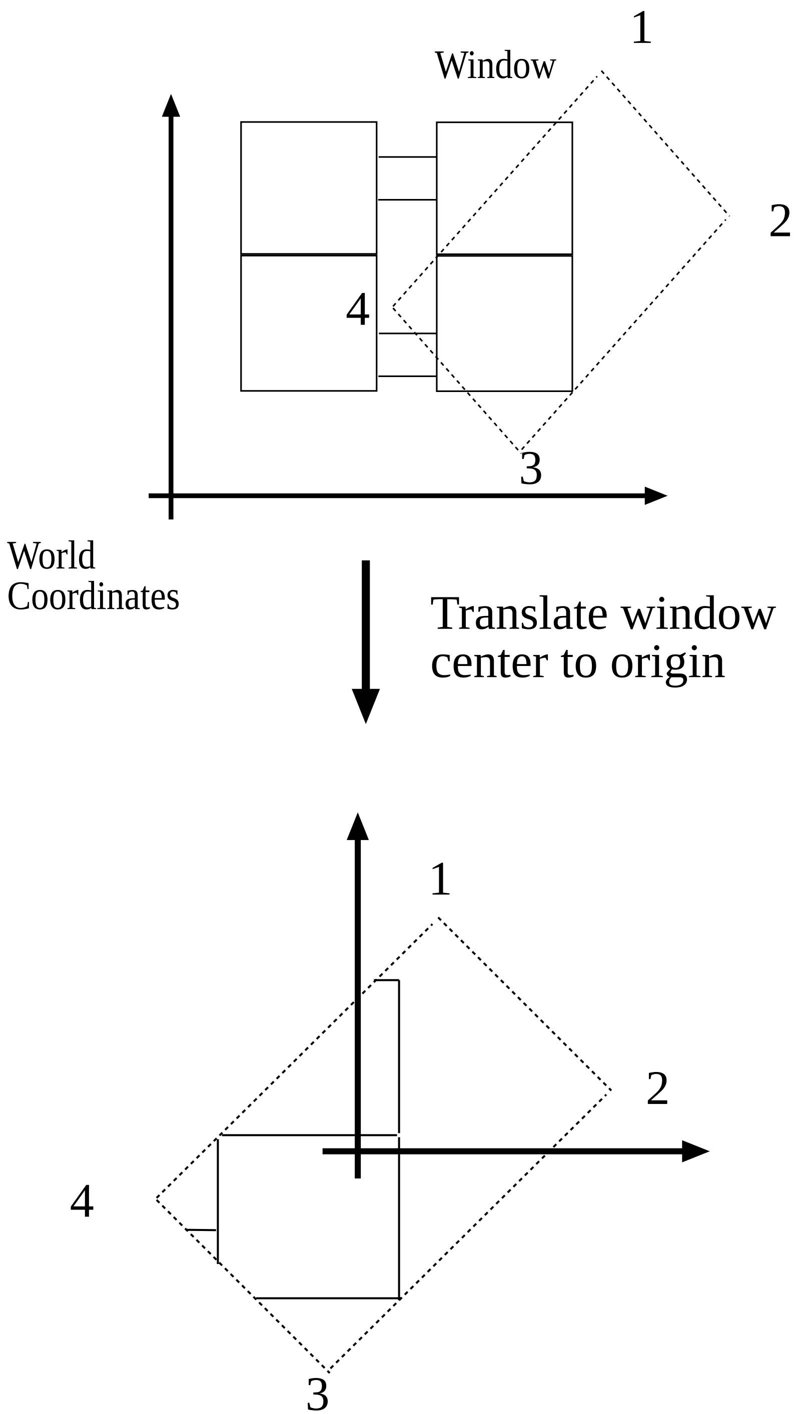

Full screen
4
3
Normalised
Device
Coordinates
Viewport
1
2


Element in modelling
coordinates
Transform into
Normalised Device
Coordinates
Transform into
Device Coordinates
Clip
Draw





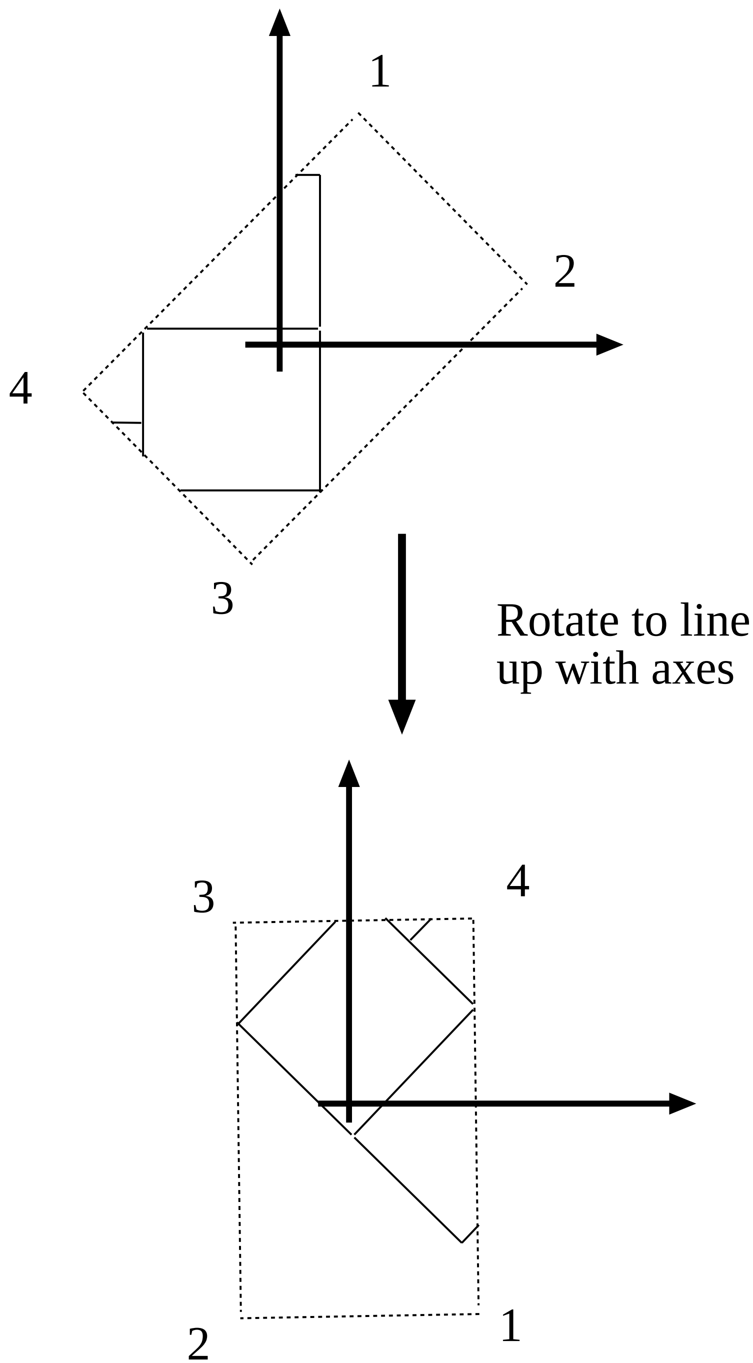
- view this as a sequence of transformations in h.c.'s, then determine each element in closed form.
- compute numerically from point correspondences.



- write (wx_i, wy_i) for coordinates of i’th point on window
- translation is:

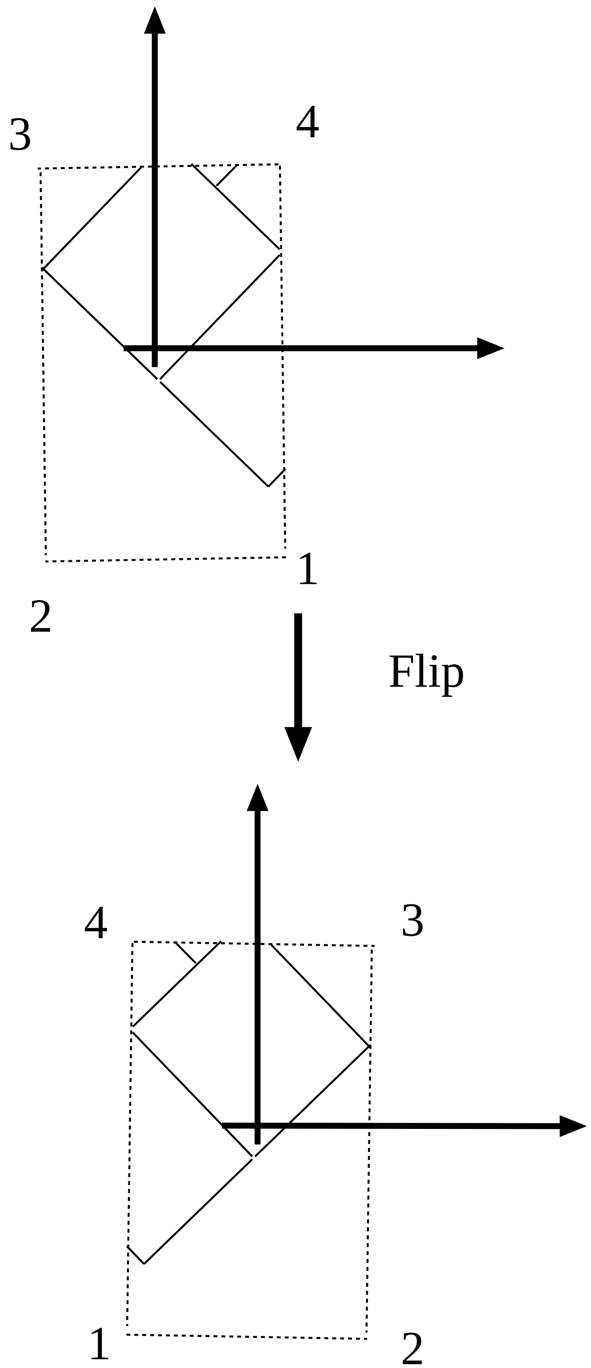
$$\begin{pmatrix}x'\\ y'\\ 1\end{pmatrix} = \begin{pmatrix}1 & 0 & -\overline{wx}\\ 0 & 1 & -\overline{wy}\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ 1\end{pmatrix}$$

(overbar denotes average over vertices, i.e., 1,2,3,4)



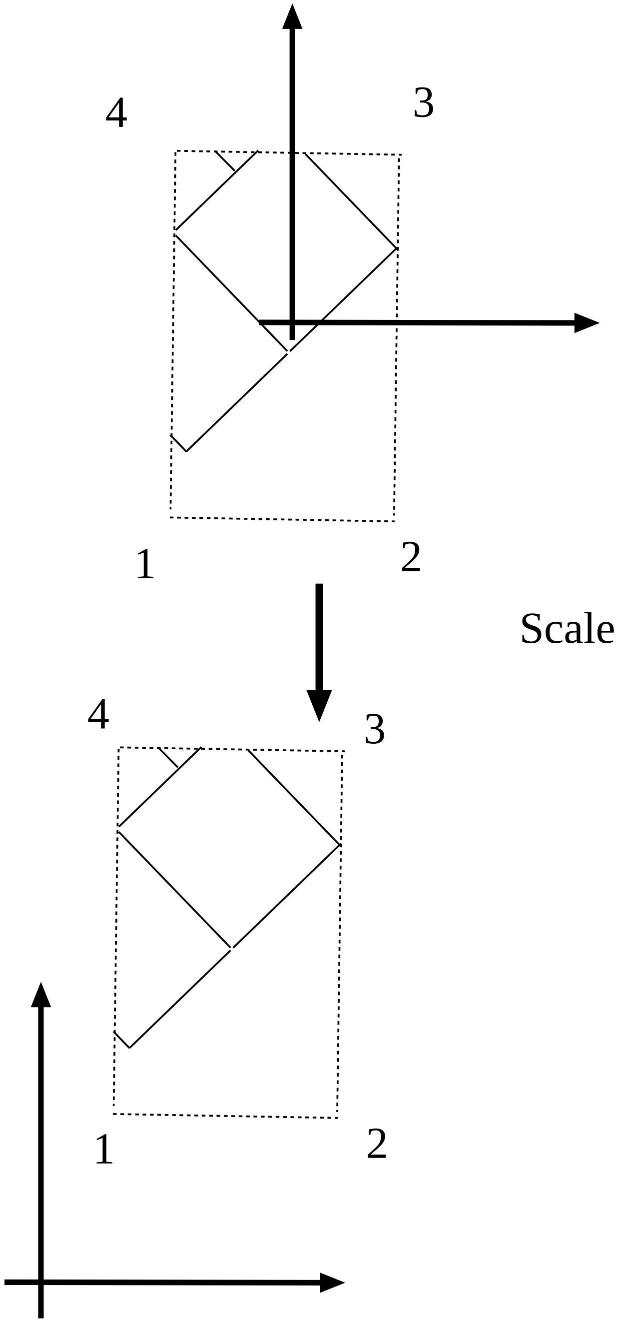
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

(Need to compute theta)



$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

(Vertex order does not correspond, need to flip)



Scale and translate

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{w_{new}}{w_{old}} & 0 & \overline{x_{new}} \\ 0 & \frac{h_{new}}{h_{old}} & \overline{y_{new}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Notice that choice of new width, height, and center give translation to either normalized device coords, or to device coordinates

- Get overall transformation by multiplying transforms.
- This gives a single transformation matrix, whose elements are functions of window/viewport coordinates.
- Notice notational advantage of homogeneous coordinates - no extra vectors hanging around.

$$x' = M_{(\text{translate origin to viewport cog, scale})}\, M_{(\text{flip})}\, M_{(\text{rotate})} M_{(\text{translate window cog->origin})} x$$

NDC's/DC's

World coords

(cog==window center of gravity)

Affine transformations

- Another approach to determining the whole transform for the pipeline; this is an affine transform.
- Matrix form:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix}$$

- Now assume that I know that $Mp_1=q_1$, $Mp_2=q_2$, $Mp_3=q_3$
- Quick way to determine transform, because this is the same as six linear equations, in six variables, which are the entries in the matrix:

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix}$$

Details

- $Mp_1=q_1$ gives first two rows
- $p_1 = (x_1, y_1, 1)^T$, $q_1 = (u_1, v_1, 1)^T$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$

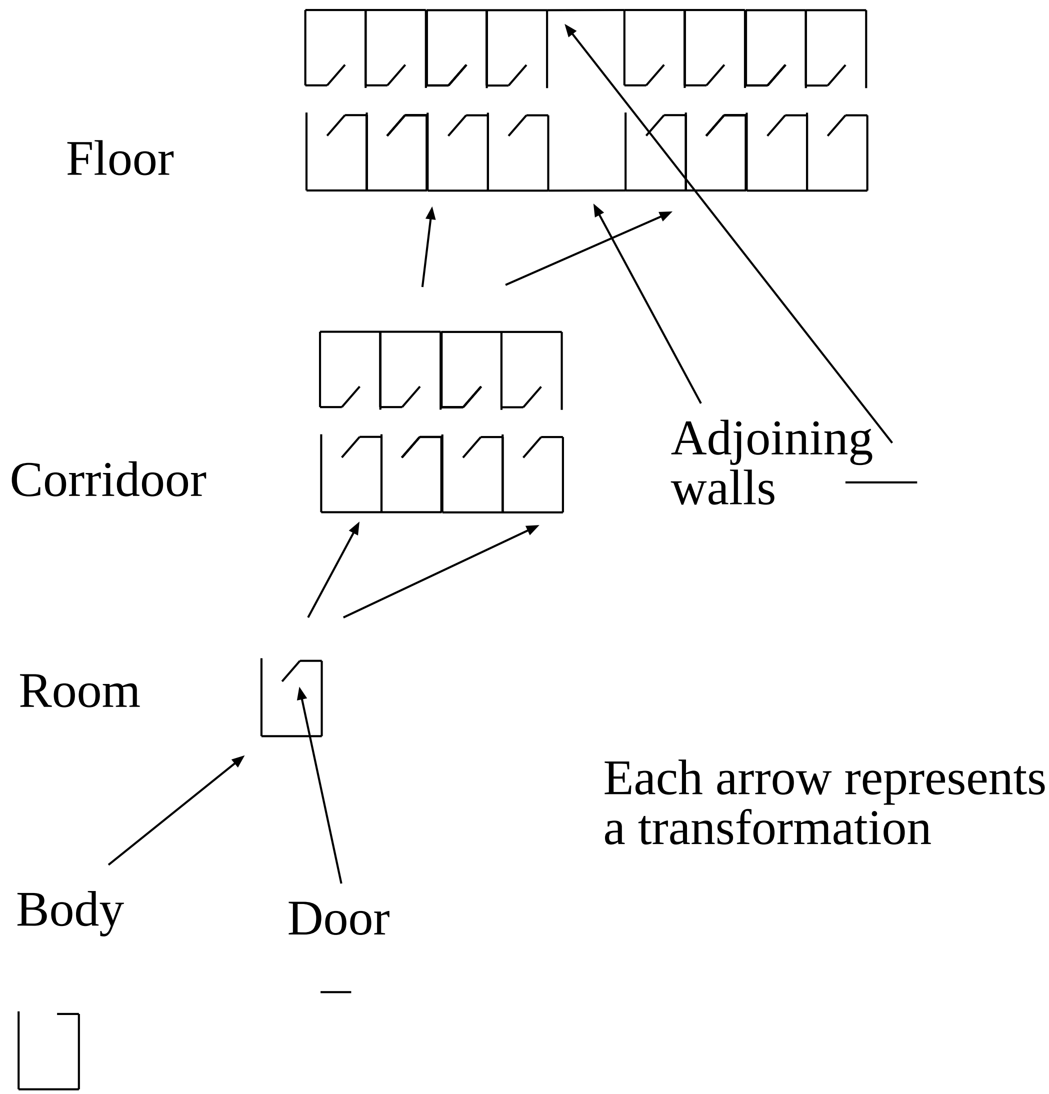
$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix}$$

$$\begin{aligned} ax_1 + by_1 + c &= u_1 \\ dx_1 + ey_1 + f &= v_1 \end{aligned}$$

$Mp_2=q_2$, $Mp_3=q_3$ give other rows

Hierarchical modeling

- Consider constructing a complex 2d drawing: e.g. an animation showing the plan view of a building, where the doors swing open and shut.
- Options:
  - specify everything in world coordinate frame; but then each room is different, and each door moves differently. (hugely difficult).
  - Exploit similarities by using repeated copies of models in different places (instancing)



Hierarchical modeling

- Model form
  - Directed acyclic graph.
  - Each node consists of 0 or more objects (lines, polygons, etc).
  - Each edge is a transformation
- Notice there can be many edges joining two nodes (e.g. in the case of the corridor - many copies of the same room model, each transformed differently).
- Every graphics API supports hierarchies - some directly (meaning you have to learn a language to express the model) some indirectly with a matrix stack

- Write the transformation from door coordinates to room coordinates as:

$$T_{room}^{door}$$

Then to render a door, use the transformation:

$$T_{device}^{world} T_{floor}^{corridoor} T_{corridoor}^{room} T_{room}^{door}$$

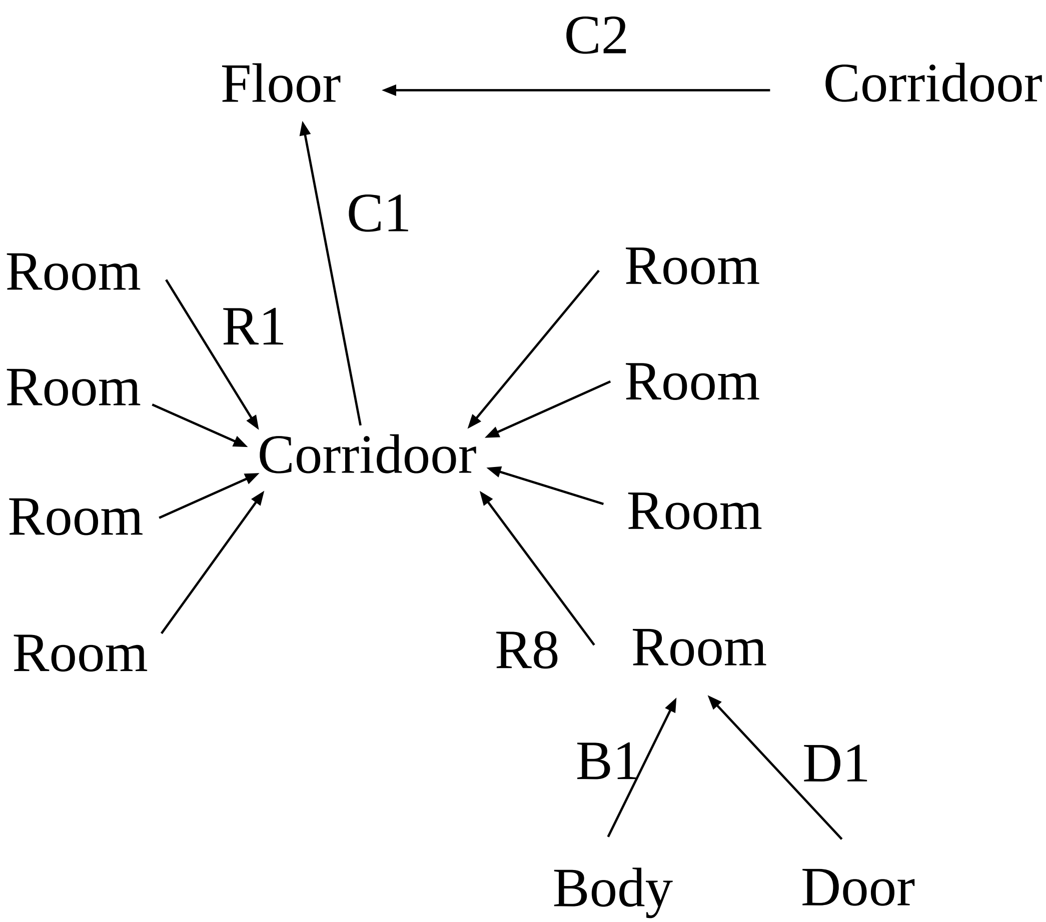
To render a body, use the transformation:

$$T_{device}^{world} T_{floor}^{corridoor} T_{corridoor}^{room} T_{room}^{body}$$

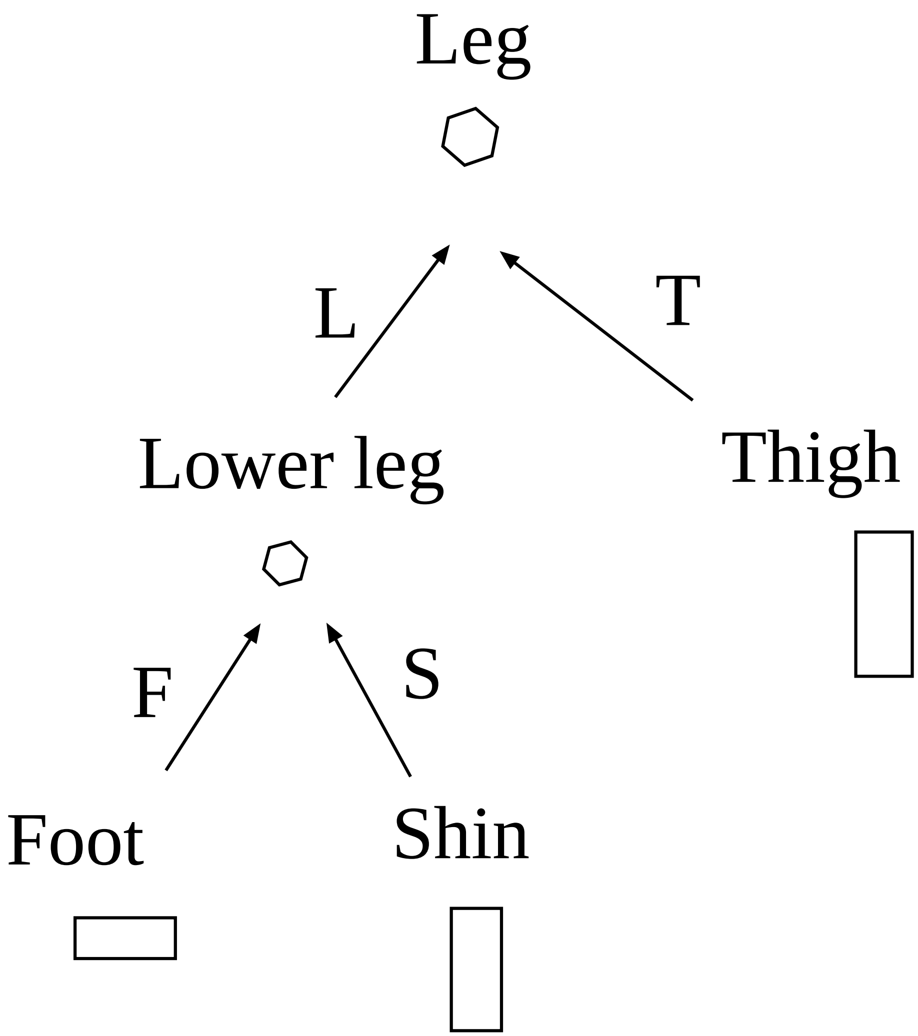
Matrix stacks and rendering

- Matrix stack:
  - stack of matrices used for rendering
  - applied in sequence.
  - Pop=remove last matrix
  - Push=append a new matrix
  - in previous example, body-device transformation comes from door-device transformation by popping door-room and pushing body-room

- Algorithm for rendering a hierarchical model:
  - stack is ________ T_{device}^{root}
  - render (root)
- Render (node)
  - for each child:
    - push transformation
    - render (child)
    - pop transformation



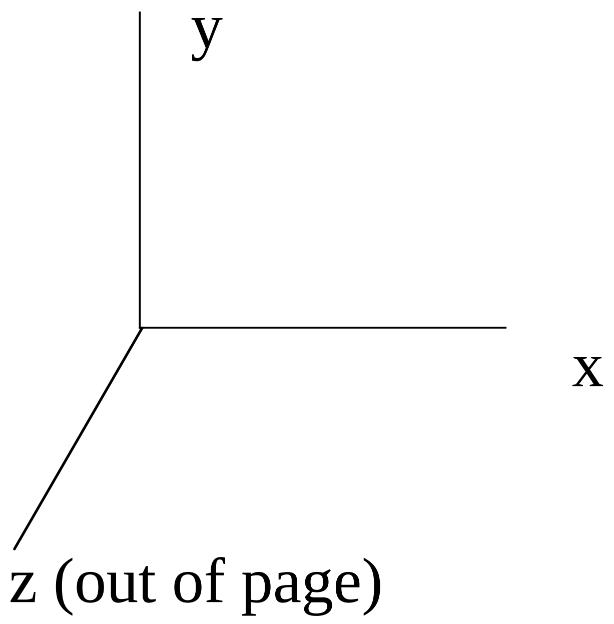
- Now to render door on first room in first corridor, stack looks like: W C1 R1 D1
- Note that we do not need two copies of corridor, or 16 copies of body; we render one copy using 16 different transformations. This is known as instancing
- Animation requires care: if D1 is a single function of time, all doors will swing open and closed at the same time.

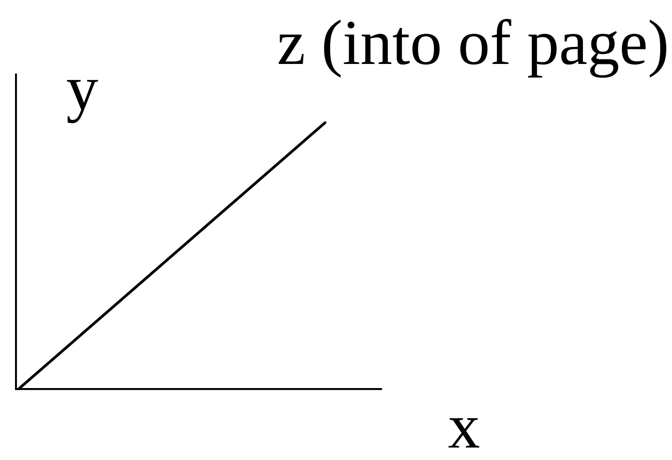


- Stack is W
- render kneecap
- Stack is W L
- render ankle
- Stack is W L F
- render foot
- Stack is W L S
- render shin
- Stack is W T
- render thigh

Transformations in 3D

- Right hand coordinate system (conventional, i.e., in math)
- In graphics a LHS is sometimes also convenient (Easy to switch between them--later).





Transformations in 3D

- Homogeneous coordinates now have four components - traditionally, (x, y, z, w)
  - ordinary to homogeneous: (x, y, z) -> (x, y, z, 1)
  - homogeneous to ordinary: (x, y, z, w) -> (x/w, y/w, z/w)
- Again, translation can be expressed as a multiplication.

Transformations in 3D

- Translation:

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D transformations

- Anisotropic scaling:

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Shear (one example):

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & a & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotations in 3D

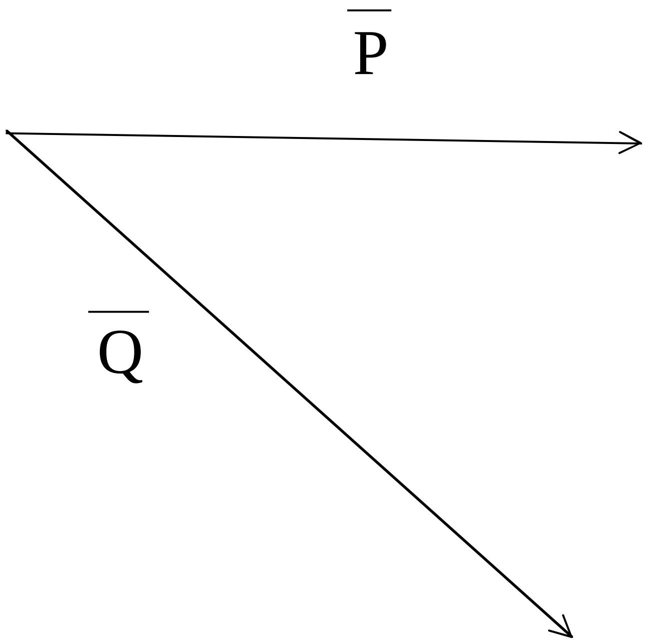
- 3 degrees of freedom
- Det(R)=1
- Orthogonal
- Many representations are possible.
- Our representation: rotate about coordinate axes in sequence.
- Sequence of axes is arbitrary, but choice does affect the angles used (cannot use same angles with different order).
- Convention: look down coordinate axis (towards origin), anticlockwise rotation is positive angle.
- Likely easier way to remember is the following Right Hand Rule--point thumb along axis in direction of increasing ordinate--then fingers curl in the direction of positive rotation).

Rotations in 3D

- About z-axis

$$M = \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Finding a Normal Vector



A vector normal to the plane of P and Q is the cross (vector) product of P and Q. No 2D analog.

Direction is into the page. (Right hand rule--if P is thumb, Q is index finger, the P x Q is middle finger.

Formula: (x_1,y_1,z_1) X (x_2,y_2,z_2) =
$(y_1z_2\text{-}z_1y2,\ z_1x_2\text{-}z_2x_1,\ x_1y_2\text{-}y_1x_2)$