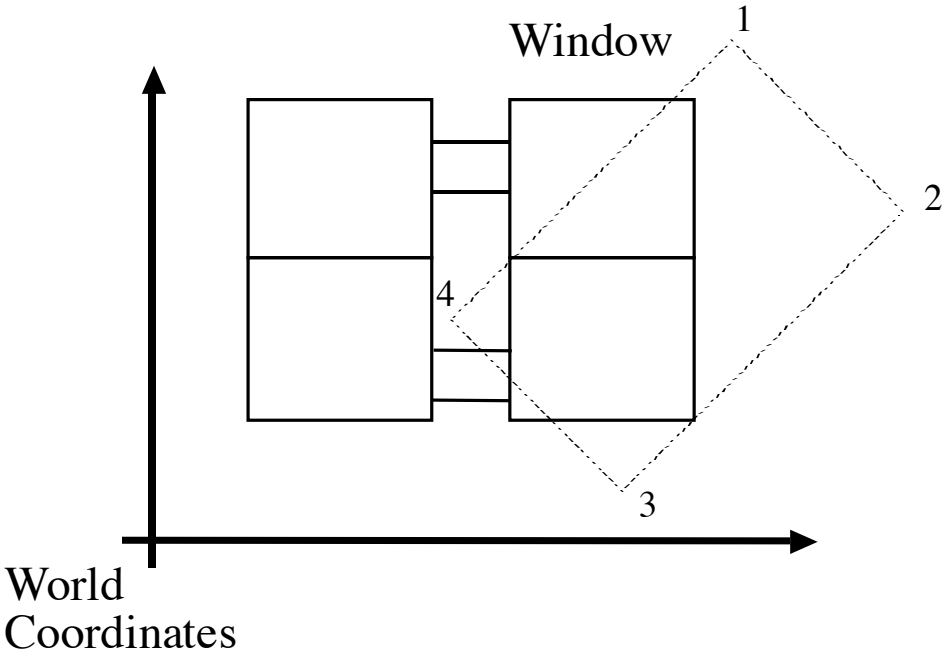
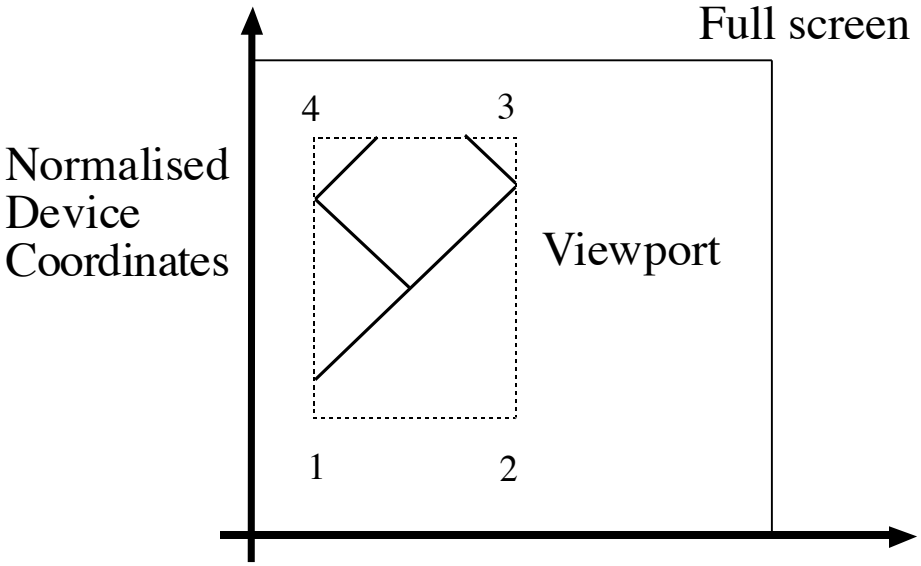
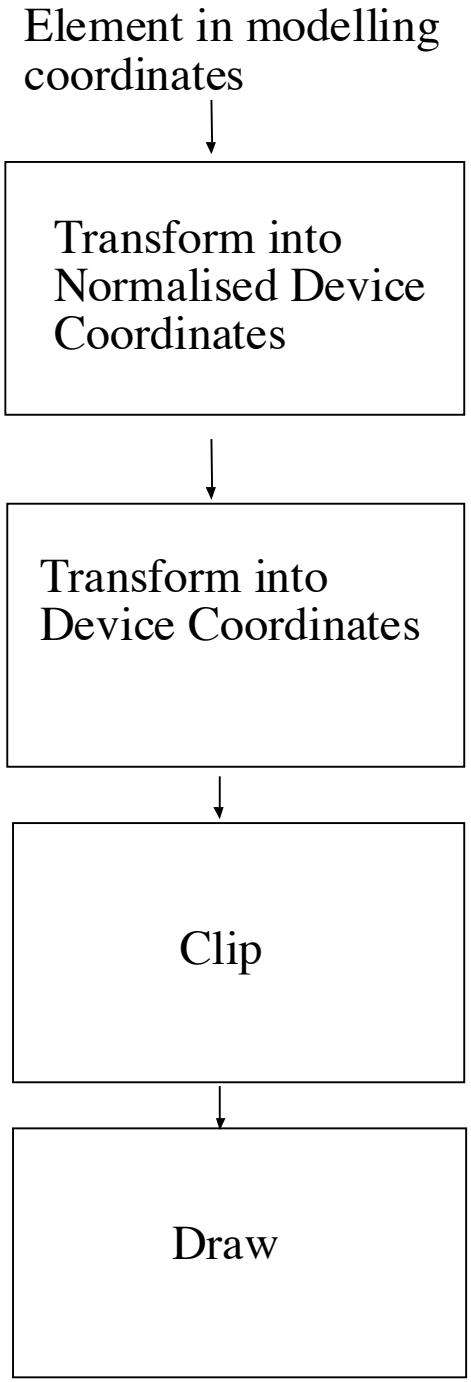
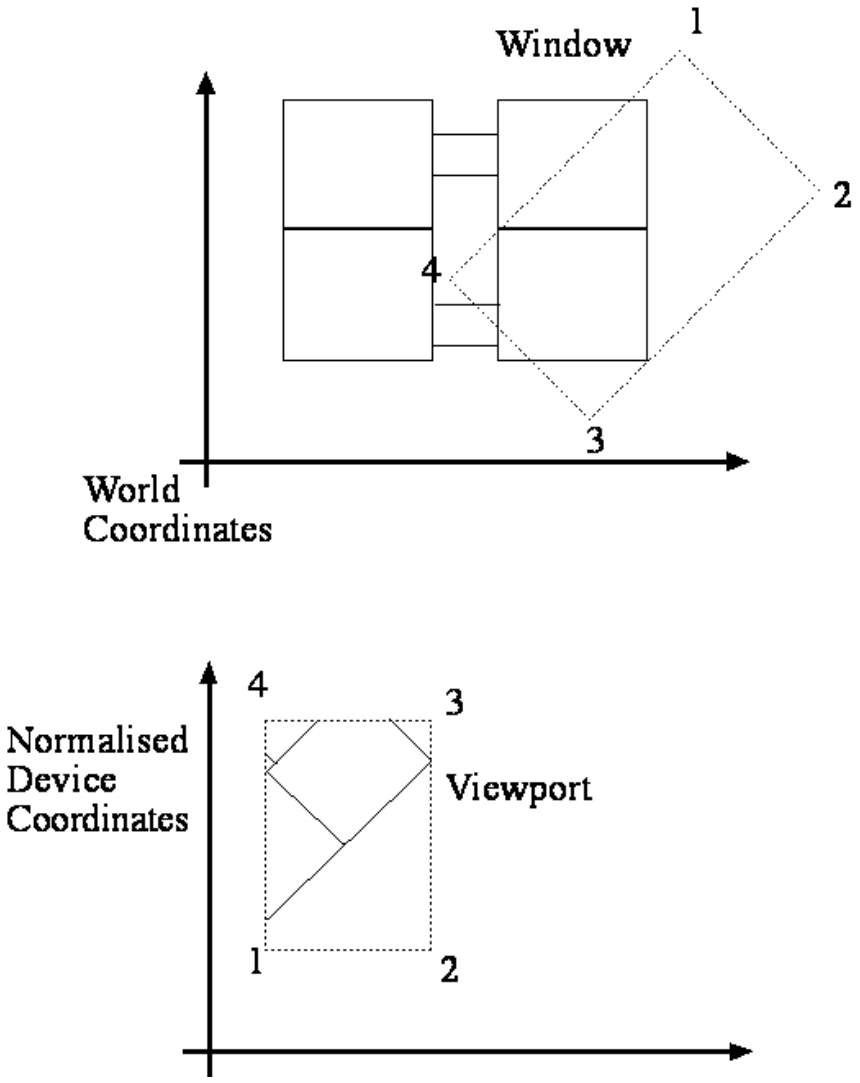
2D viewing

- Three coordinate systems are common in graphics
  - World coordinates or modeling coordinates - where the model is defined (meters, miles, etc.)
  - Normalized device coordinates; usually (0-1) in each variable.
  - Device coordinates: the actual coordinates of the pixels on the frame-buffer or the printer
- Need to construct transformations between coordinate systems
- Terminology:
  - window = region on drawing that will be displayed (rectangle)
  - viewport = region in NDC's/DC's where this rectangle is displayed (often simply entire screen).

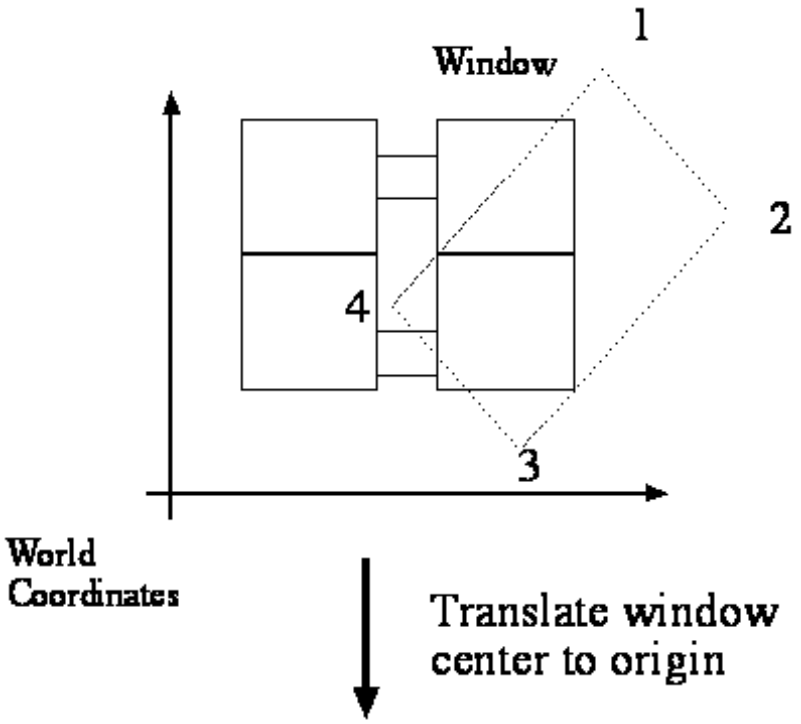

Window
1
2
3
4
World
Coordinates

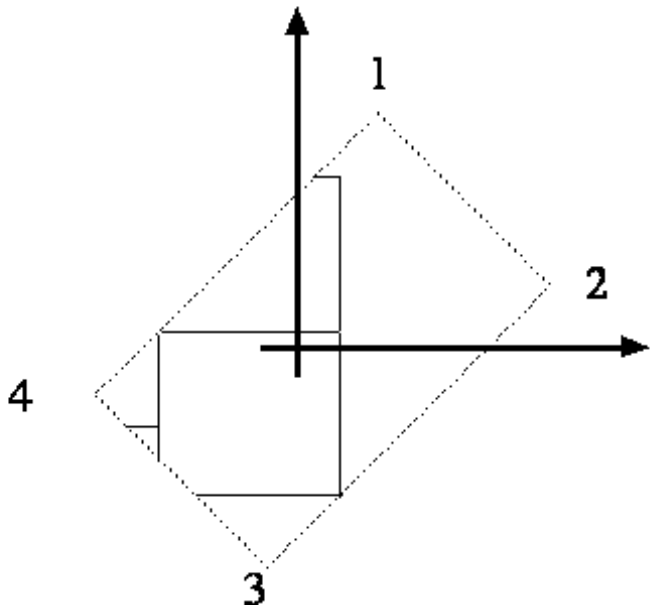

Full screen
4
3
Normalised
Device
Coordinates
Viewport
1
2


Element in modelling coordinates
Transform into Normalised Device Coordinates
Transform into Device Coordinates
Clip
Draw



- View this as a sequence of transformations in homogenous coords, then determine each element in closed form.
- Compute numerically from point correspondences.

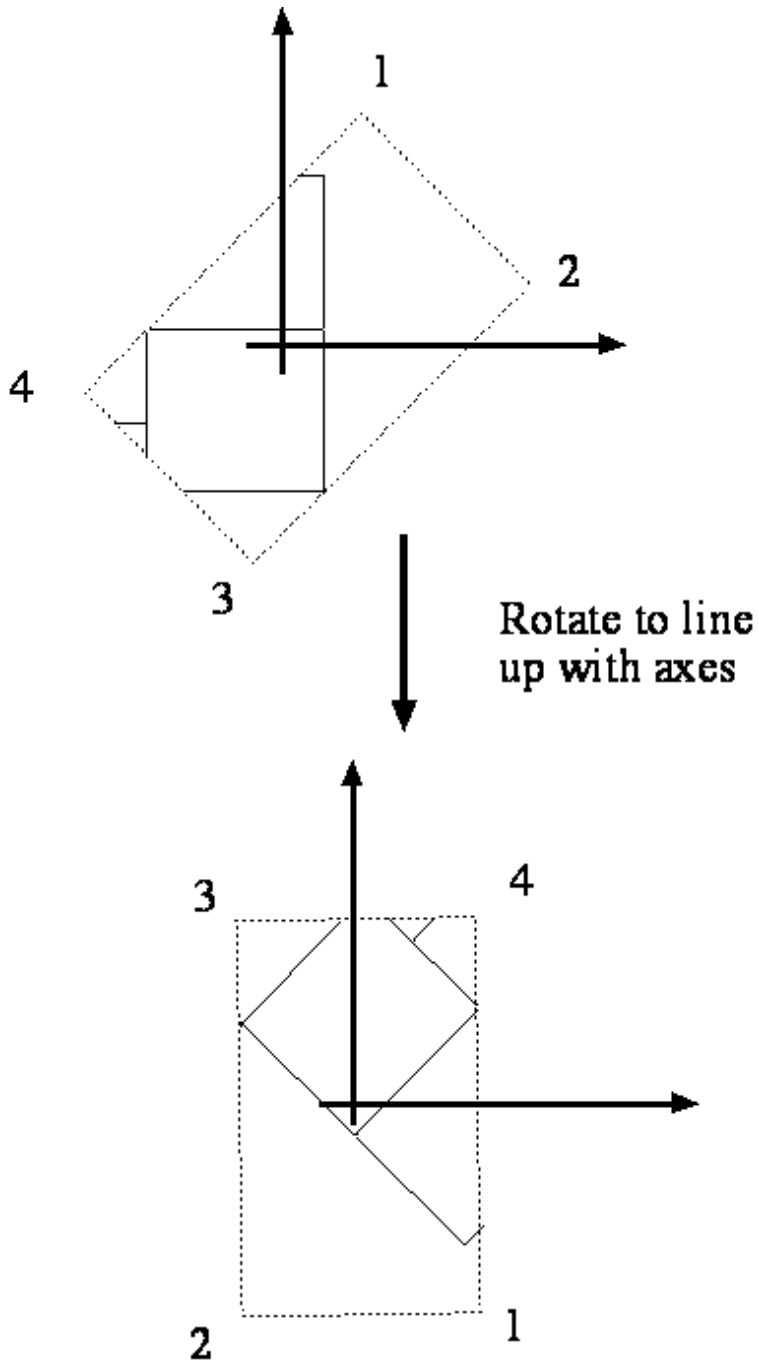




- write (wx_i, wy_i) for coordinates of i'th point on window
- translation is:

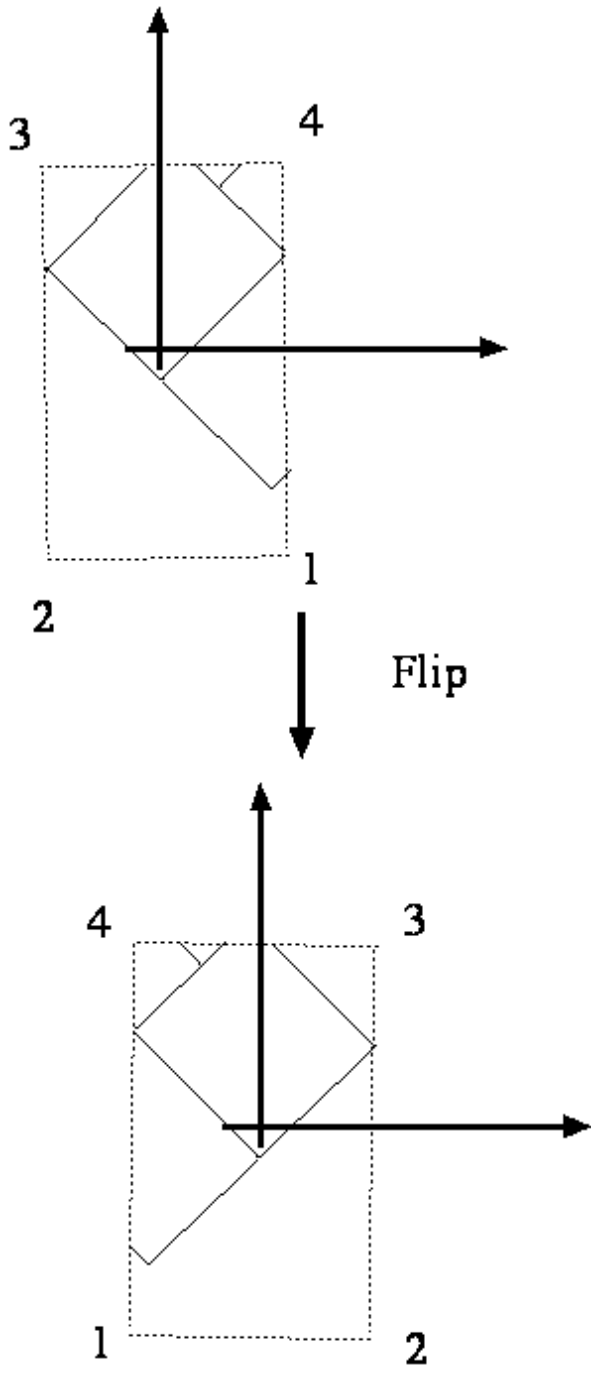
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\overline{wx} \\ 0 & 1 & -\overline{wy} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(overbar denotes average over vertices, i.e., 1,2,3,4)



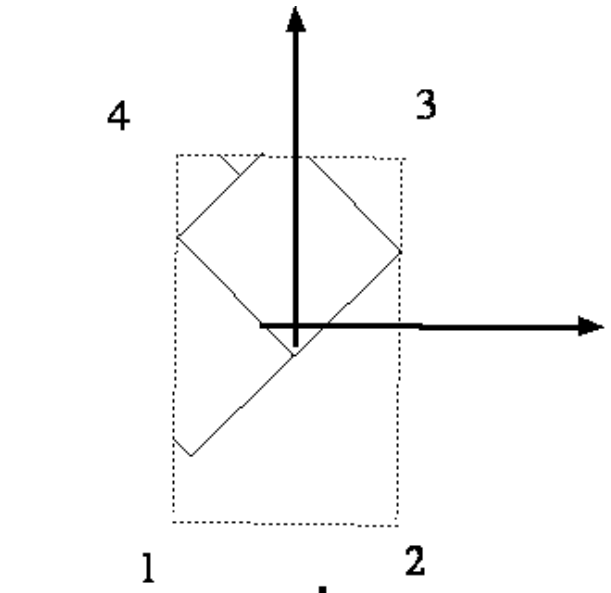
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(Need to compute theta)

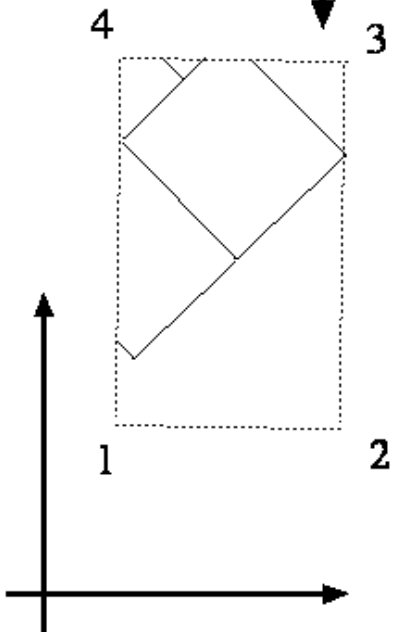


$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(Vertex order does not correspond, need to flip)



Scale and translate



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{w_{new}}{w_{old}} & 0 & \overline{x}_{new} \\ 0 & \frac{h_{new}}{h_{old}} & \overline{y}_{new} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Notice that choice of new width, height, and center give translation to either normalized device coords, or to device coordinates

- Get overall transformation by multiplying transforms.
- This gives a single transformation matrix, whose elements are functions of window/viewport coordinates.

$$x' = M_{(\text{translate origin to viewport cog, scale})}\, M_{(\text{flip})}\, M_{(\text{rotate})} M_{(\text{translate window cog->origin})} x$$

NDC's/DC's

World coords

(cog==window center of gravity)

Affine transformations

- Another approach to determining the whole transform for the pipeline; this is an affine transform.
- Matrix form:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

- Now assume that we know that $Mp_1=q_1$, $Mp_2=q_2$, $Mp_3=q_3$
- Quick way to determine transform, because this is the same as six linear equations, in six variables, which are the entries in the matrix:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

Details

- $Mp_1=q_1$ gives first two rows
- $p_1 = (x_1, y_1, 1)^T$, $q_1 = (u_1, v_1, 1)^T$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

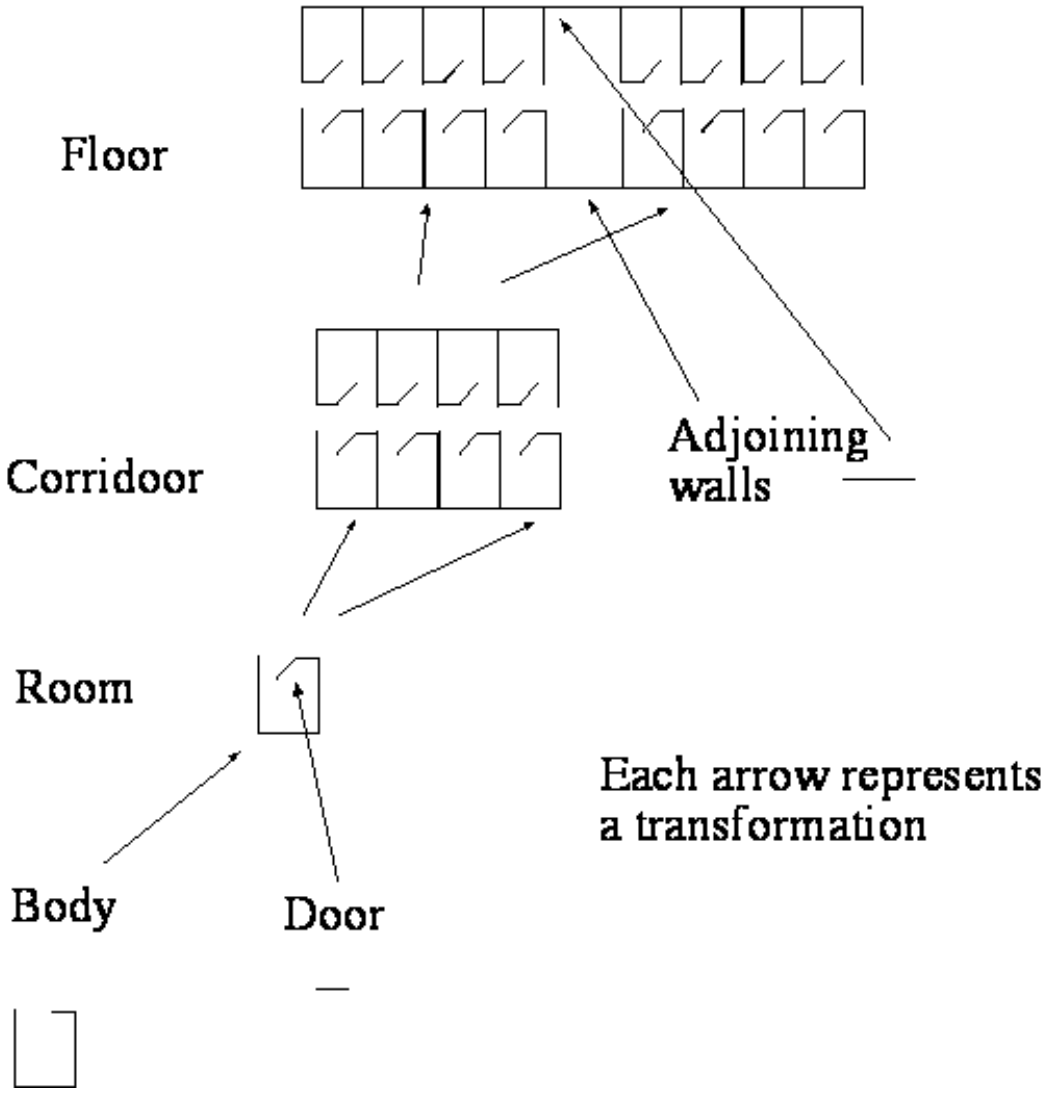
$$\begin{aligned} ax_1 + by_1 + c &= u_1 \\ dx_1 + ey_1 + f &= v_1 \end{aligned}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

$Mp_2=q_2$, $Mp_3=q_3$ give other rows

Hierarchical modeling

- Consider constructing a complex 2d drawing: e.g. an animation showing the plan view of a building, where the doors swing open and shut.
- Options:
  - specify everything in world coordinate frame; but then each room is different, and each door moves differently. (hugely difficult).
  - Exploit similarities by using repeated copies of models in different places (instancing)



Hierarchical modeling

- Model form
  - Directed acyclic graph.
  - Each node consists of 0 or more objects (lines, polygons, etc).
  - Each edge is a transformation
- There can be many edges joining two nodes (e.g. in the case of the corridor - many copies of the same room model, each transformed differently).
- Every graphics API supports hierarchies - some directly (meaning you have to learn a language to express the model) some indirectly with a matrix stack

- Write the transformation from door coordinates to room coordinates as:

$$T_{room}^{door}$$

Then to render a door, use the transformation:

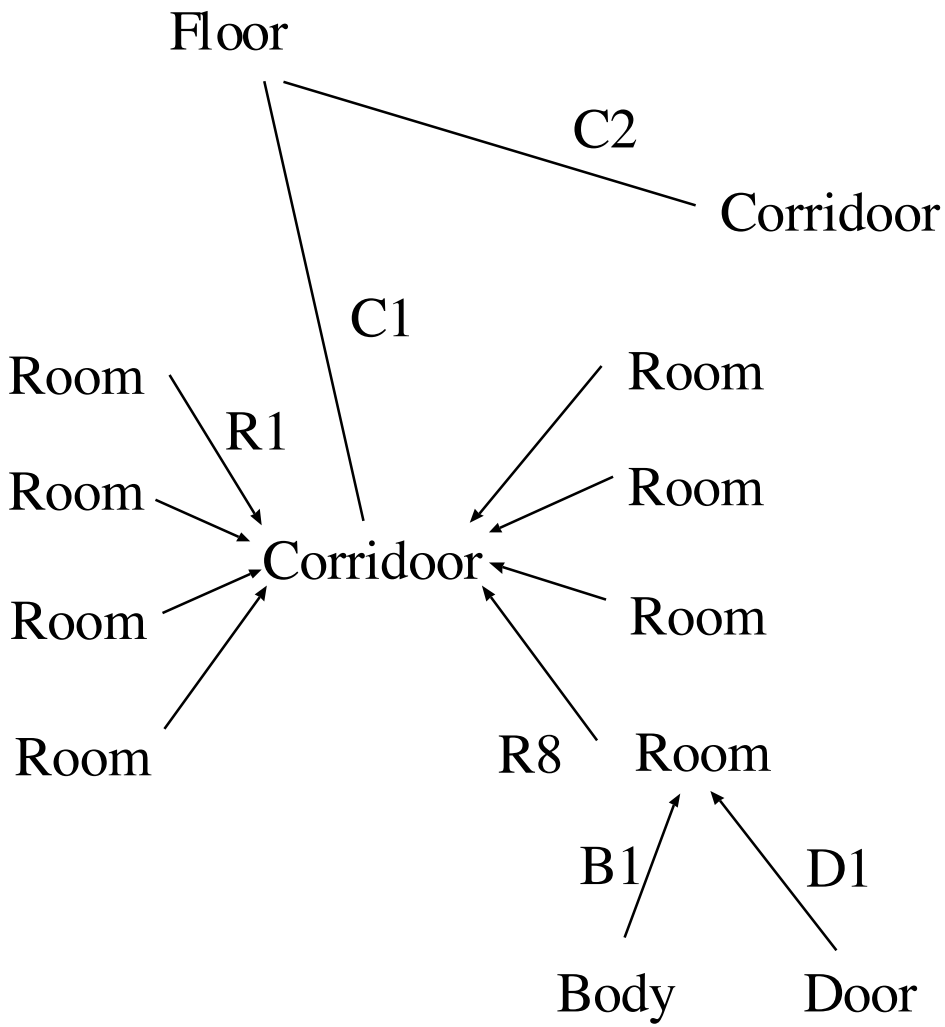
$$T_{device}^{world} T_{floor}^{corridoor} T_{corridoor}^{room} T_{room}^{door}$$

To render a body, use the transformation:

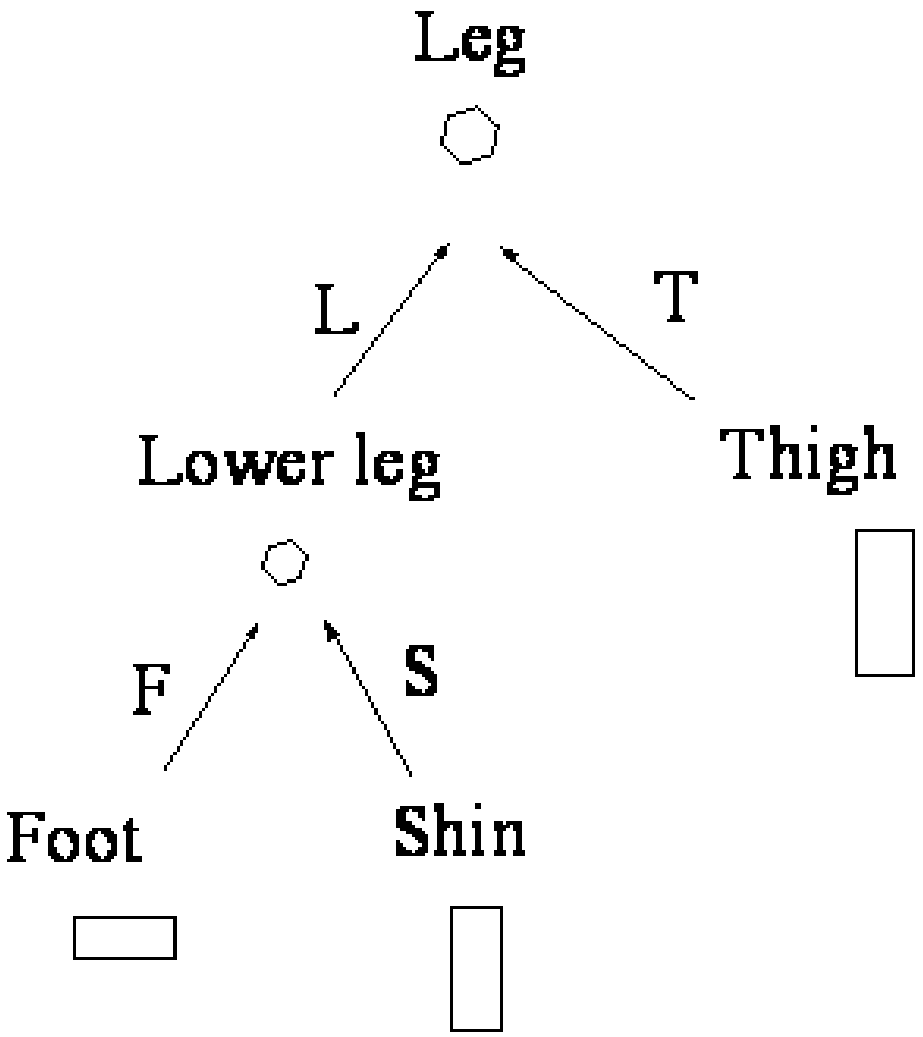
$$T_{device}^{world} T_{floor}^{corridoor} T_{corridoor}^{room} T_{room}^{body}$$

Matrix stacks and rendering

- Matrix stack:
  - Stack of matrices used for rendering
  - Applied in sequence.
  - Pop=remove last matrix
  - Push=append a new matrix
  - In previous example, body-device transformation comes from door-device transformation by popping door-room and pushing body-room
- Algorithm for rendering a hierarchical model:
  - stack is T_{device}^{root}
  - render (root)
- Render (node)
  - for each child:
    - push transformation
    - render (child)
    - pop transformation



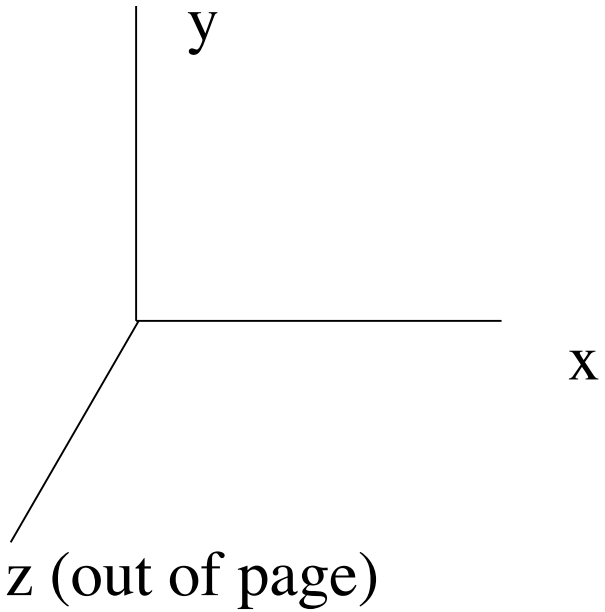
- Now to render door on first room in first corridor, stack looks like: W C1 R1 D1
- For efficiency we would store "running" products, IE, the stack contains: W, W*C1, W*C1*R1, W*C1*R1*D1.
- We do not need two copies of corridor, or 16 copies of body; we render one copy using 16 different transformations. This is known as instancing
- Animation requires care: if D1 is a single function of time, all doors will swing open and closed at the same time.



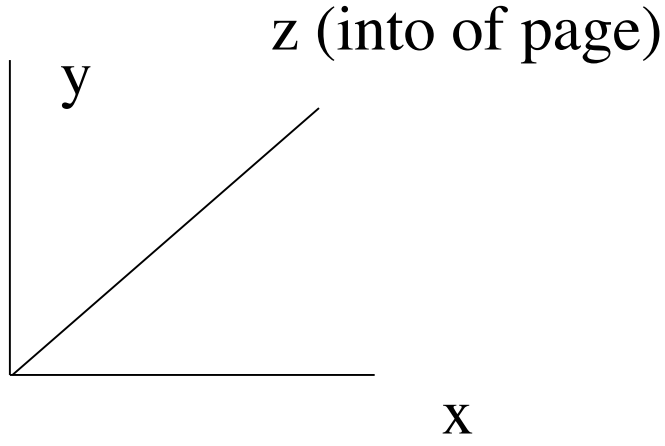
- Stack is W
- render kneecap
- Stack is W L
- render ankle
- Stack is W L F
- render foot
- Stack is W L S
- render shin
- Stack is W T
- render thigh

Transformations in 3D (Watt chapter 1)

- Right hand coordinate system (conventional, i.e., in math)



- In graphics a LHS is sometimes also convenient (Easy to switch between them--later).



Transformations in 3D

- Homogeneous coordinates now have four components - traditionally, (x, y, z, w)
  - ordinary to homogeneous: (x, y, z) -> (x, y, z, 1)
  - homogeneous to ordinary: (x, y, z, w) -> (x/w, y/w, z/w)
- Again, translation can be expressed as a multiplication.

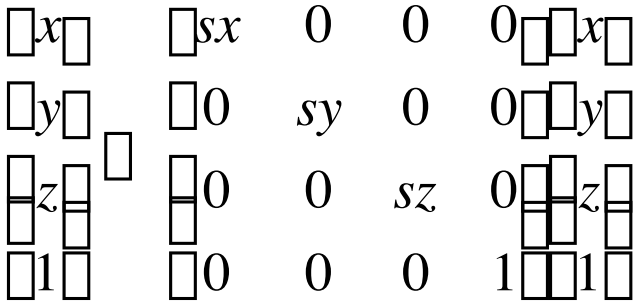
Transformations in 3D

- Translation:

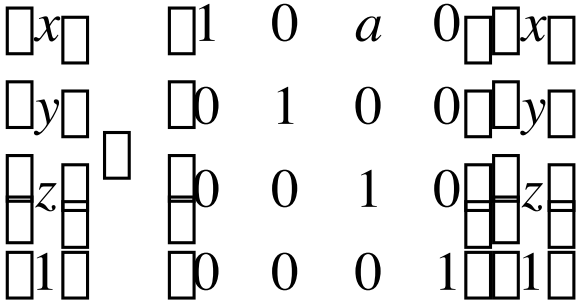
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D transformations

- Anisotropic scaling:



- Shear (one example):



Rotations in 3D

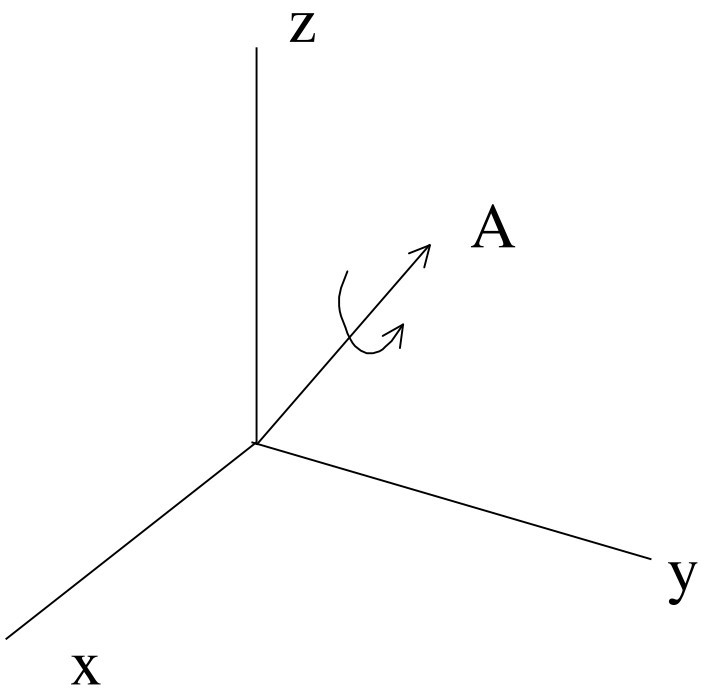
- 3 degrees of freedom
- Det(R)=1
- Orthogonal
- Many representations are possible.
- Our representation: rotate about coordinate axes in sequence.
- Sequence of axes is arbitrary, but choice does affect the angles used (cannot use same angles with different order).
- Sign of rotation follows the Right Hand Rule--point thumb along axis in direction of increasing ordinate--then fingers curl in the direction of positive rotation).

Rotations in 3D

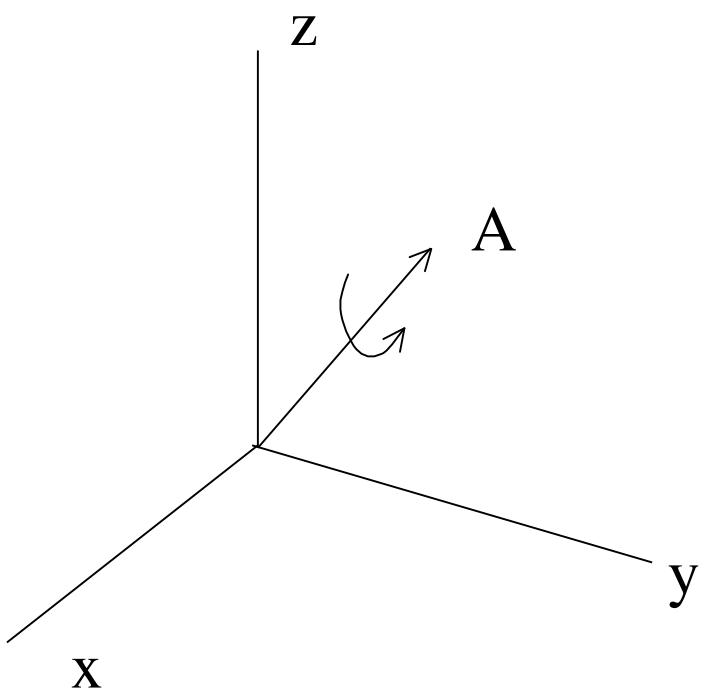
- About z-axis

$$M = \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Rotation about an arbitrary axis (likely to be needed for an assignment!)

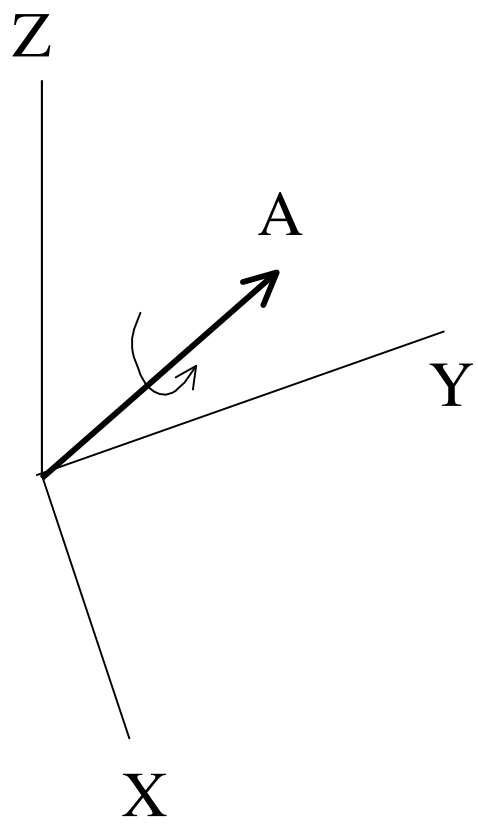


Rotation about an arbitrary axis



Strategy--rotate A to Z axis, rotate about Z axis, rotate Z back to A.

Rotation about an arbitrary axis

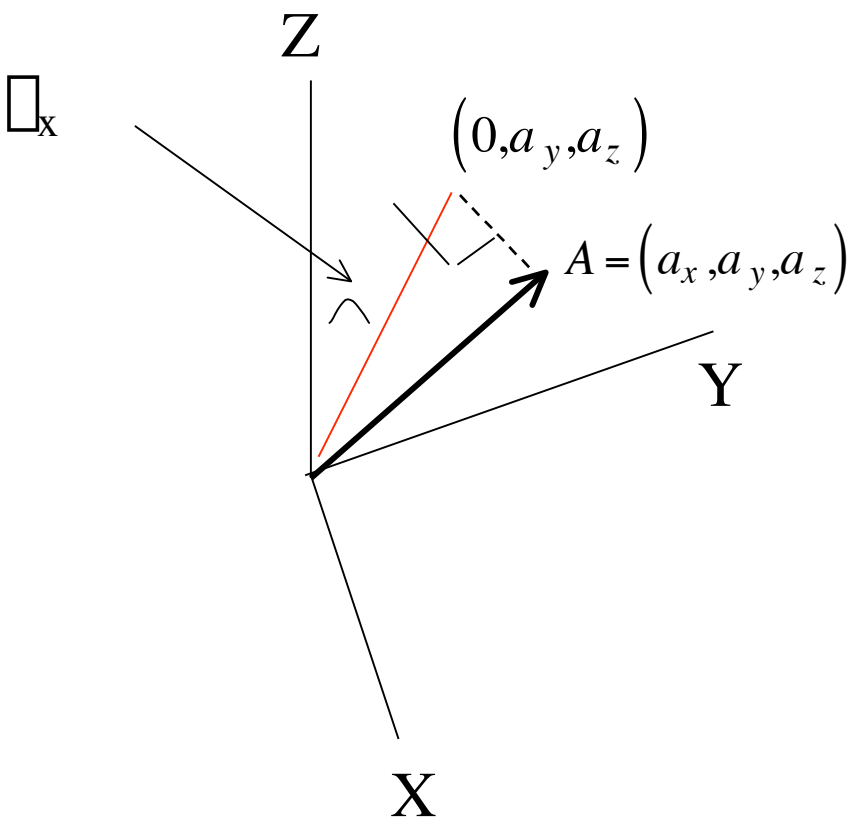


Tricky part:
rotate A to Z axis

Two steps.

1) Rotate about x to xz plane
2) Rotate about y to Z axis.

Rotation about an arbitrary axis (assignment hint)



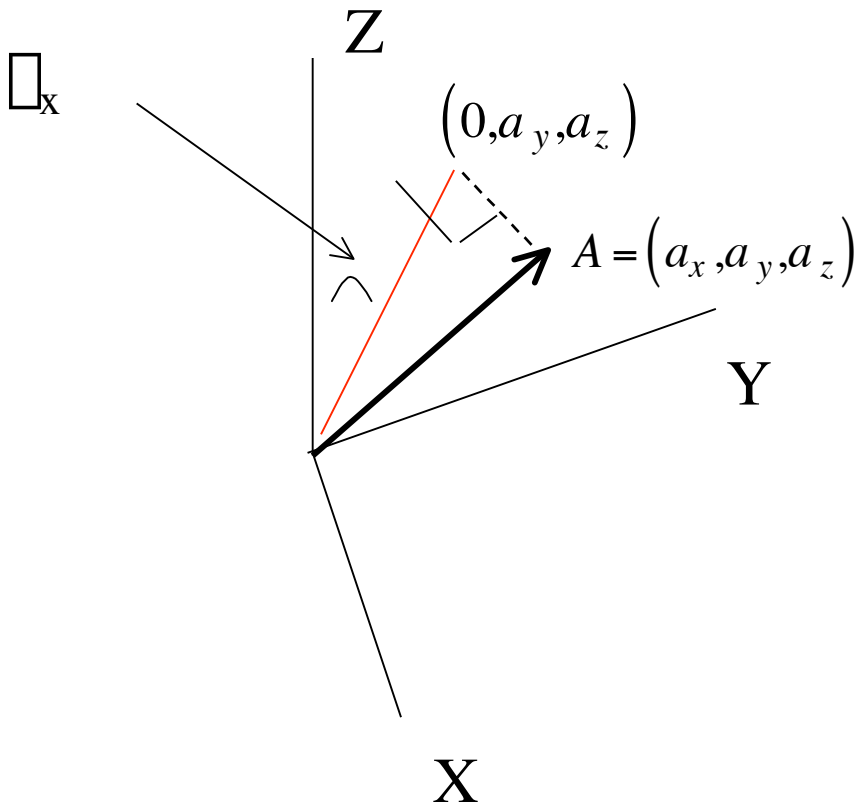
Tricky part:

rotate A to Z axis

Two steps.

1) Rotate about X to xz plane
2) Rotate about Y to Z axis.

Rotation about an arbitrary axis



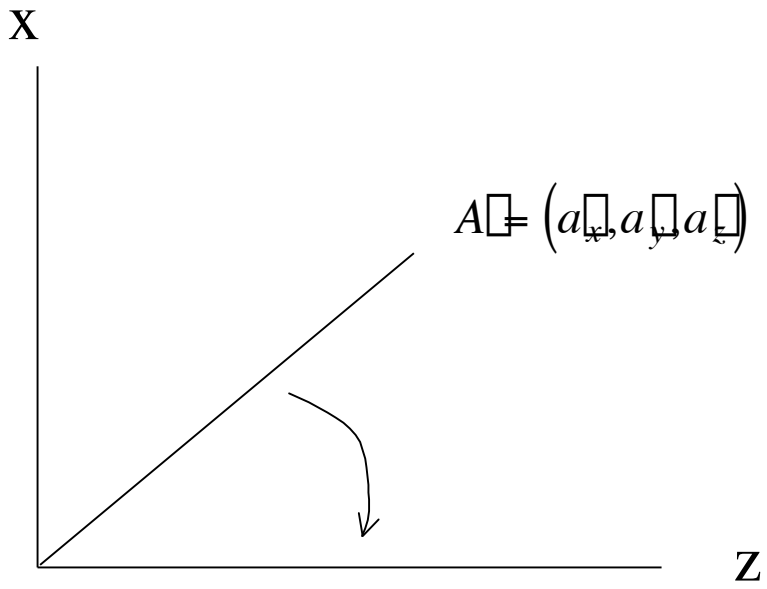
$$d = \sqrt{a_y^2 + a_z^2}$$

$$\sin\alpha_x = a_y / d$$

$$\cos\alpha_x = a_z / d$$

No need to compute angles, just put sines and cosines into rotation matrices

Rotation about an arbitrary axis



Apply $R_x(\theta_x)$ to A and renormalize to get A'

$R_y(\theta_y)$ should be easy, but note that it is clockwise.

Rotation about an arbitrary axis

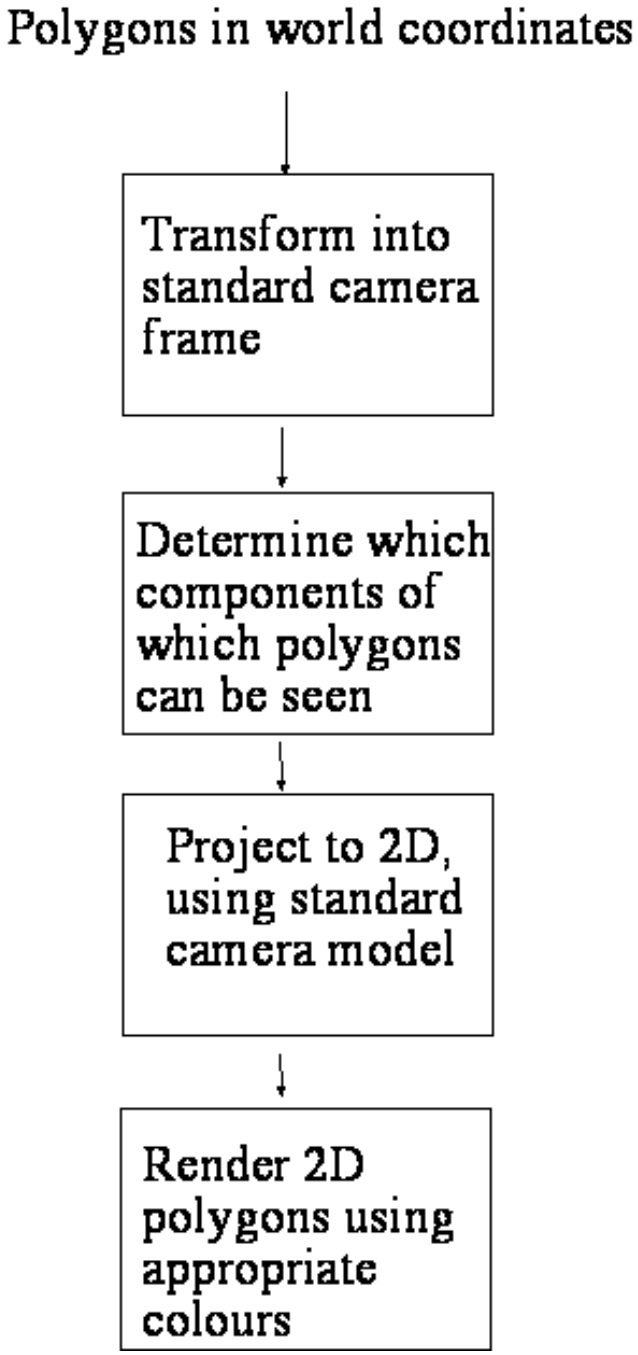
Final form is

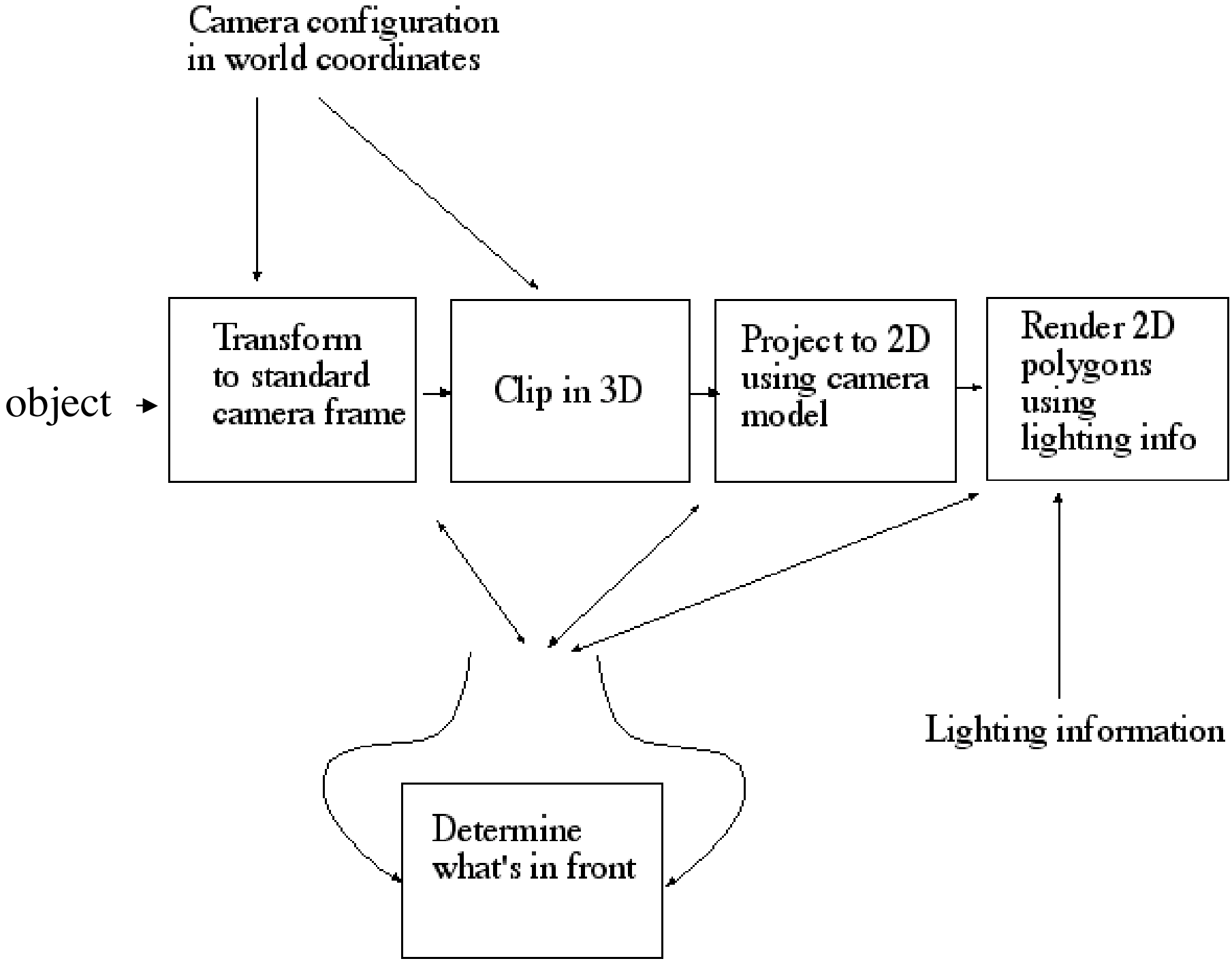
$$R_x(-\theta_x)R_y(-\theta_y)R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

3D Graphics Concepts

(Watt ch. 5, Foley et al ch. 6)

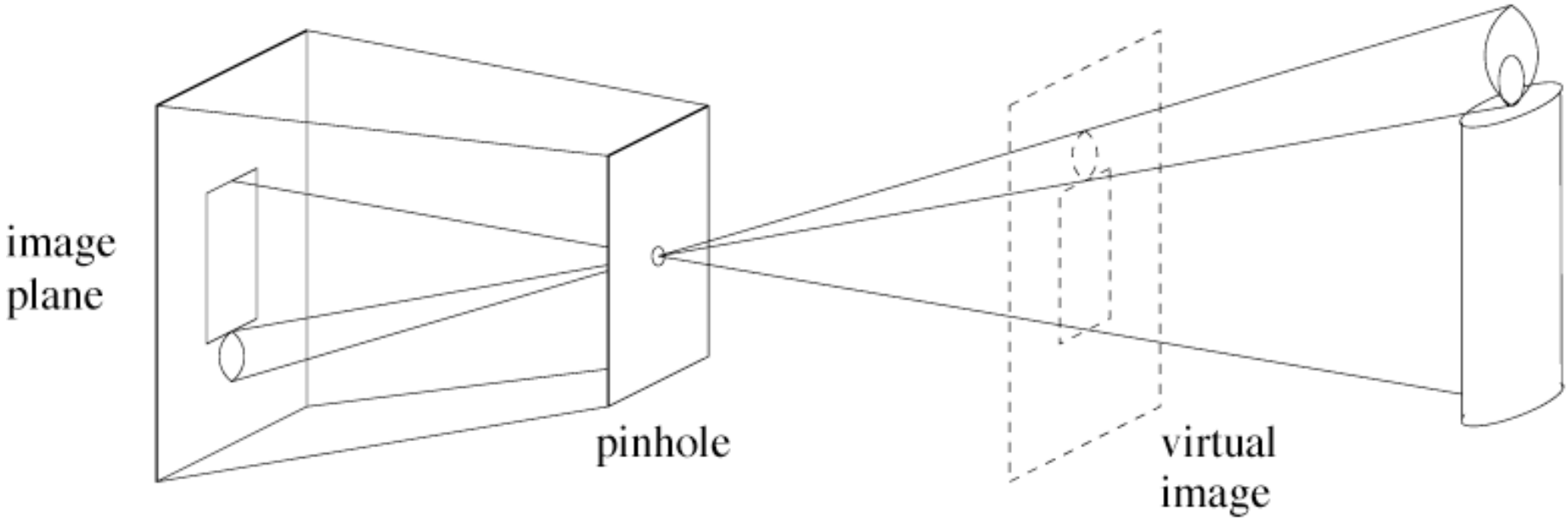
- Modeling: For now, objects will be collections of polygons in 3D. Complex shapes will be many small polygons.
- Issues:
  - Which polygons can be seen? (some polygons hide others, and some are outside the relevant volume of space and need to be clipped).
  - Where do they go in the 2D image? (key abstraction is a virtual camera)
  - How bright should they be? (for example, to make it look as if we are looking at a real surface)



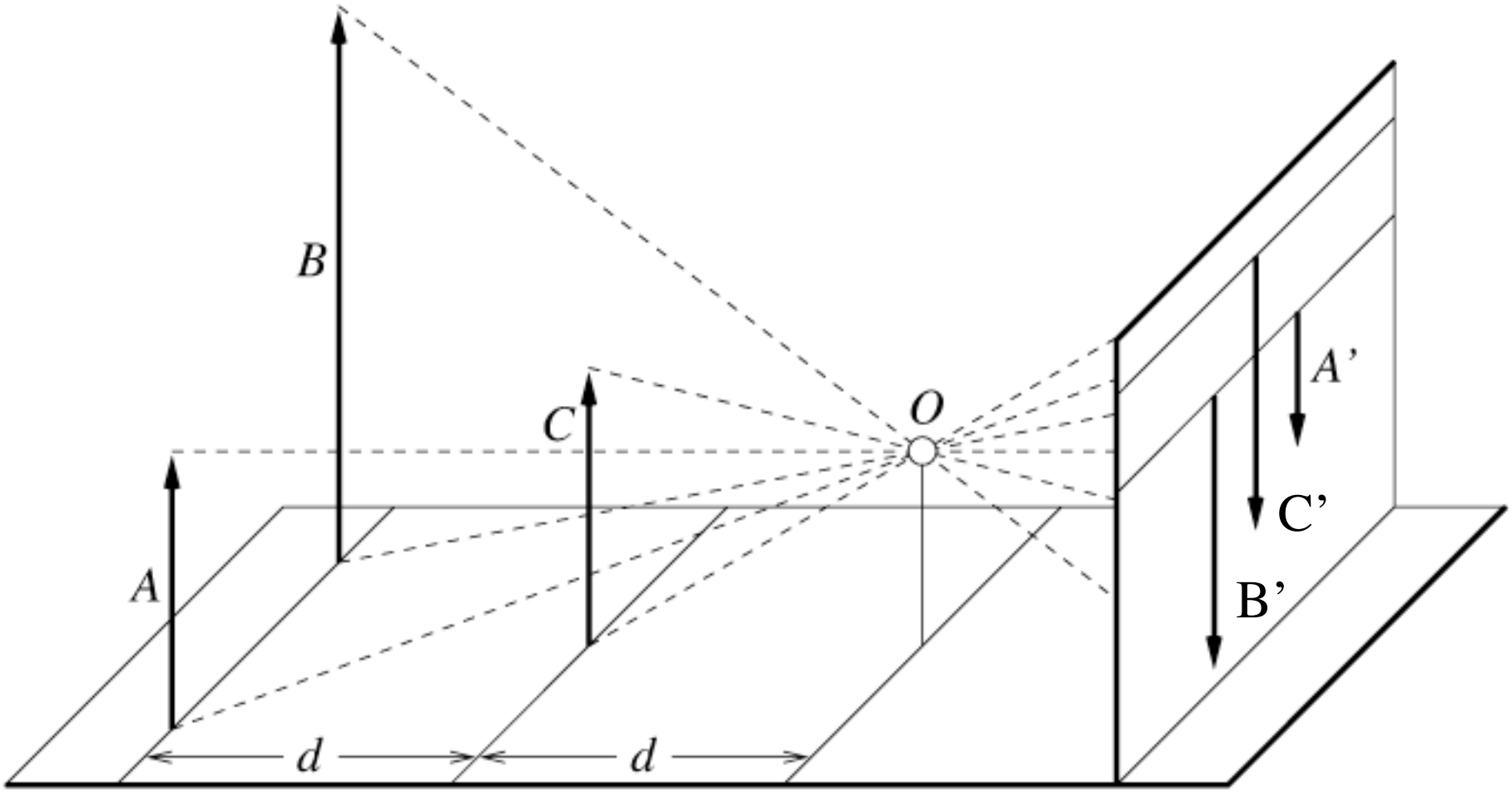

Camera configuration
in world coordinates
object
Transform
to standard
camera frame
Clip in 3D
Project to 2D
using camera
model
Render 2D
polygons
using
lighting info
Lighting information
Determine
what's in front

Pinhole cameras

- Abstract camera model--box with a small hole in it
- Pinhole cameras work for deriving algorithms--a real camera needs a lens

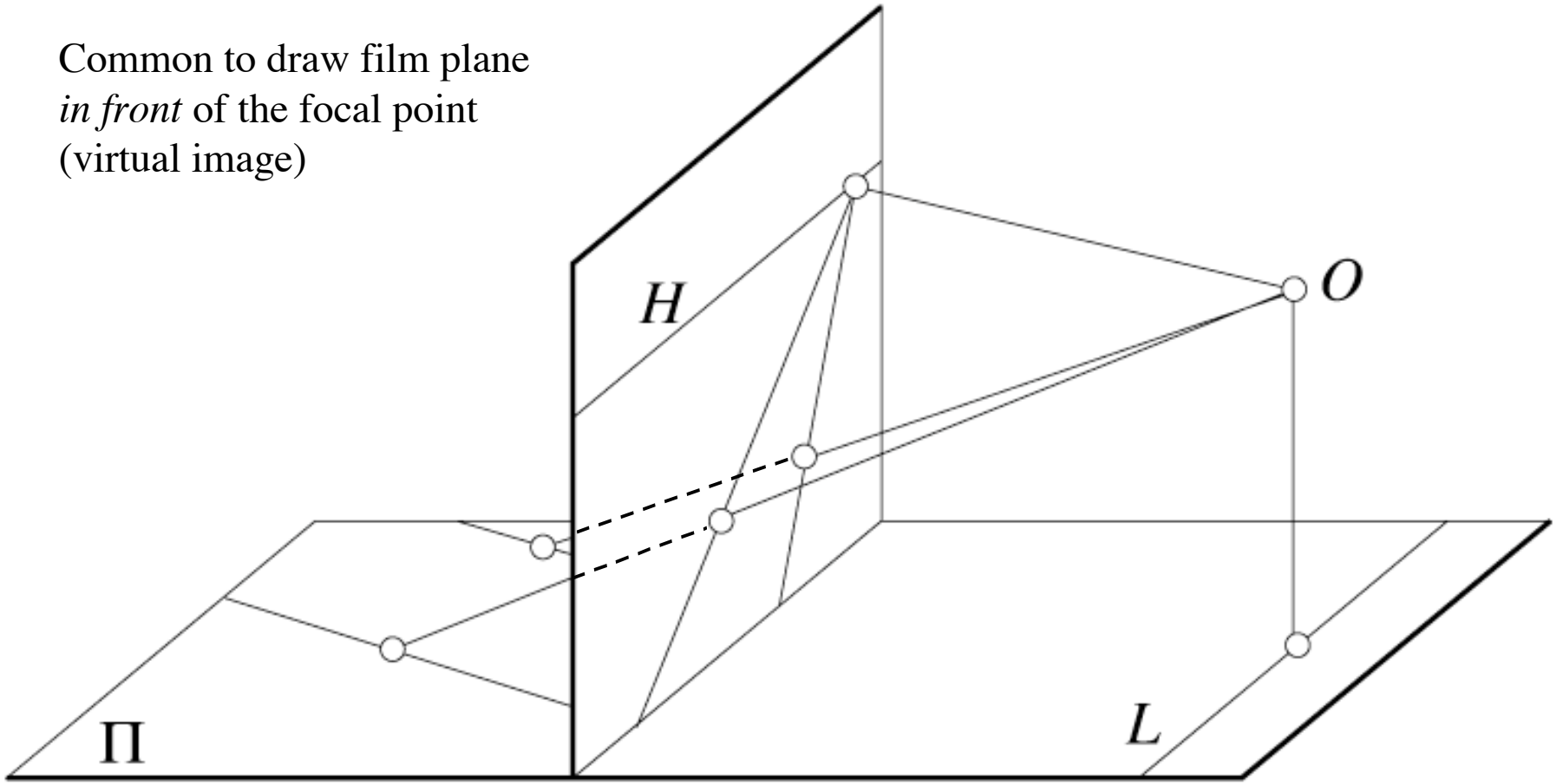


Distant objects are smaller



Parallel lines meet*

Common to draw film plane
in front of the focal point
(virtual image)



*Exceptions?

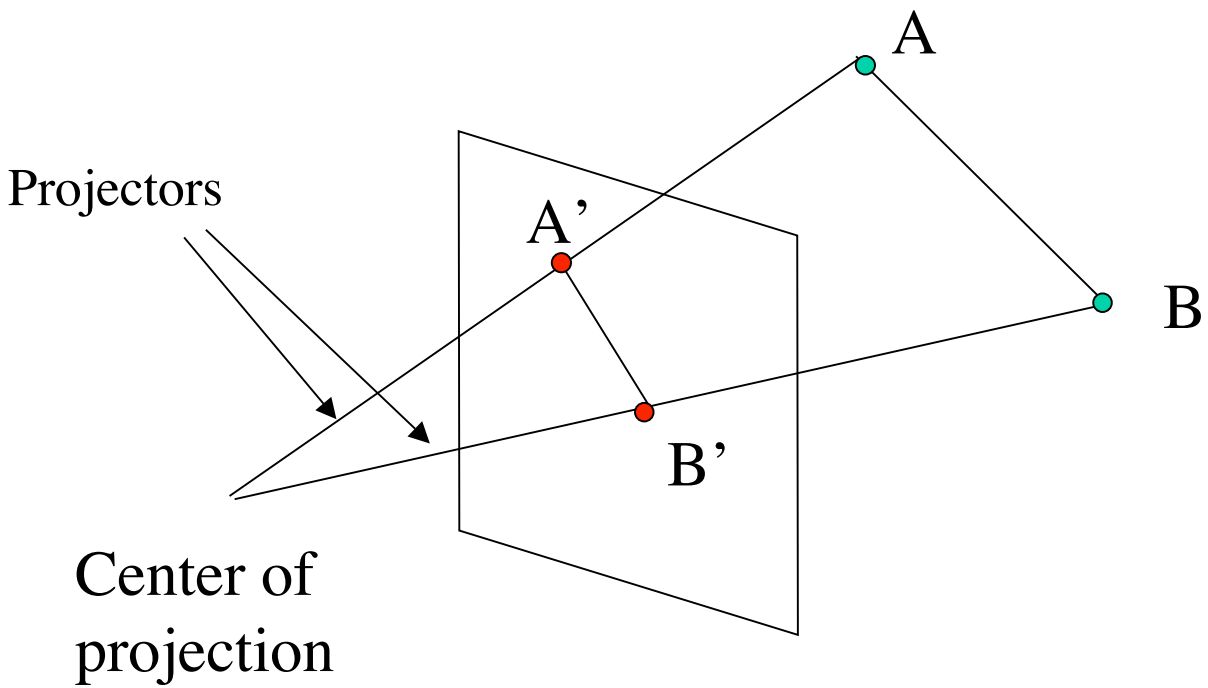
Vanishing points

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
  - Standard horizon is the horizon of the ground plane.
- One way to spot fake images

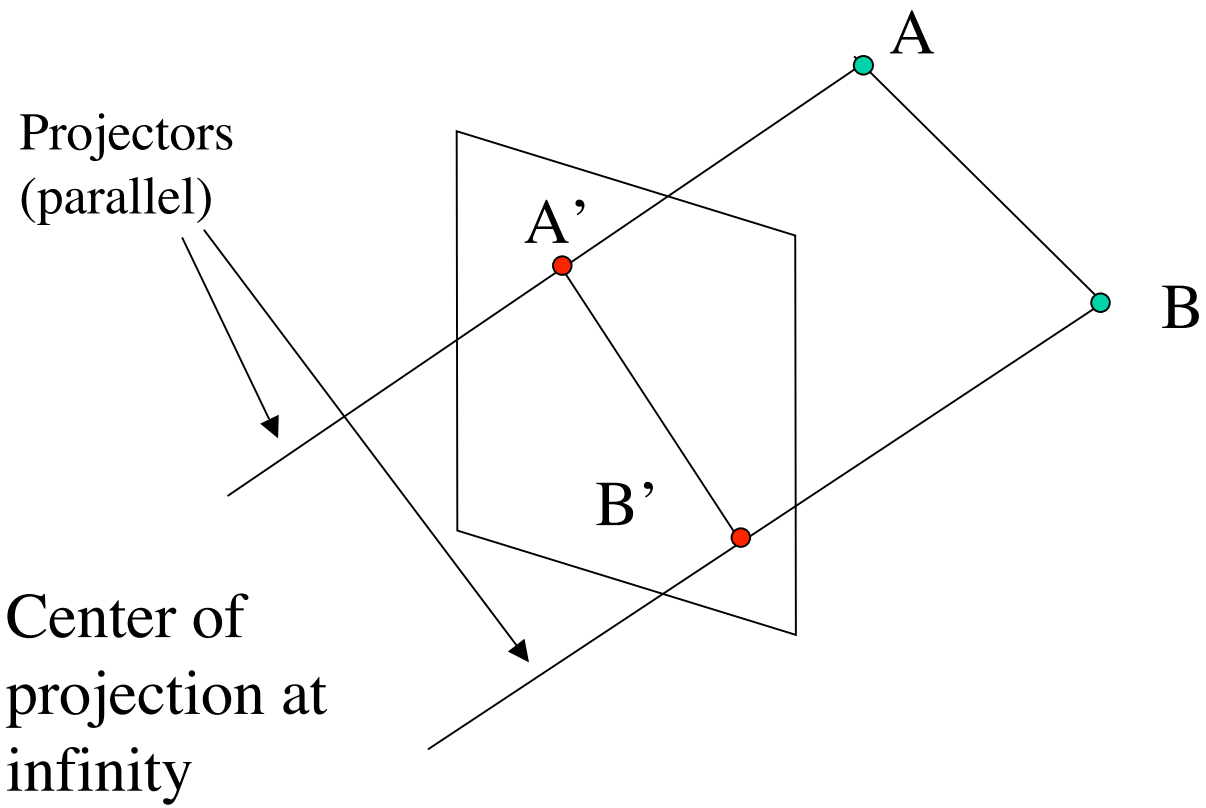
Projections

- Mathematical definition of a projection: PP=P
- (Doing it a second time has no effect).
- Generally rank deficient (non-invertable)--exception is P=I
- Transformation looses information (e.g., depth)
- Given a 2D image, there are many 3D worlds that could have lead to it.

Projections



Parallel Projection



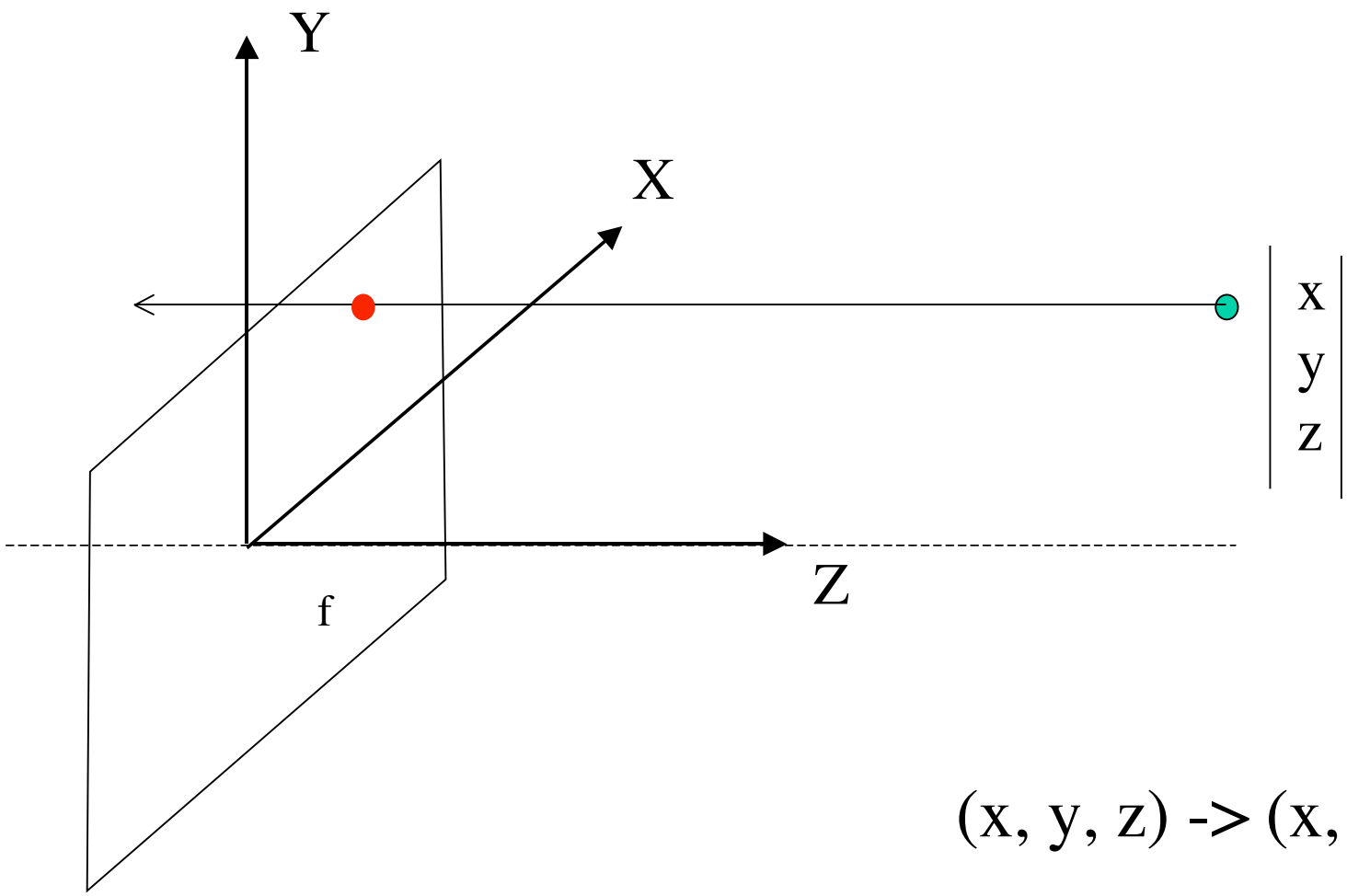
Parallel Projection

Parallel lines remain parallel, some 3D measurements can be made using 2D picture

Does not give realistic 3D view because eye is more like perspective projection.

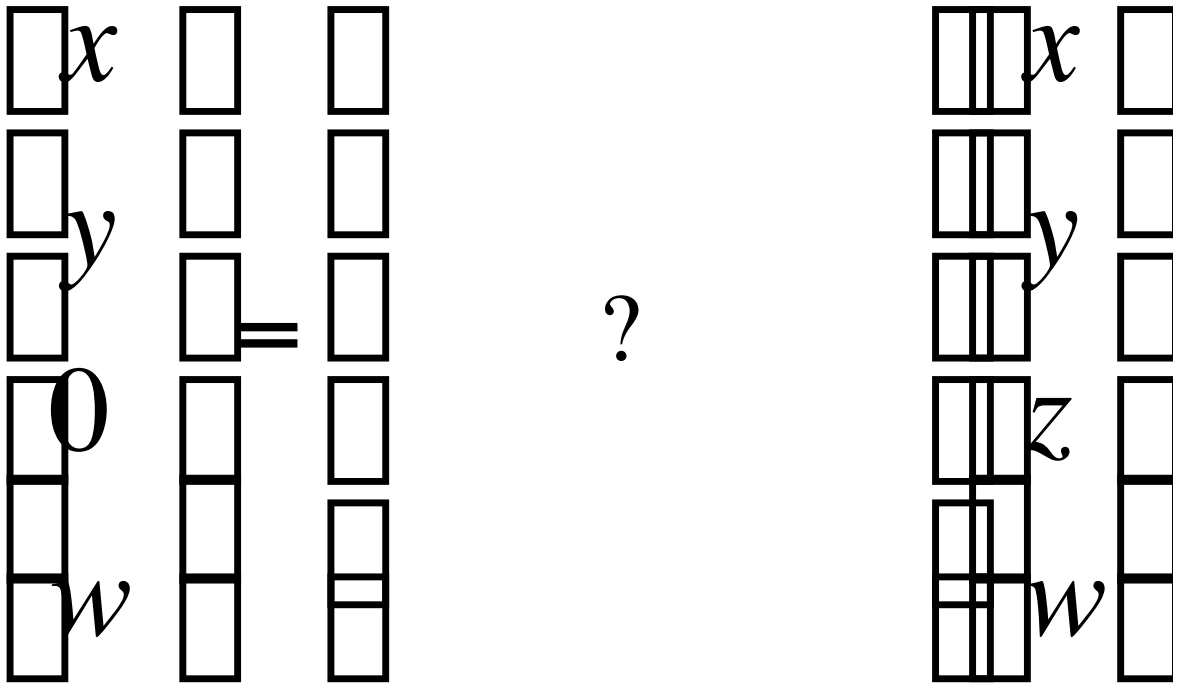
If projection plane is perpendicular to projectors the projection is orthographic (e.g., top view, side view, front view)

Orthographic example (onto z=0)

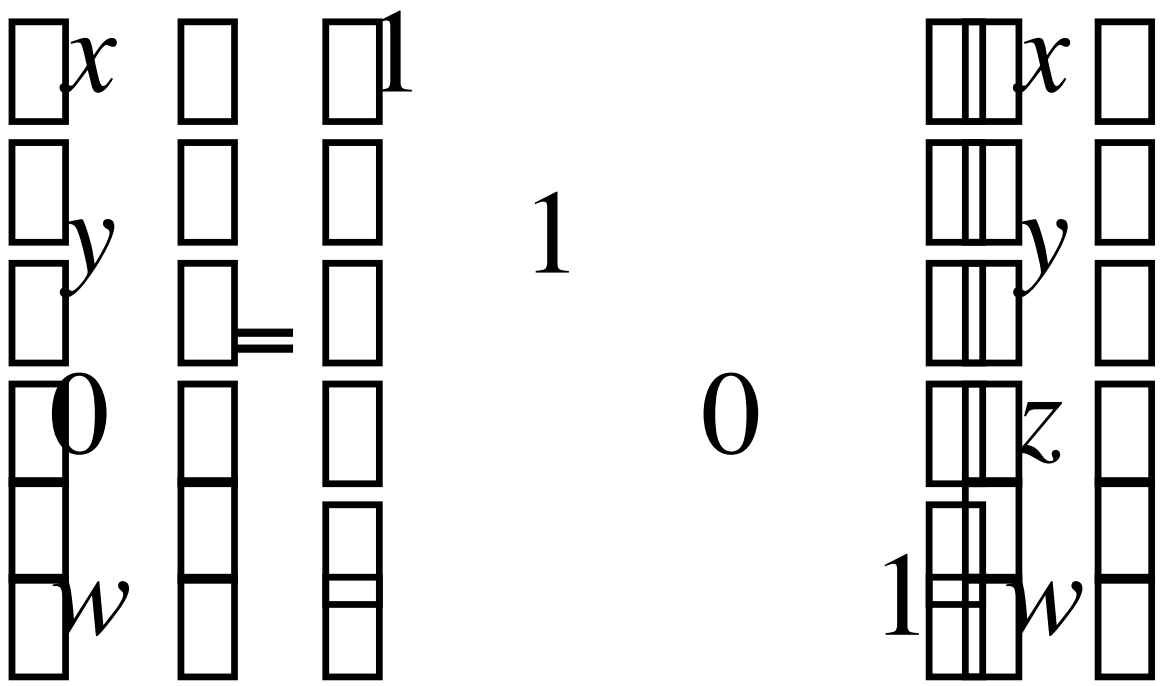


(x, y, z) -> (x, y, 0)

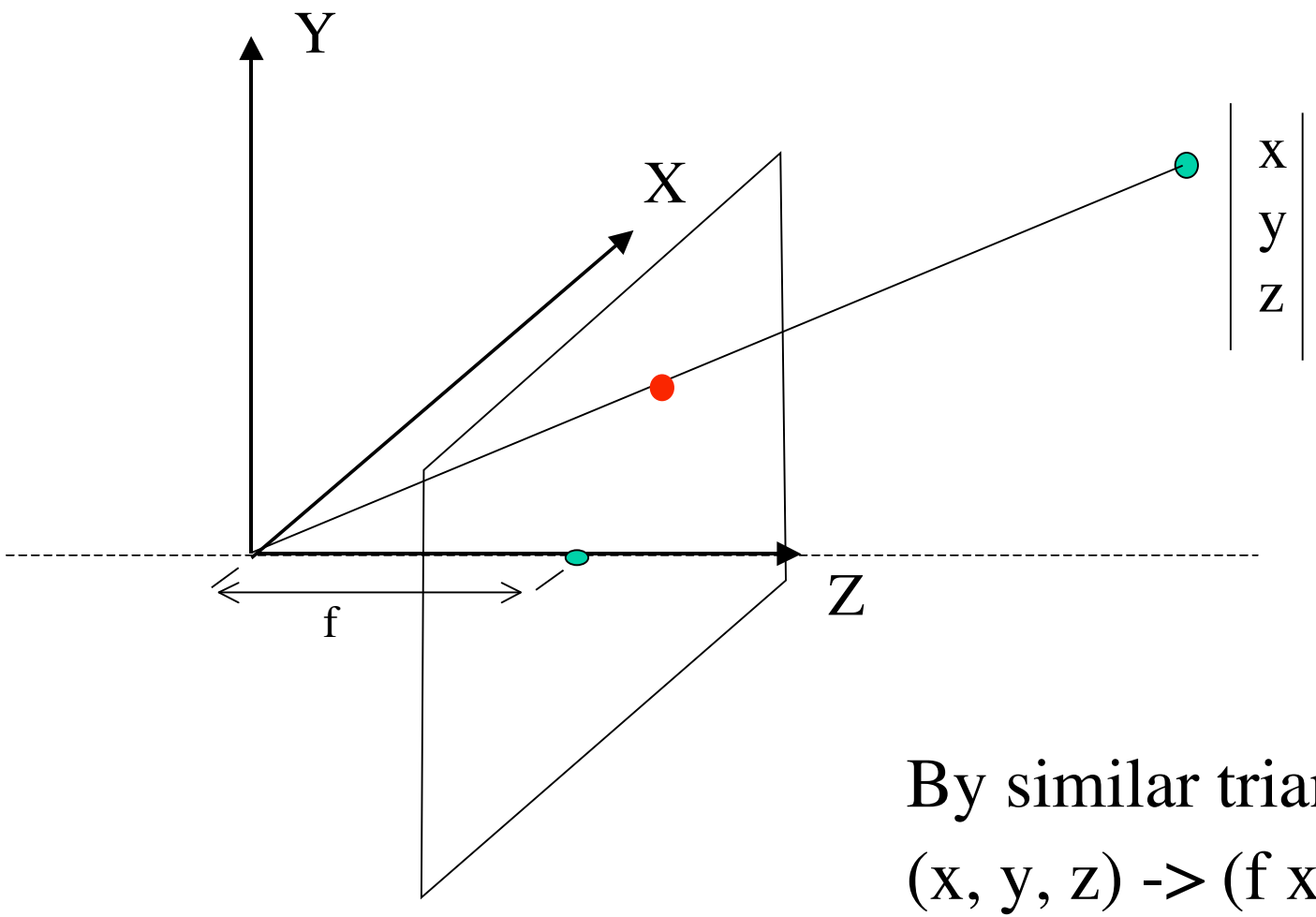
The camera matrix



The camera matrix



Perspective example (onto z=f)



By similar triangles,

(x, y, z) -> (f x/z, f y/z, f)

The equation of projection

- In homogeneous coordinates

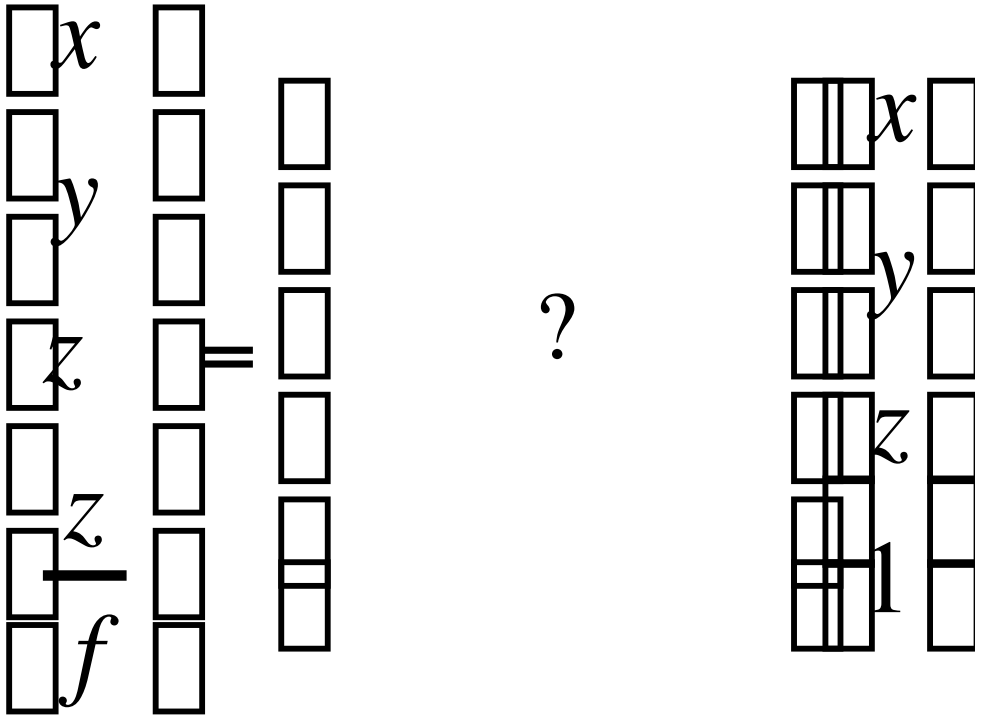
$$(x,\ y,\ z,\ 1) \quad \square \quad (f\frac{x}{z},\ f\frac{y}{z},\ f,\ 1)$$

- Equivalently

$$(x,\ y,\ z,\ 1) \quad \square \quad (x,\ y,\ z,\ \frac{z}{f})$$

- (Now H.C. are being used to store foreshortening)

The camera matrix



The camera matrix