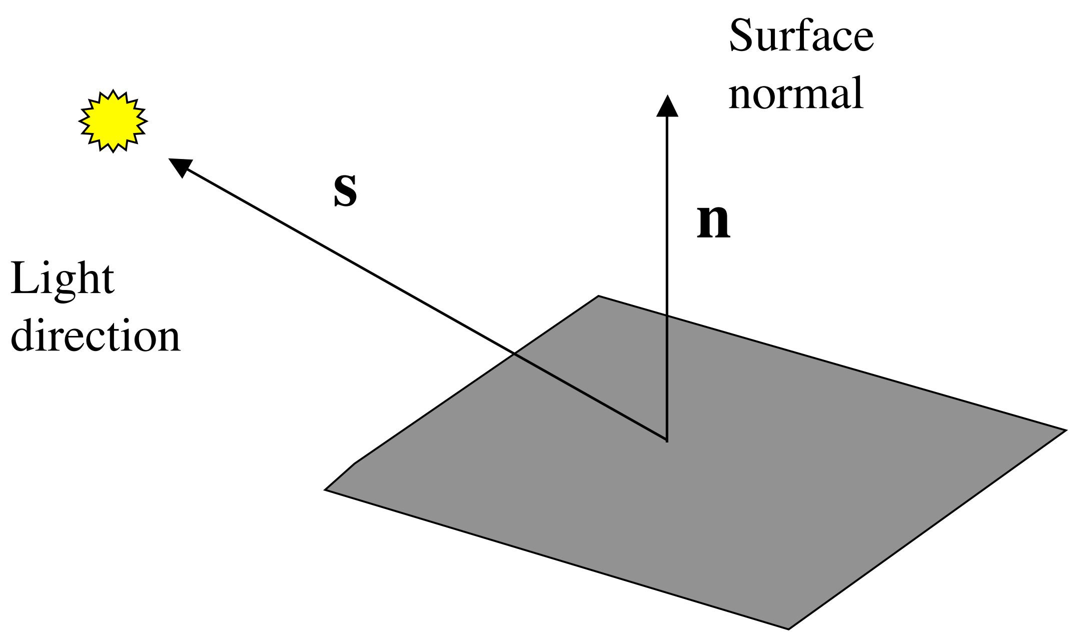
# Lambertian Reflection



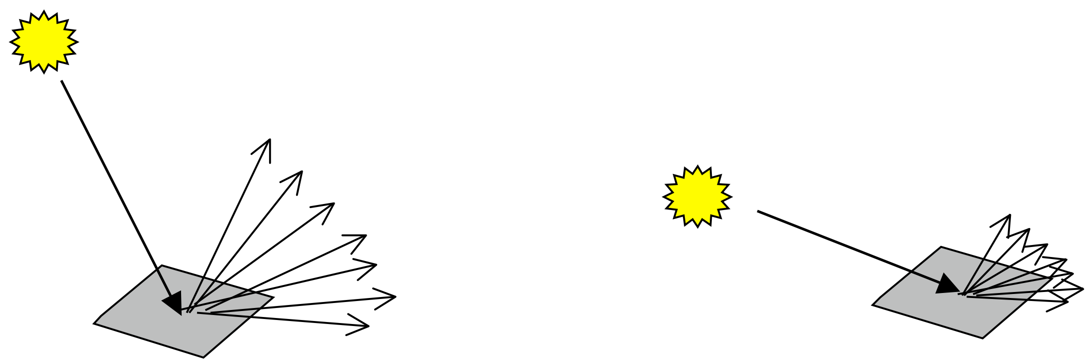

Why is brightness proportional to **n•s** ?

What about more than one light?

# Lambertian Reflection

Why is brightness proportional to **n•s** ?

Intuitive argument: The surface scatters light in all directions equally, but as the angle of the light becomes oblique, the amount of light per unit area is reduced (foreshortening) by a factor of the cosine of the angle.



# Lambertian Reflection

What about more lights?

If they are point sources, just add them up. Note that this means that extended sources can be approximated by multiple point sources and/or integration.

Applies to non-Lambertian surfaces also.

Special cases to be handled later: Very long thin source and large, planer source.

# Lambertian Reflection

Most the world is not Lambertian

Lambertian assumption failures

- Rough surfaces--important example--the moon is not Lambertian
- Dielectrics (plastics, many paints)
- Metallic surfaces
- Skin

# More General Reflection

- Many effects when light strikes a surface -- could be:
  - absorbed (could depend on incoming angle)
  - transmitted
  - reflected
  - scattered (in a variety of directions!)
- Typically assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e., they are not sources)
  - all the light leaving a point is due to that arriving at that point

# More General Reflection

- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- This is the ratio of what comes out to what came in
- What comes out <--> “radiance”
- What goes in <--> “irradiance”
- Both are characterized by two angles
- Thus BRDF is a function of four angles
- Technical discussion included in notes for interest only

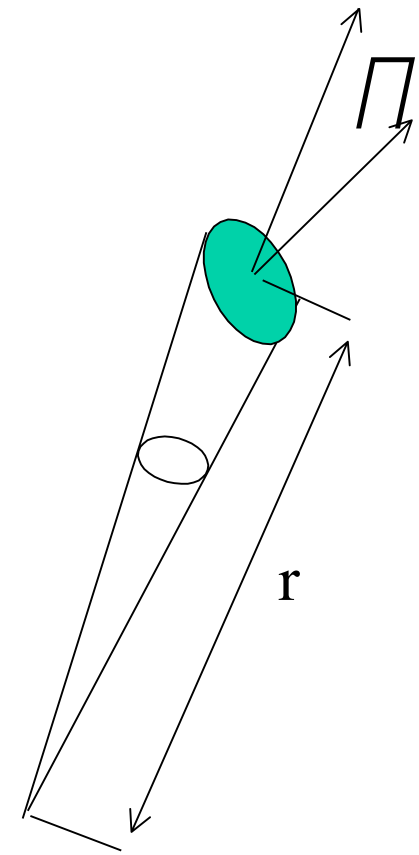
Optional

# Solid Angle

- Analogous to measuring angles radians
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA \cos\theta}{r^2}$$

- Units are steradians (sr)

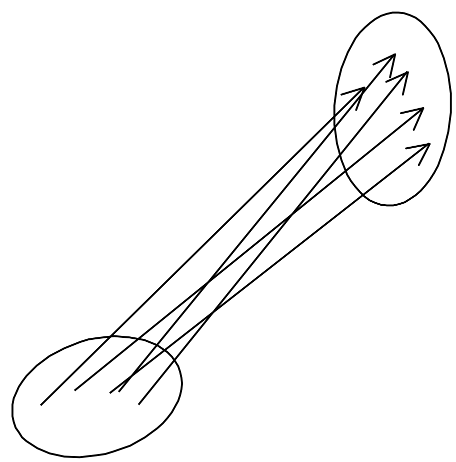


Optional

# Radiance

- Amount of light at a point in a particular direction
- Think of a small area either emitting or collecting the light
- Property is: *Radiant power per unit foreshortened area per unit solid angle*
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

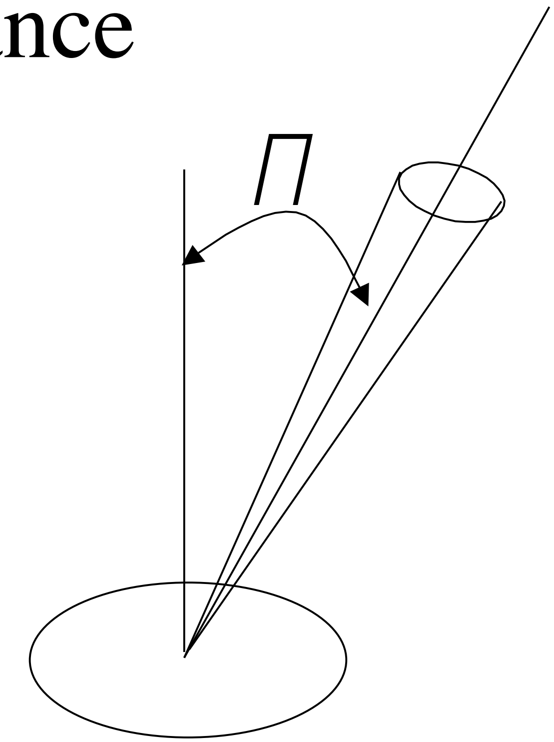
$$L(\underline{x}, \square, \square)$$



Optional

# Radiance

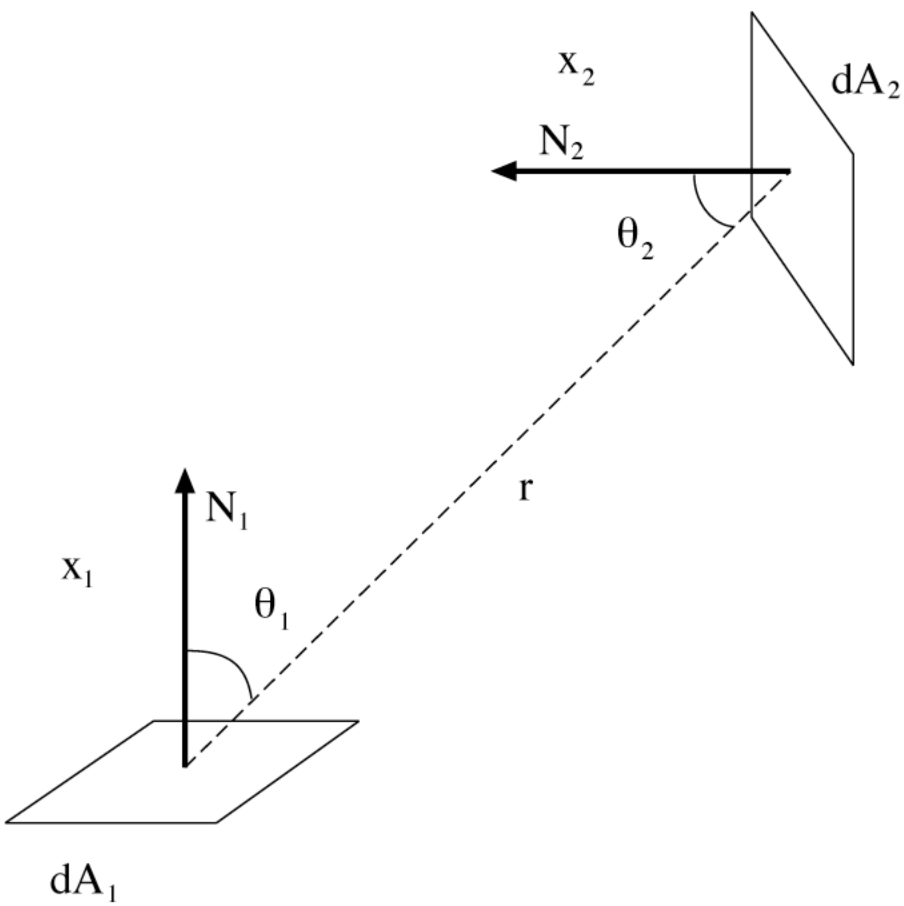
$$L(\underline{x},\theta,\phi) = \frac{d^2 P(\underline{x})}{d\omega \, dA \cos\theta}$$



- Crucial property: In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p

Optional

# Radiance is constant along straight lines




- Power 1->2, leaving 1:

$$L(\underline{x}_1,\theta,\phi)\left(dA_1\cos\theta_1\right)\left(\frac{dA_2\cos\theta_2}{r^2}\right)$$

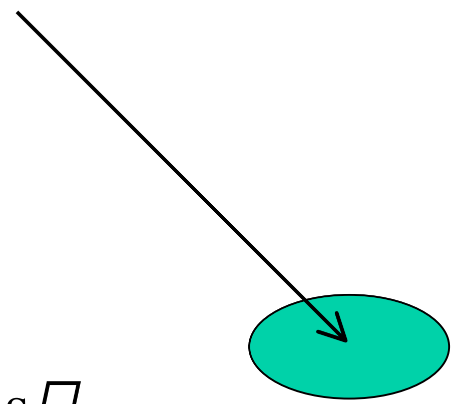
- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\theta,\phi)\left(dA_2\cos\theta_2\right)\left(\frac{dA_1\cos\theta_1}{r^2}\right)$$

Optional

# Irradiance

- Irradiance is the amount of light (power) falling on a surface per unit area.
- Units are watts/$m^2$
- Generally a function of direction



$$E(\underline{x},\theta,\phi) = \frac{\Delta P(\underline{x})}{\Delta A} = L(\underline{x},\theta,\phi)\ \Delta\omega\ \cos\theta$$

Optional

# Irradiance

- Note that irradiance is the incident power per unit area *not foreshortened*.
- A surface experiencing radiance L(x,θ,φ) coming from dω experiences irradiance

$$L(\underline{x},\theta,\phi)\cos\theta \quad d\omega$$

- Total power arriving at the surface is given by adding irradiance over all incoming angles.
- Total power is:

$$\int_{\Omega} L(\underline{x},\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi$$

For integration in polar coords

Optional

# BRDF (Bidirectional reflectance distribution function)

- The irradiance at a point due to a particular angle is

$$L_i\left(\underline{x},\theta_i,\phi_i\right)\cos\theta_i\, d\omega$$

- The energy leaving (reflected) in a particular outgoing direction is given by:

$$L_o\left(\underline{x},\theta_o,\phi_o\right)$$

- The BRDF is simply the ratio of the output to input.

$$\rho_{bd}\left(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i,\right) = \frac{L_o\left(\underline{x},\theta_o,\phi_o\right)}{L_i\left(\underline{x},\theta_i,\phi_i\right)\cos\theta_i\, d\omega}$$

Optional

# BRDF

- Units are inverse steradians ($sr^{-1}$)
- Symmetric in incoming and outgoing directions
- The “distribution” part of the name is a hint that we need to integrate the function to get some light.
- To compute the brightness of a surface viewed from a given direction, we add up the contributions from all the input directions:

$$\int_{\Omega} \rho_{bd}\left(\underline{x}, \theta_o, \phi_o, \theta_i, \phi_i,\right) L_i\left(\underline{x}, \theta_i, \phi_i\right) \cos \theta_i \, d\omega_i$$

Optional

# BRDF

- Note that what we have developed so far is mostly notation, definitions, and descriptions.
- Two approaches to obtaining BRDF's--measure and model.
- Measuring BRDF is painful (but there is some data available on-line (and more clever ways to collect the have been proposed).
- Developing physics based approximations for the BRDF for simple classes of surfaces is complicated but possible--this is still an active research area.
- Adding color to the BRDF is easy (one more variable). The full form has additional variables for fluorescence and polarization.

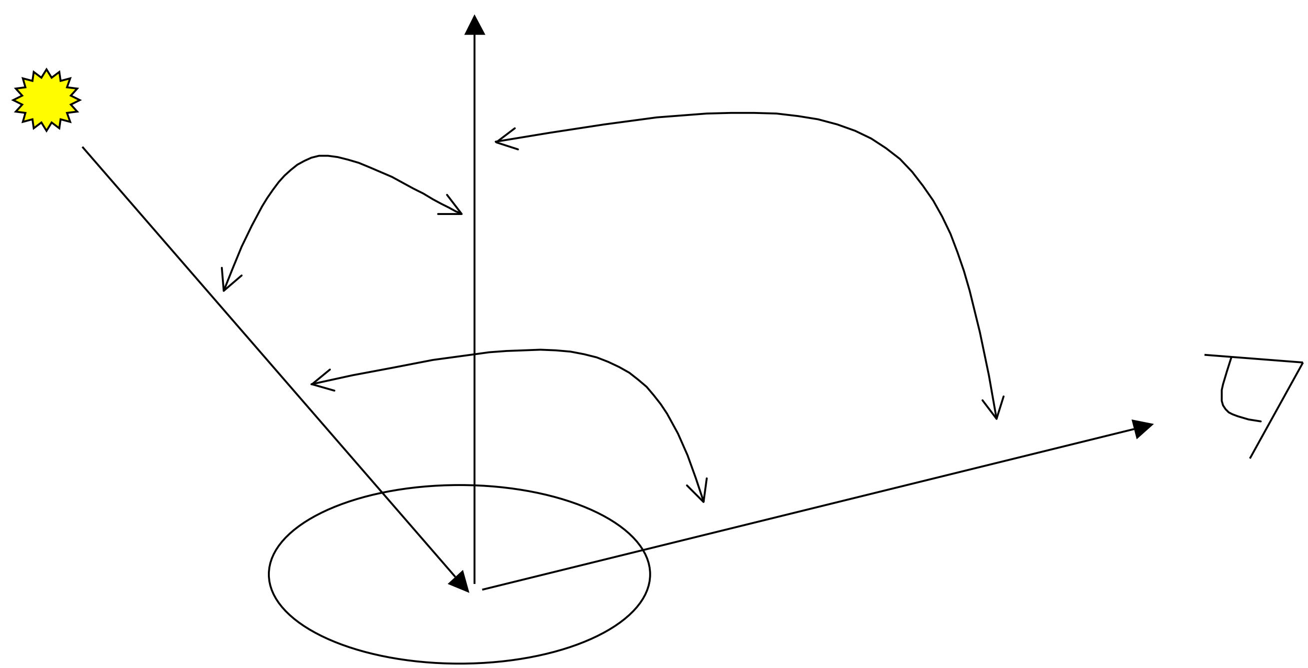
Optional

# BRDF

- So why do we care about the BRDF?
  - If you have it, then you can compute the effect of any illumination distribution--a photograph only tells you the effect of one illumination distribution
  - Useful abstraction--surface reflection can be quite complex!

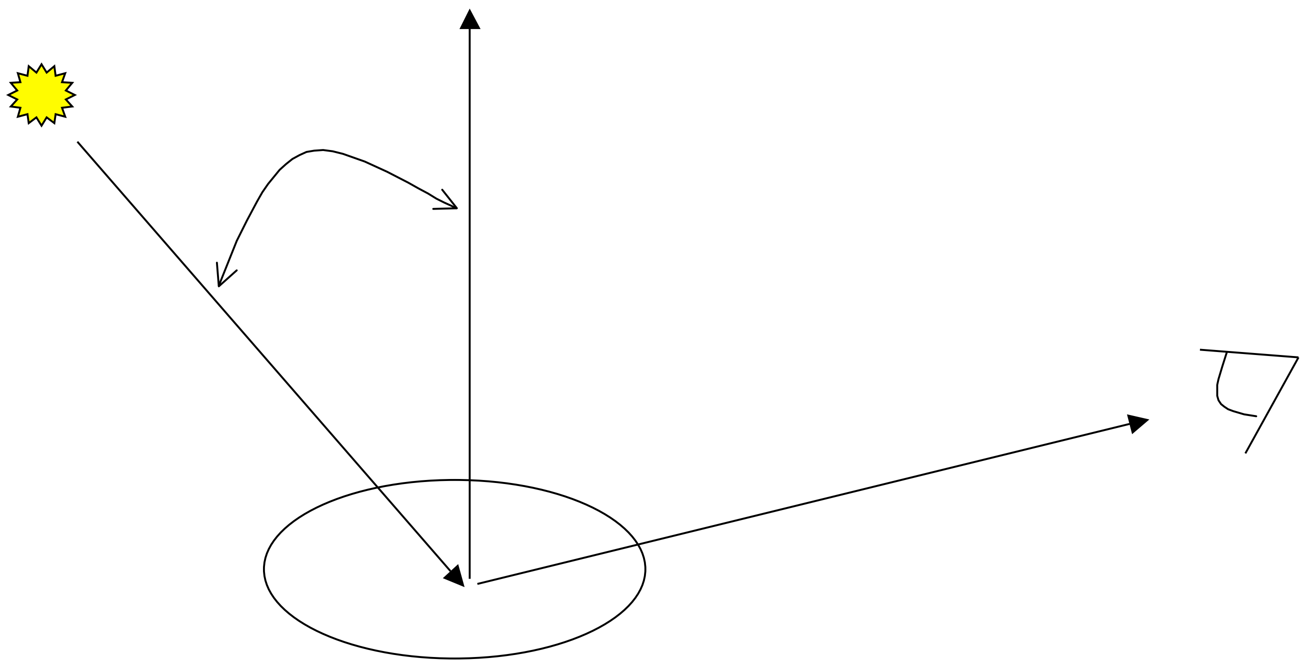
# Isotropic surfaces

The BRDF for many surfaces can be well approximated as a function of 3 variables (angles), not 4. In this case, turning the surface around the normal has no effect. The surface is said to be *isotropic*.



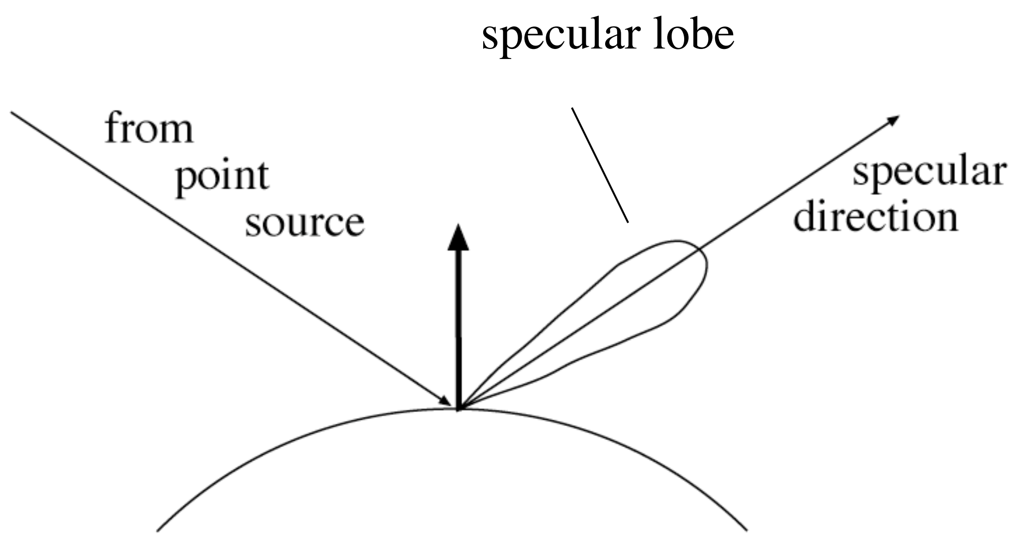
# Lambertian surfaces

- Even simpler case--the BRDF does not depend on the viewing (output) direction (e.g., Lambertian).



# Specular surfaces

- Another important class of surfaces is specular (somewhat mirror-like).
  - specular surfaces reflect a significant amount of energy in the specular (mirror) direction
  - a significant amount may also be reflected in a direction roughly in the mirror direction (specular lob)
  - typically there is a diffuse component as well
  - writing a BRDF approximation is possible, but beyond the scope of this course



# Lambertian surfaces and albedo

- For some surfaces, the percentage of arriving light that leaves is independent of direction in which it arrived
- Lambertian surfaces / ideal diffuse surfaces
  – cotton cloth, carpets, matte paper, matte paints, etc.
- Use radiosity as a unit to describe light leaving the surface (def'n next slide)
- Percentage of light leaving the surface is often called diffuse reflectance, or *albedo* for a Lambertian surface.

# Radiosity

- Again, in many situations, we do not need angle coordinates at all
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface ($Wm^{-2}$)
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \theta, \phi) \cos\theta \, d\omega$$

Optional

# Sources and Exitance

- Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- A source will have both
  - radiosity, because it reflects
  - exitance, because it emits

Radiosity leaving = Exitance + Radiosity due to incoming light