Ruled surfaces -1

- Popular, because it’s easy to build a curved surface out of straight segments - e.g. pavilions, etc.
- Take two space curves, and join corresponding points - same s - with line segment.
- Even if space curves are lines, the surface is usually curved.

\[
(x(s, t), y(s, t), z(s, t)) = (1 - t)(x_1(s), y_1(s), z_1(s)) + t(x_2(s), y_2(s), z_2(s))
\]
Ruled Surfaces - 2

Easy to explain, hard to draw!
Normals

- Normal is cross product of tangent in t direction and s direction.

\[
\begin{align*}
\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t} & \quad \frac{\partial x}{\partial s}, \frac{\partial y}{\partial s}, \frac{\partial z}{\partial t} \\
\end{align*}
\]

- Cylinder: normal is cross-product of curve tangent and direction vector

- Surface of revolution: take curve normal and spin round axis
Rendering

- Cylinders: small steps along curve, straight segments along t generate polygons; exact normal is known.
Rendering

• Cone: small steps in $s$ generate straight edges, join with vertex to get triangles, normals known exactly except at vertex.
Rendering

- Surface of revolution: small steps in s generate strips, small steps in t along the strip generate edges; join up to form triangles. Normals known exactly.
• Ruled surface: steps in $s$ generate polygons, join opposite sides to make triangles - otherwise “non planar polygons” result. Normals known exactly.
Specifying Curves from Points

- Want to modulate curves via “control” points.
- Strategy depends on application. Possibilities:
  - Force a polynomial of degree N-1 through N points (Lagrange interpolate)
  - Specify a combination of “anchor” points and derivatives (Hermite interpolate)
  - Other “blends” (Bezier, B-splines)--more useful than Lagrange/Hermite
Specifying Curves from Points-II

- **Issues:**
  - Continuity of curve and derivatives (geometric, parametric)
  - Local versus global control
  - Polynomials versus other forms
  - Higher polynomial degree versus stitching lower order polynomials together
  - Polynomial degree (usually 3--fewer is not flexible enough, and higher gives hard to control wiggles).
Parametric vs Geometric Continuity

- **Parametric continuity:**
  - The curve and derivatives up to $k$ are continuous *as a function of parameter value*
  - $C^k$
  - Useful for (for example) animation

- **Geometric continuity**
  - curve, derivatives up to $k$’th are the same for equivalent parameter values
  - $D^k$
  - i.e. there exists a reparametrisation that would achieve parametric continuity
  - Useful, because we often don’t require parametric continuity,
Lagrange Interpolate

- Construct a parametric curve that passes through (interpolates) a set of points.
- Lagrange interpolate:
  - give parameter values associated with each point
  - use Lagrange polynomials (one at the relevant point, zero at all others) to construct curve
  - curve is:

\[ \sum_{i=1}^{n} p_i \delta_i^{(l)}(t) \]

- Degree is (#pts-1)
  - e.g. line through two points
  - quadratic through three.
- Functions phi are known as “blending functions”
- Standard special case: 4 points, 4 degree 3 functions.
Hermite curves

- **Hermite interpolate**
  - Curve passes through specified points and has specified derivatives at those points.
  - Curve is:

\[
p_i p_i^{(h)}(t) + v_i v_i^{(hd)}(t)
\]

- Use Hermite polynomials to construct curve
  - one at some parameter value and zero at others or
  - derivative one at some parameter value, and zero at others

- Standard special case: 2 points, 2 derivatives at those points, a total of 4 functions of degree 3.