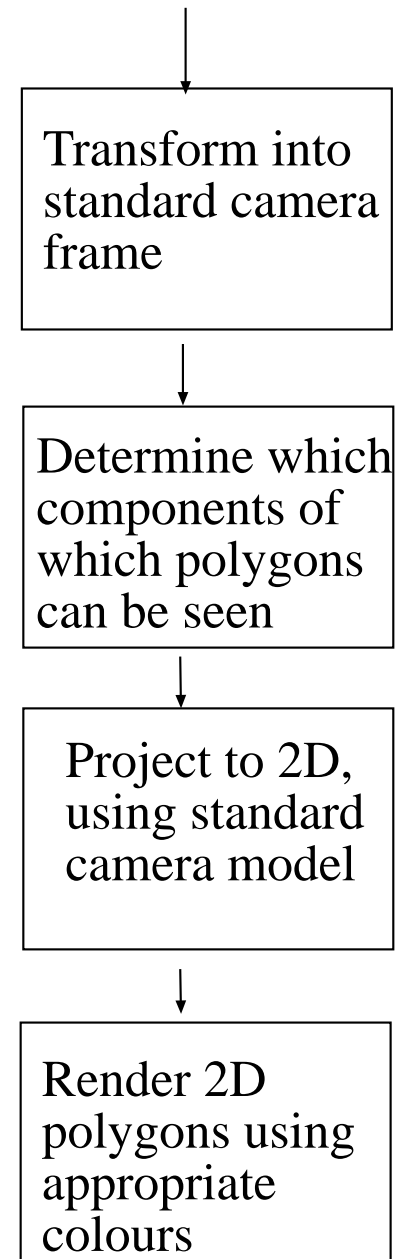
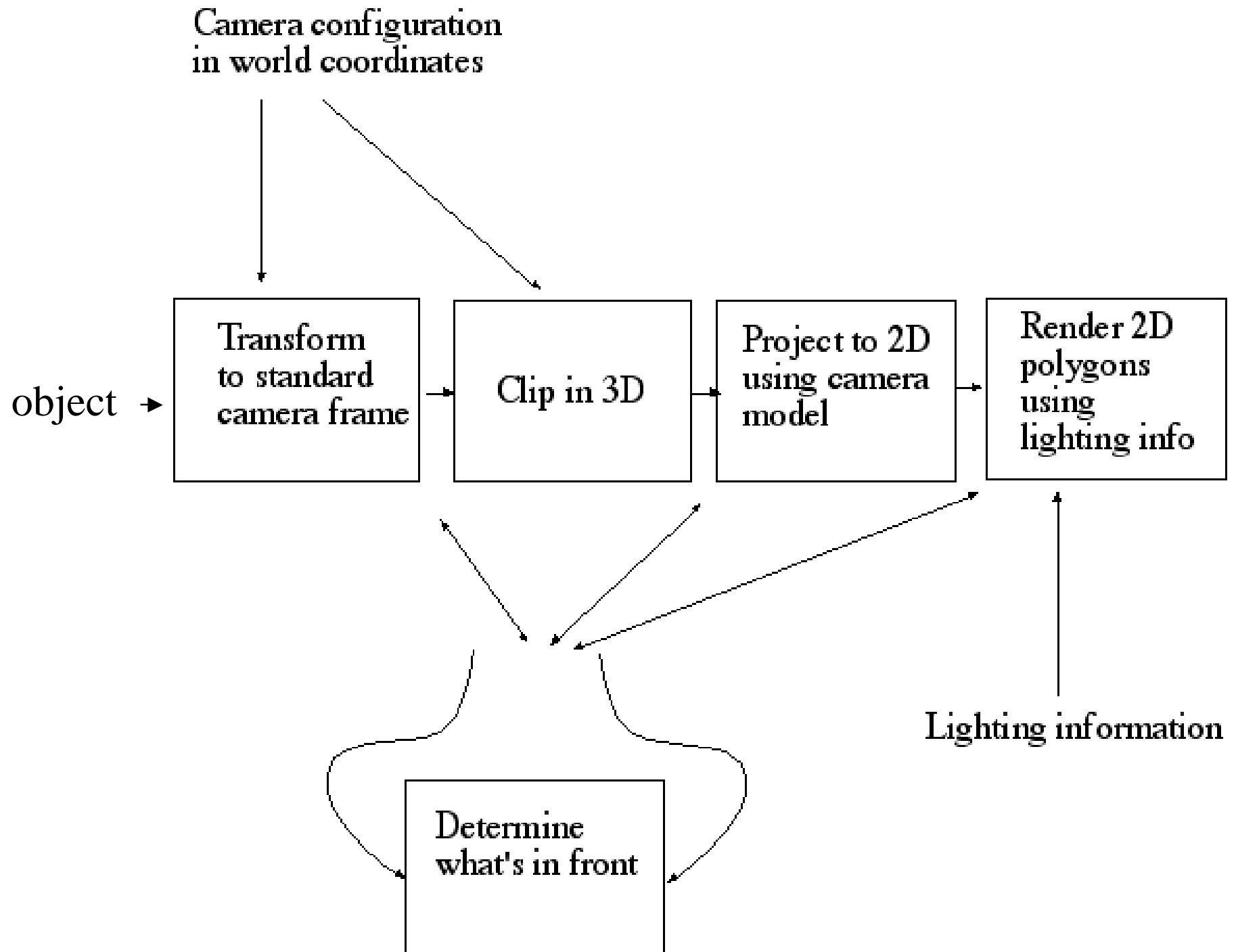


# 3D Graphics Concepts

- Modeling: For now, objects will be collections of polygons in 3D. Complex shapes will be many small polygons.
- Issues:
  - Which polygons can be seen? (some polygons hide others, and some are outside the relevant volume of space and need to be clipped).
  - Where do they go in the 2D image? (key abstraction is a virtual camera)
  - How bright should they be? (for example, to make it look as if we are looking at a real surface)

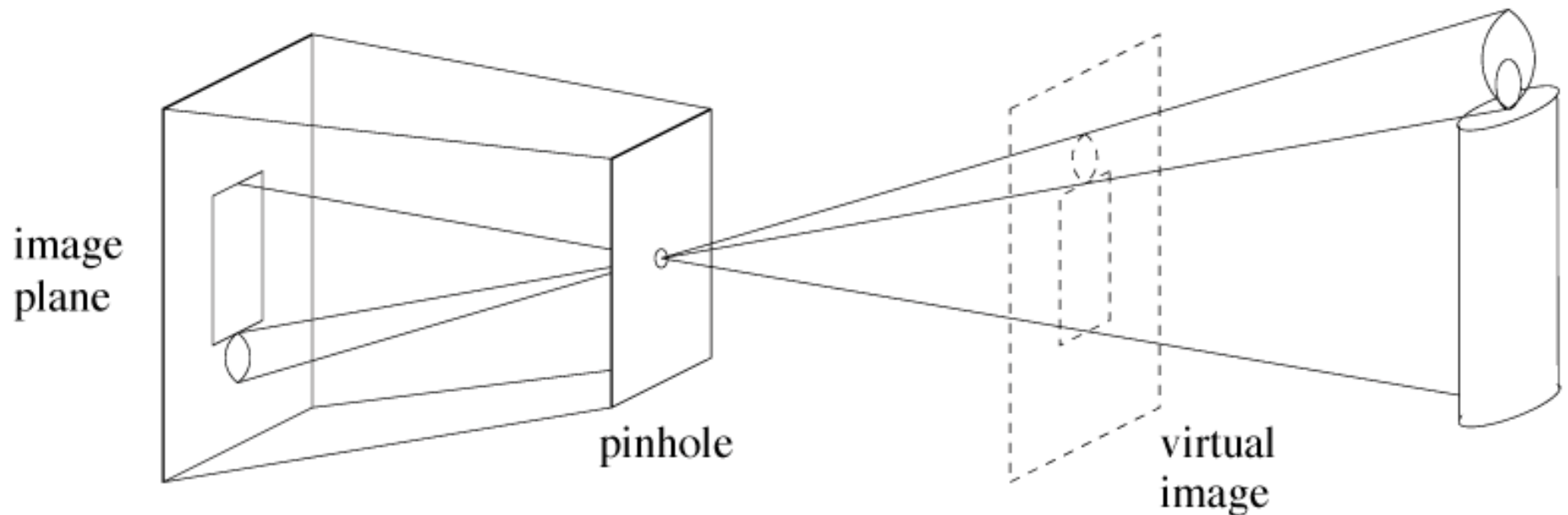
Polygons in world coordinates



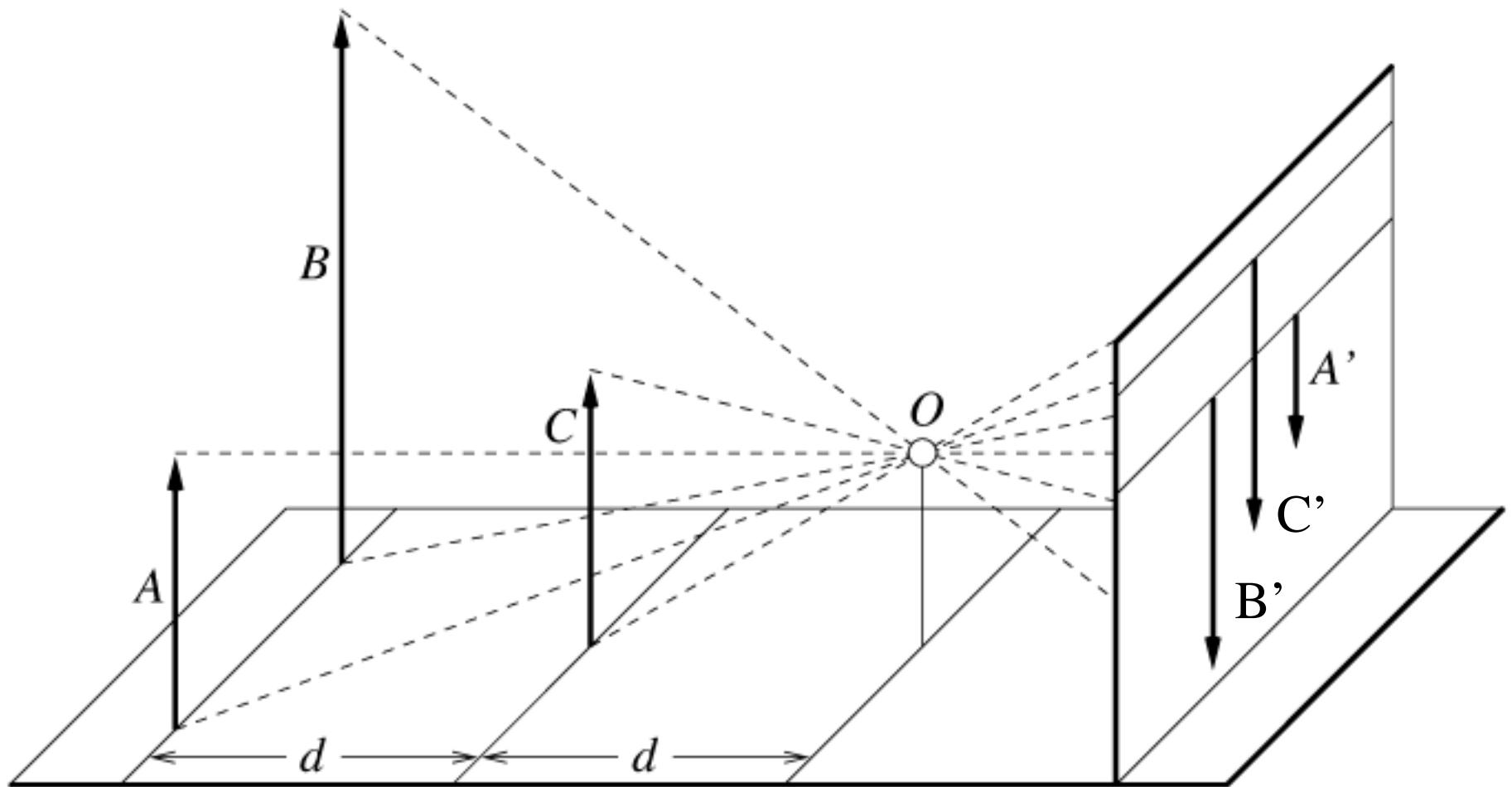


# Pinhole cameras

- Abstract camera model--box with a small hole in it
- Pinhole cameras work for deriving algorithms--a real

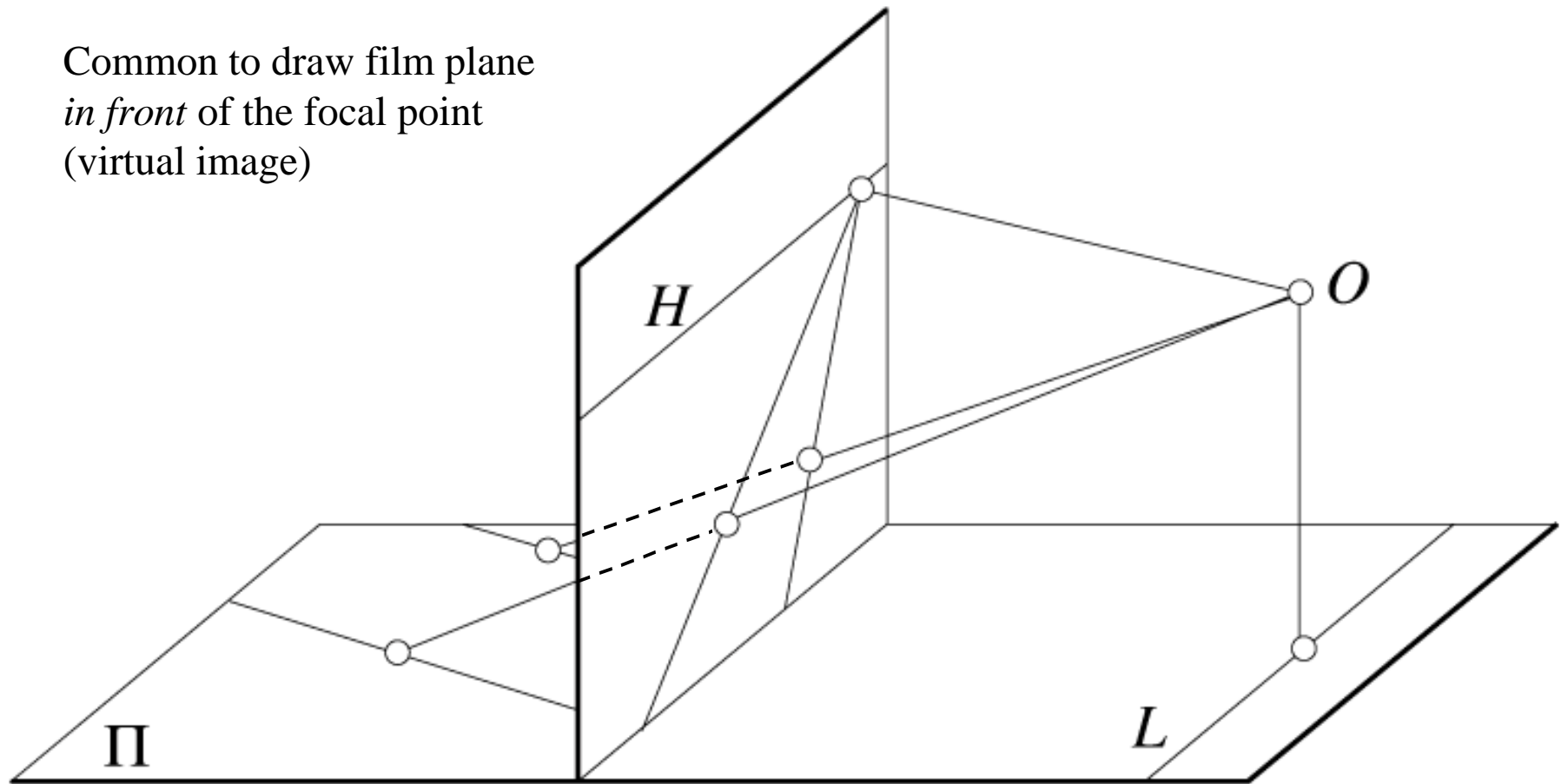


# Distant objects are smaller



# Parallel lines meet\*

Common to draw film plane  
*in front* of the focal point  
(virtual image)



\*Exceptions?

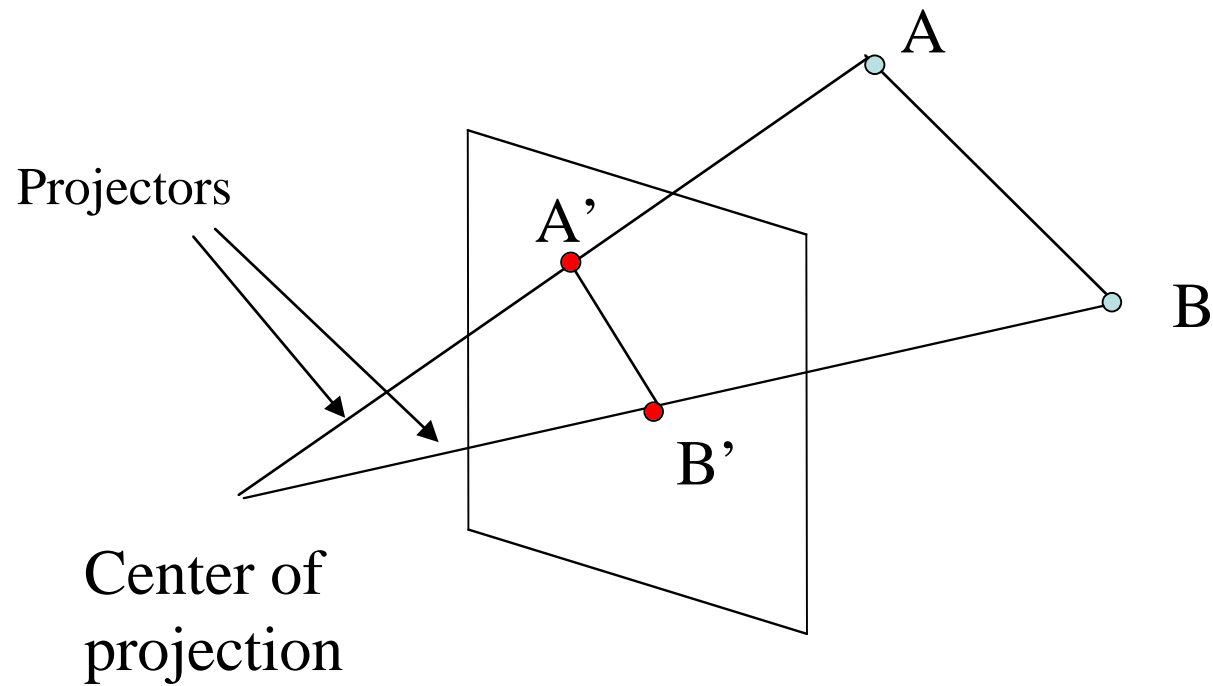
# Vanishing points

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
  - Standard horizon is the horizon of the ground plane.
- One way to spot fake images

# Projections

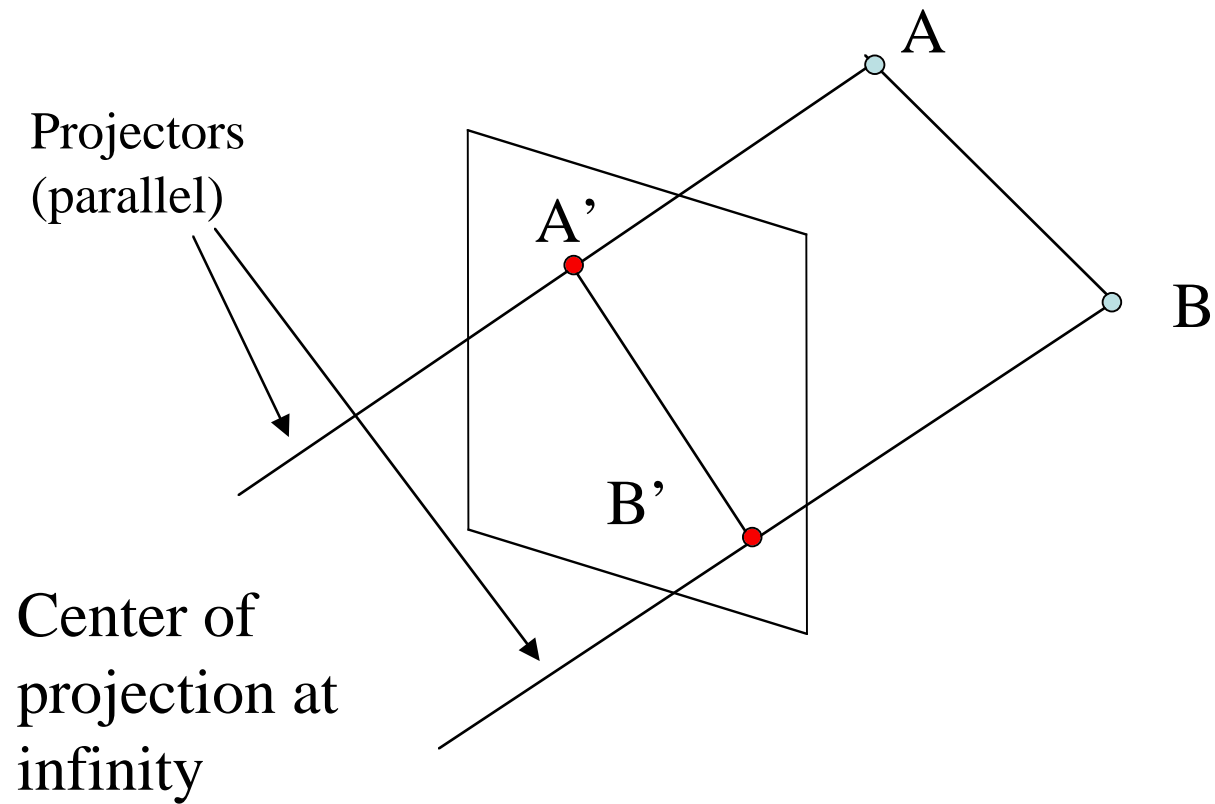
- Mathematical definition of a projection:  $PP=P$
- (Doing it a second time has no effect).
- Generally rank deficient (non-invertable)--  
exception is  $P=I$
- Transformation loses information (e.g., depth)
- Given a 2D image, there are many 3D worlds that  
could have lead to it.

# Projections (§6.2)





# Parallel Projection



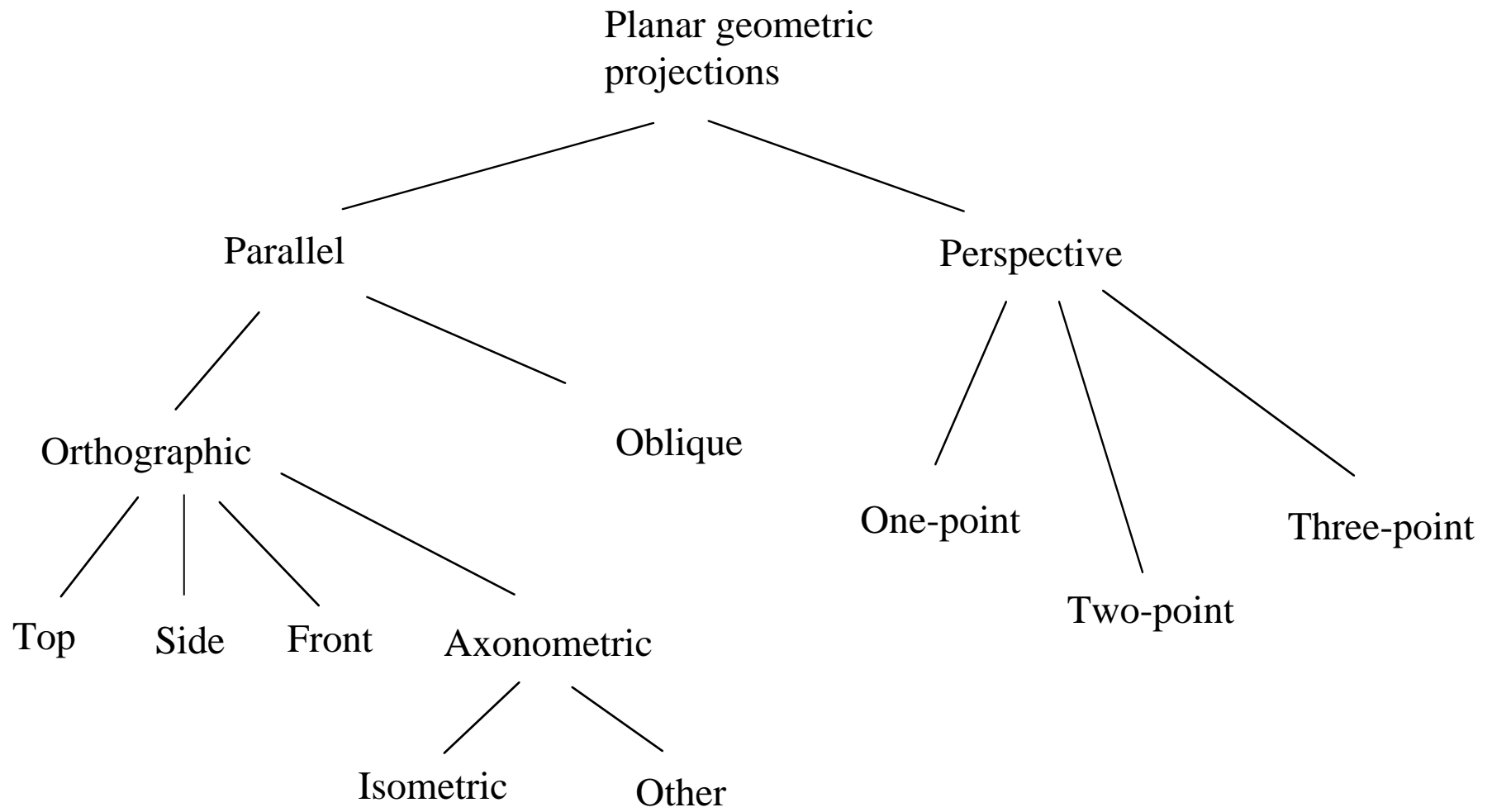
# Parallel Projection

Parallel lines remain parallel, some 3D measurements can be made using 2D picture

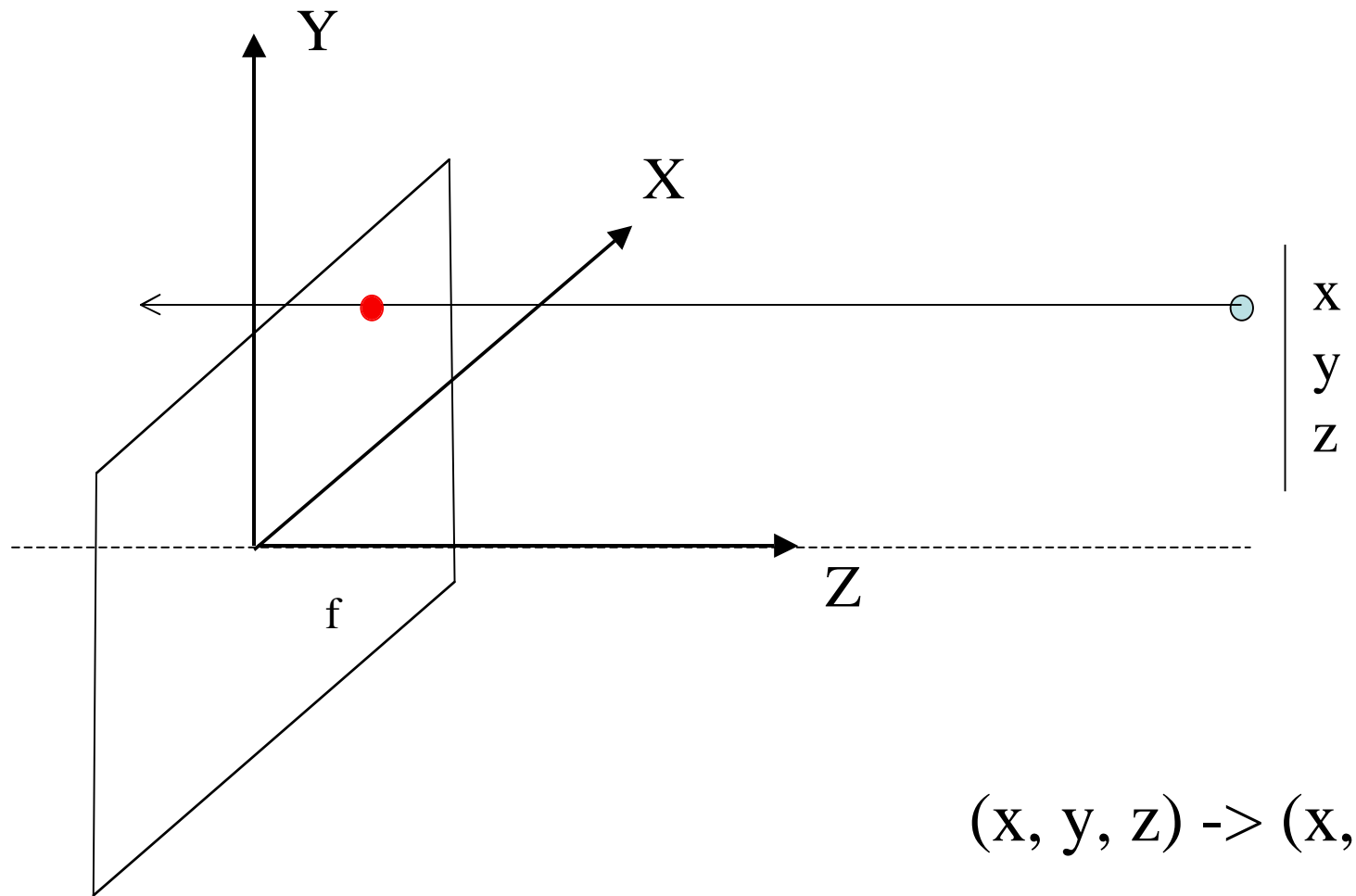
Does not give realistic 3D view because eye is more like perspective projection.

If projection plane is perpendicular to projectors the projection is orthographic (e.g., top view, side view, front view)

# Projection Taxonomy (Figure 6.10)



# Orthographic example (onto $z=0$ )



$$(x, y, z) \rightarrow (x, y, 0)$$

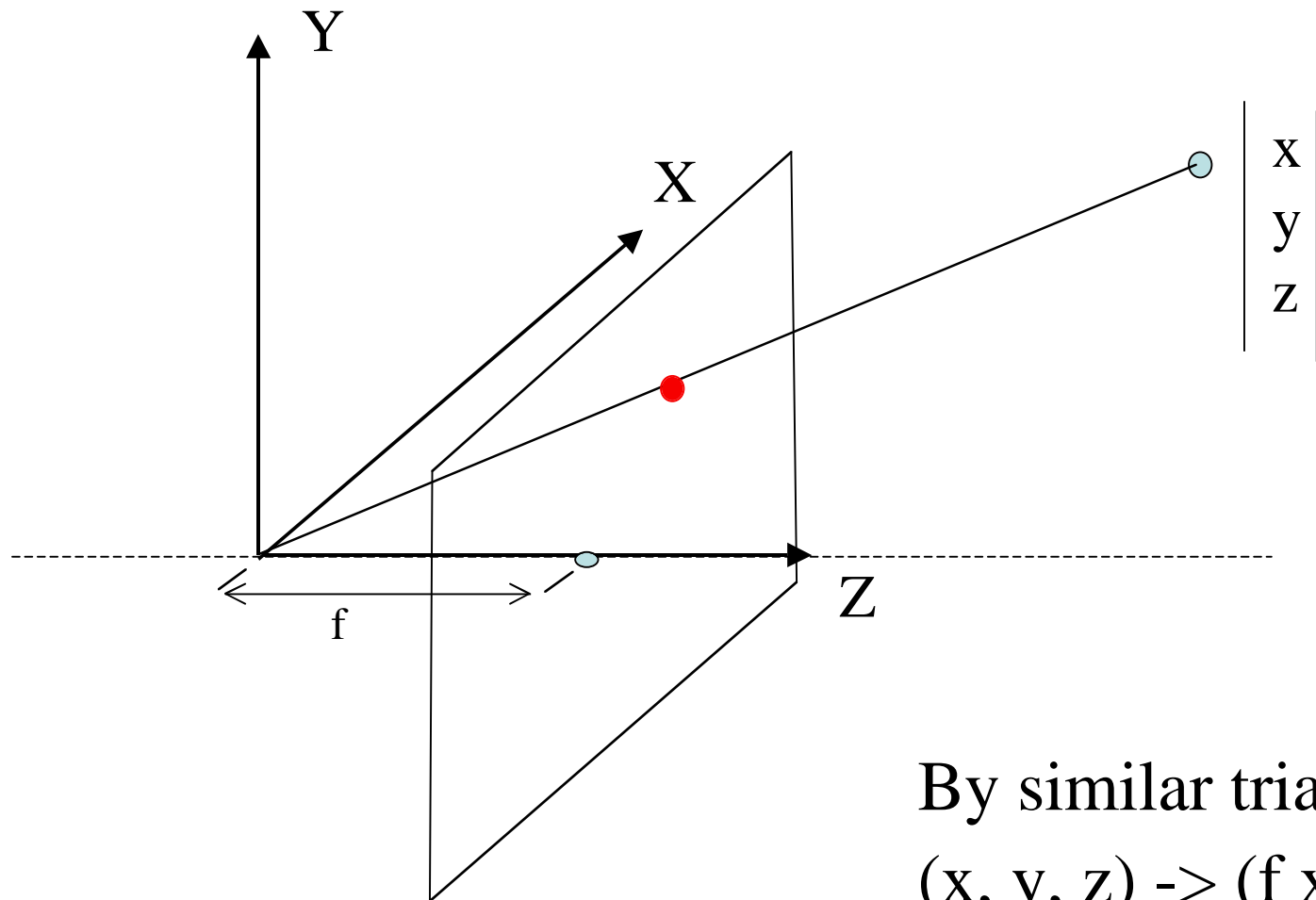
# The camera matrix

$$\begin{pmatrix} x \\ y \\ 0 \\ w \end{pmatrix} = \begin{matrix} \text{?} \end{matrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

# The camera matrix

$$\begin{pmatrix} x \\ y \\ 0 \\ w \end{pmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

# Perspective example (onto $z=f$ )



By similar triangles,  
 $(x, y, z) \rightarrow (f x/z, f y/z, f)$

# The equation of projection

- In homogeneous coordinates

$$(x, y, z, 1) \Rightarrow (f \frac{x}{z}, f \frac{y}{z}, f, 1)$$

- Equivalently

$$(x, y, z, 1) \Rightarrow (x, y, z, \frac{z}{f})$$

- (Now H.C. are being used to store foreshortening)



# The camera matrix

$$\begin{pmatrix} x \\ y \\ z \\ \frac{z}{f} \end{pmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

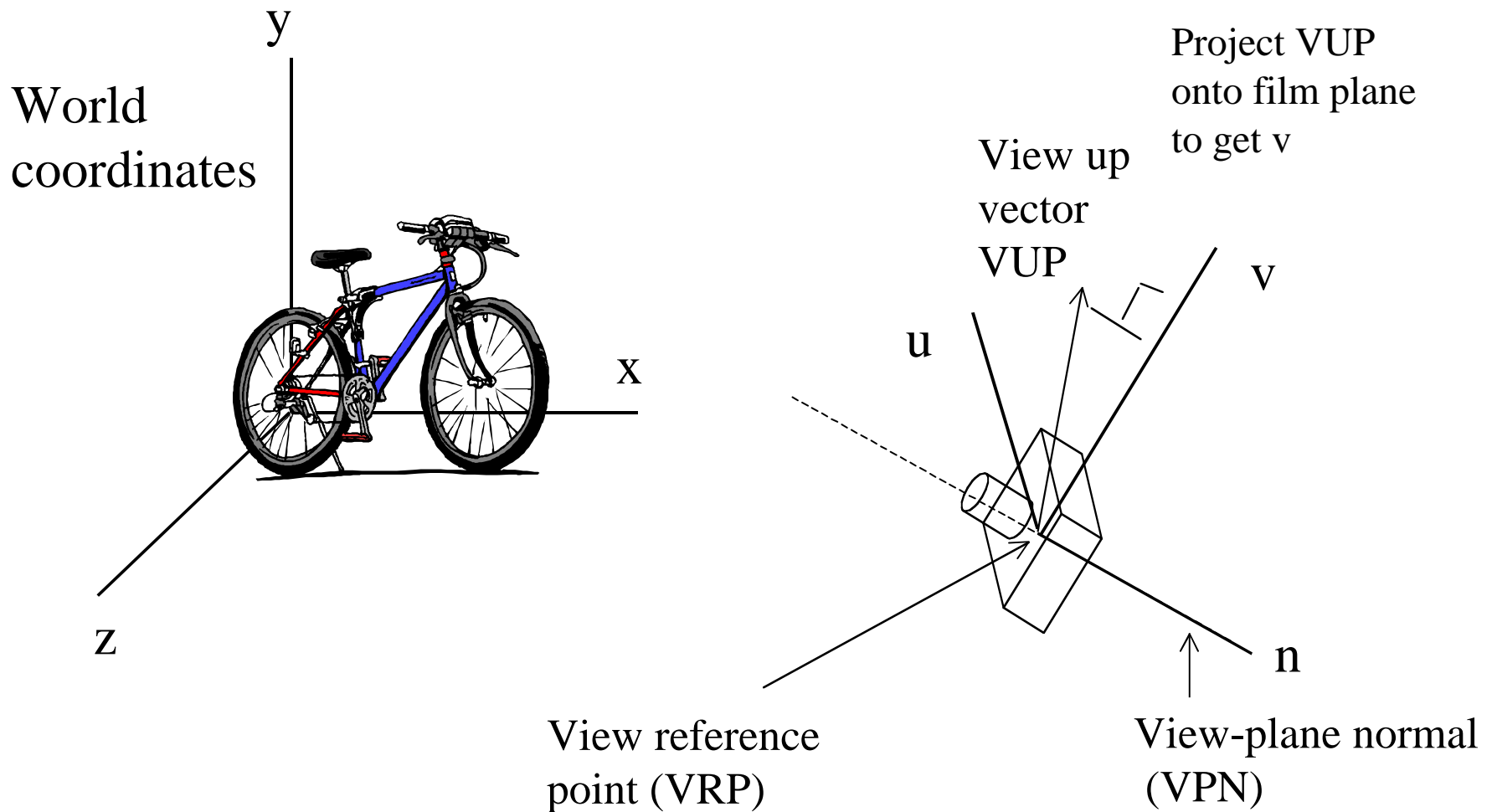
# The camera matrix

$$\begin{pmatrix} x \\ y \\ z \\ \frac{z}{f} \end{pmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & \frac{1}{f} & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

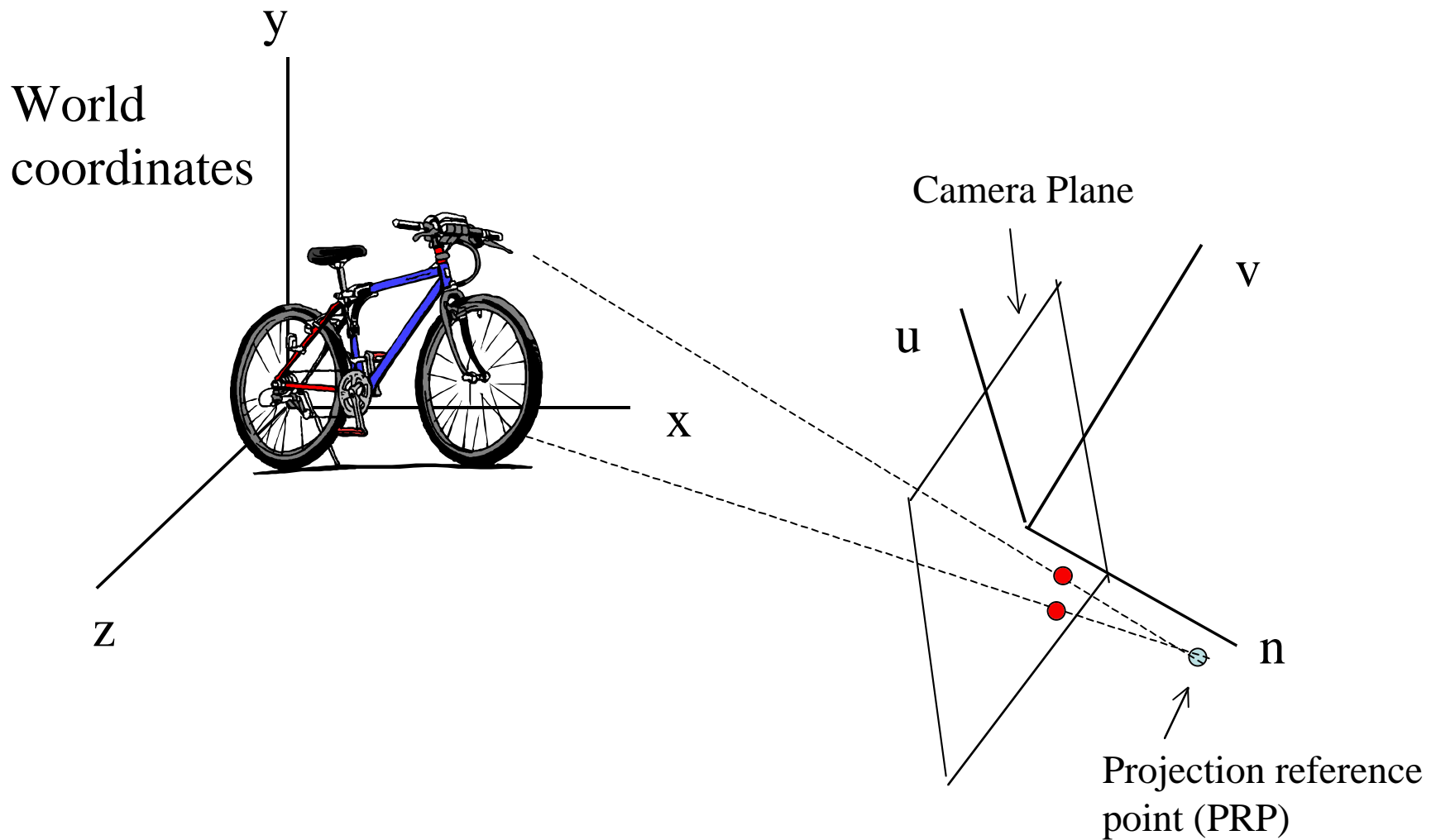
# Specifying a camera

- Camera
  - Tell rendering system where camera is in world coordinates
  - Need to specify focal point and film plane.
  - Convenient to construct a coordinate system for the camera with origin on film plane
- Clipping volume
  - we render only a window in the film plane
  - Things beyond any of four sides don't get rendered
  - Things that are too far away don't get rendered
  - Things that are too near don't get rendered.

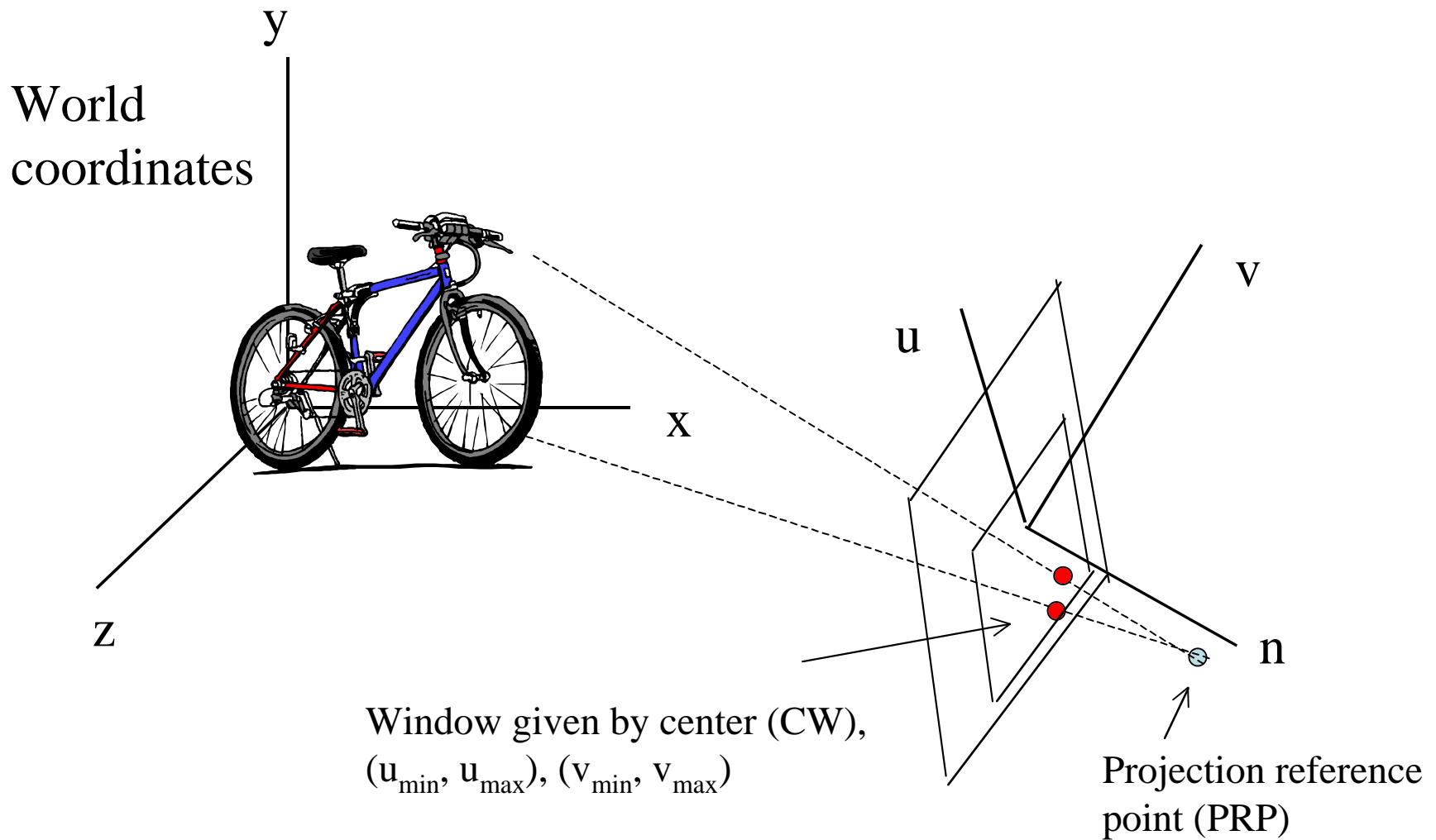
# Specifying a camera



# Specifying a camera



# Specifying a camera

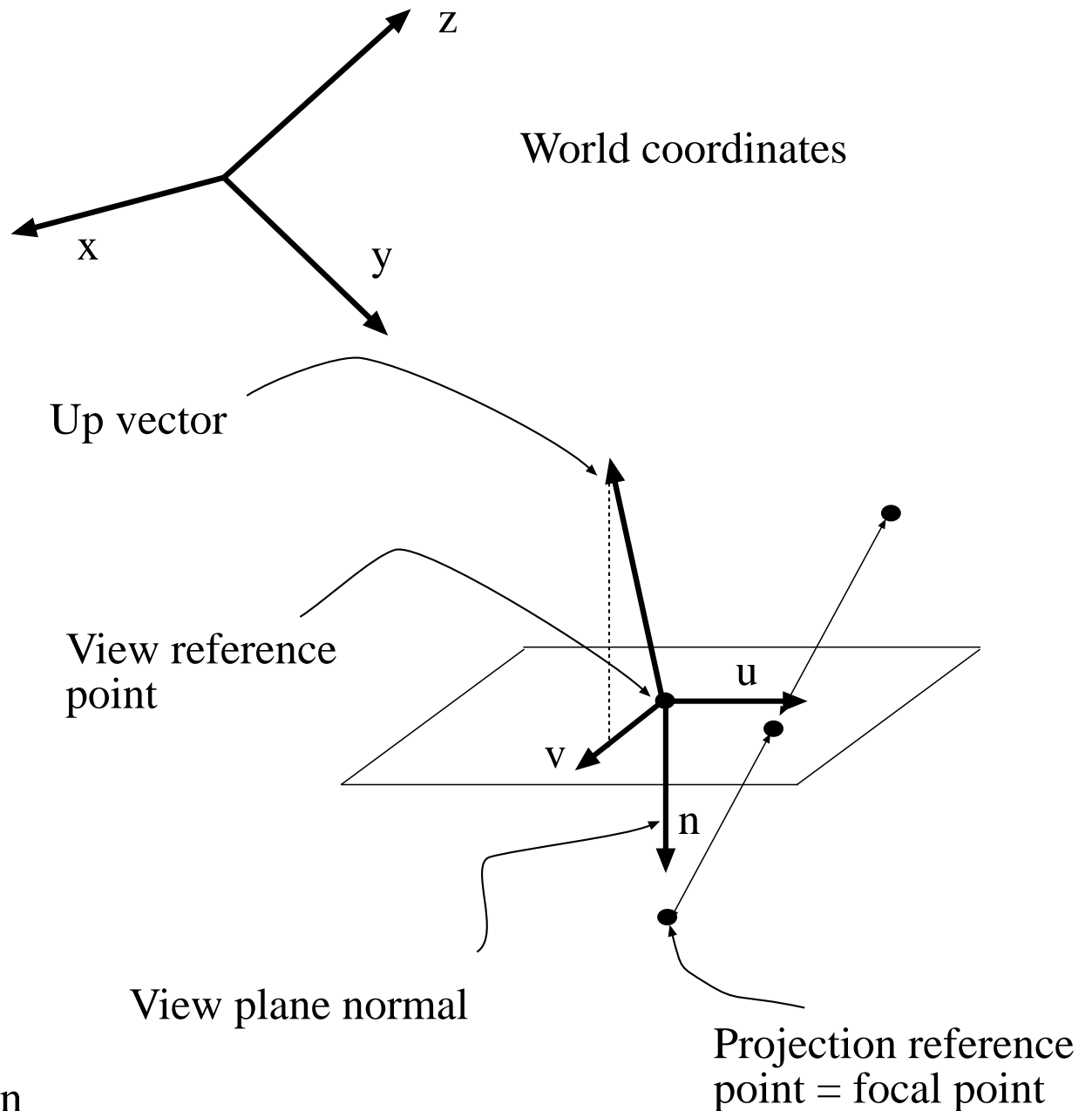


View reference point  
and view plane normal  
specify film plane.

Up vector gives an “up”  
direction in the film plane.  
vector  $v$  is projection of  
up vector into film plane  
( $= \mathbf{n} \times \mathbf{VUP} \times \mathbf{n}$ ).

$u$  is chosen so that  $(u, v, n)$   
is a right handed coordinate  
system; i.e. it is possible to  
rotate  $(x \rightarrow u, y \rightarrow v, z \rightarrow n)$   
(and we’ll do this shortly).

VRP, VPN, VUP must be in  
world coords; PRP could be in  
world coords or in camera coords



U, V can be used to specify a window in the film plane; only this section of film ends up on the screen.

This window defines four planes; points outside these planes are not rendered.

Hither and yon clipping planes, which are always given in terms of camera coordinates, and always parallel to the film plane, give a volume - known as the view frustum.

Orthographic case: - view frustum is cuboid (i.e. all angles right angles, but edges not necessarily of equal length).

