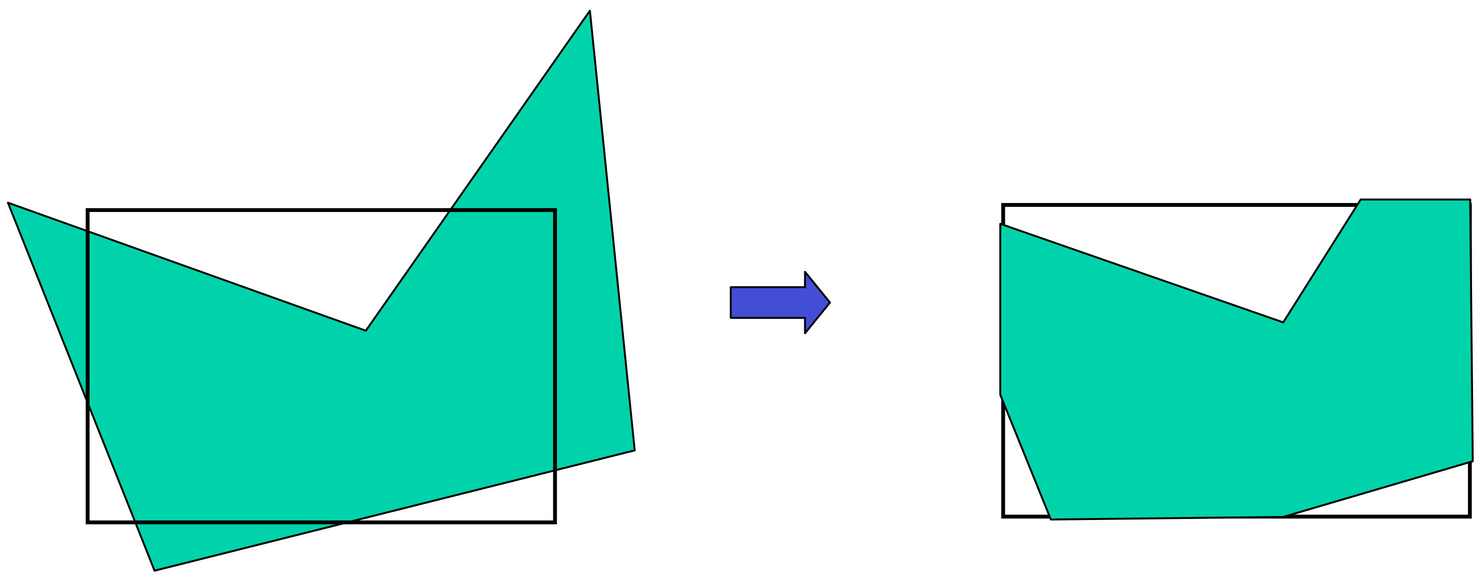
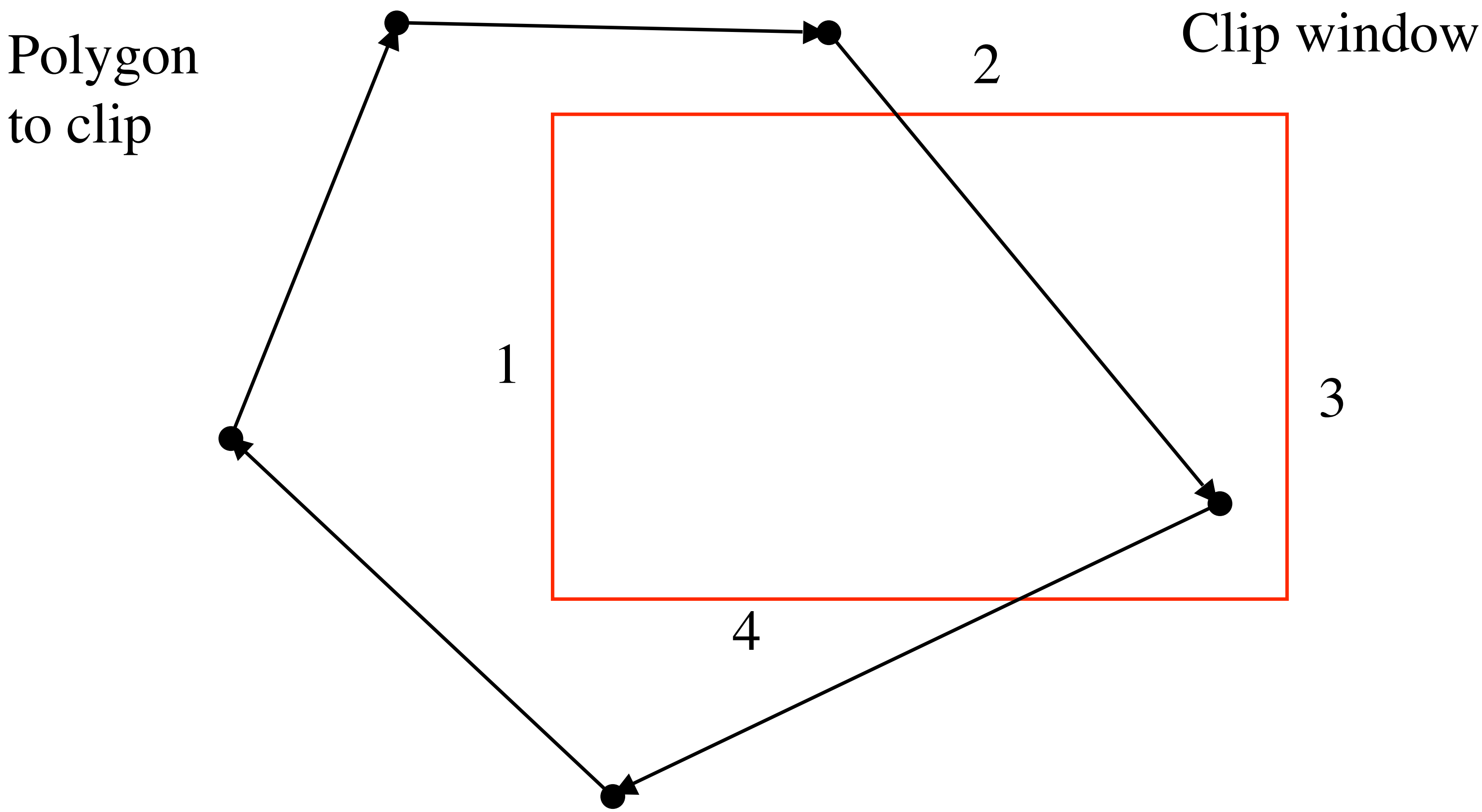
Polygon clip (against convex polygon)



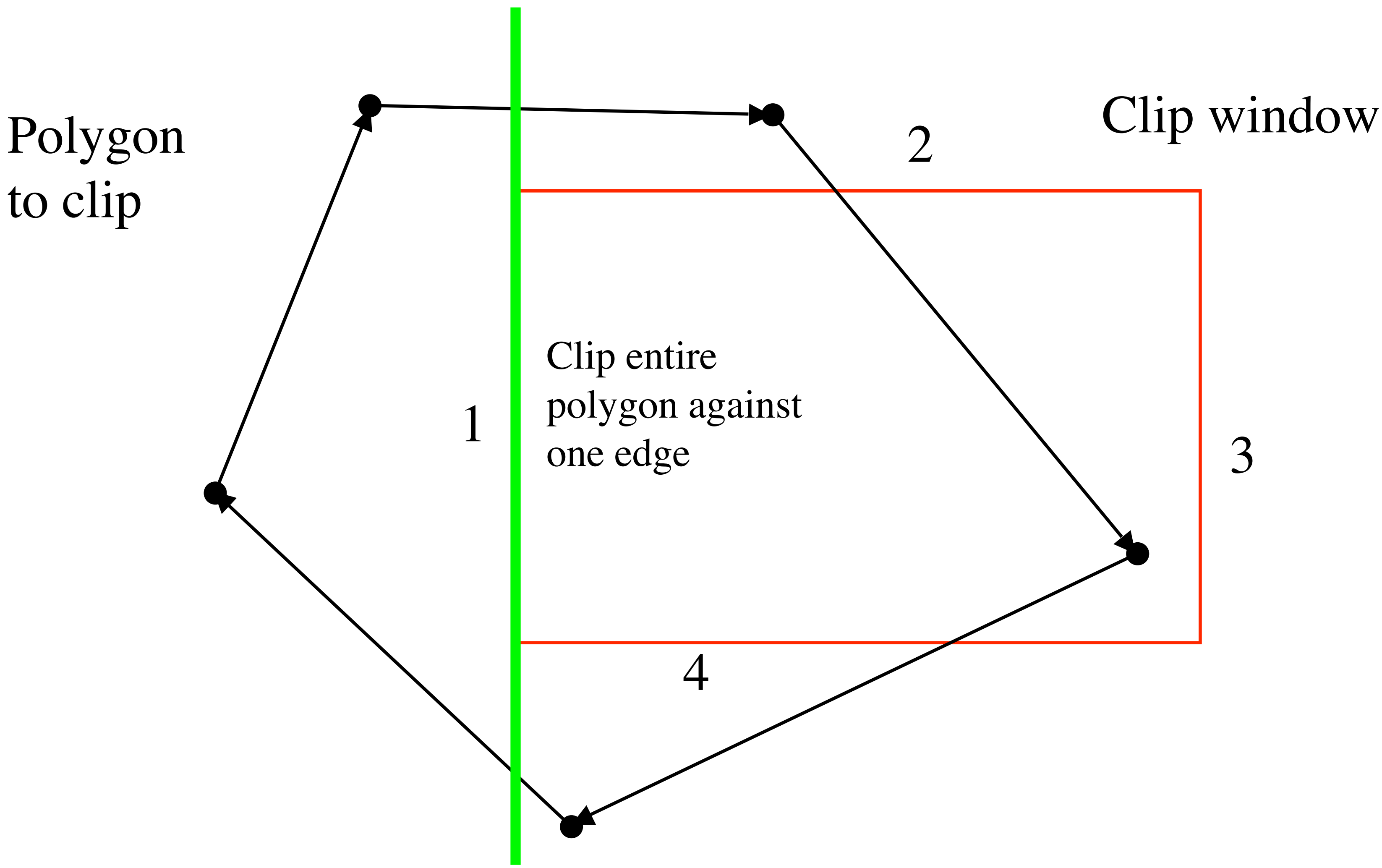
Sutherland-Hodgeman polygon clip

- Recall: polygon is convex if any line joining two points inside the polygon, also lies inside the polygon; implies that a point is inside if it is on the right side of each edge.
- Clipping each edge of a given polygon doesn't make sense - how do we reassemble the pieces? We want to arrange doing so on the fly.
- Clipping the polygon against each edge of the clip window in *sequence* works if the clip window is *convex*.
- (Note similarity to Sutherland-Cohen line clipping)

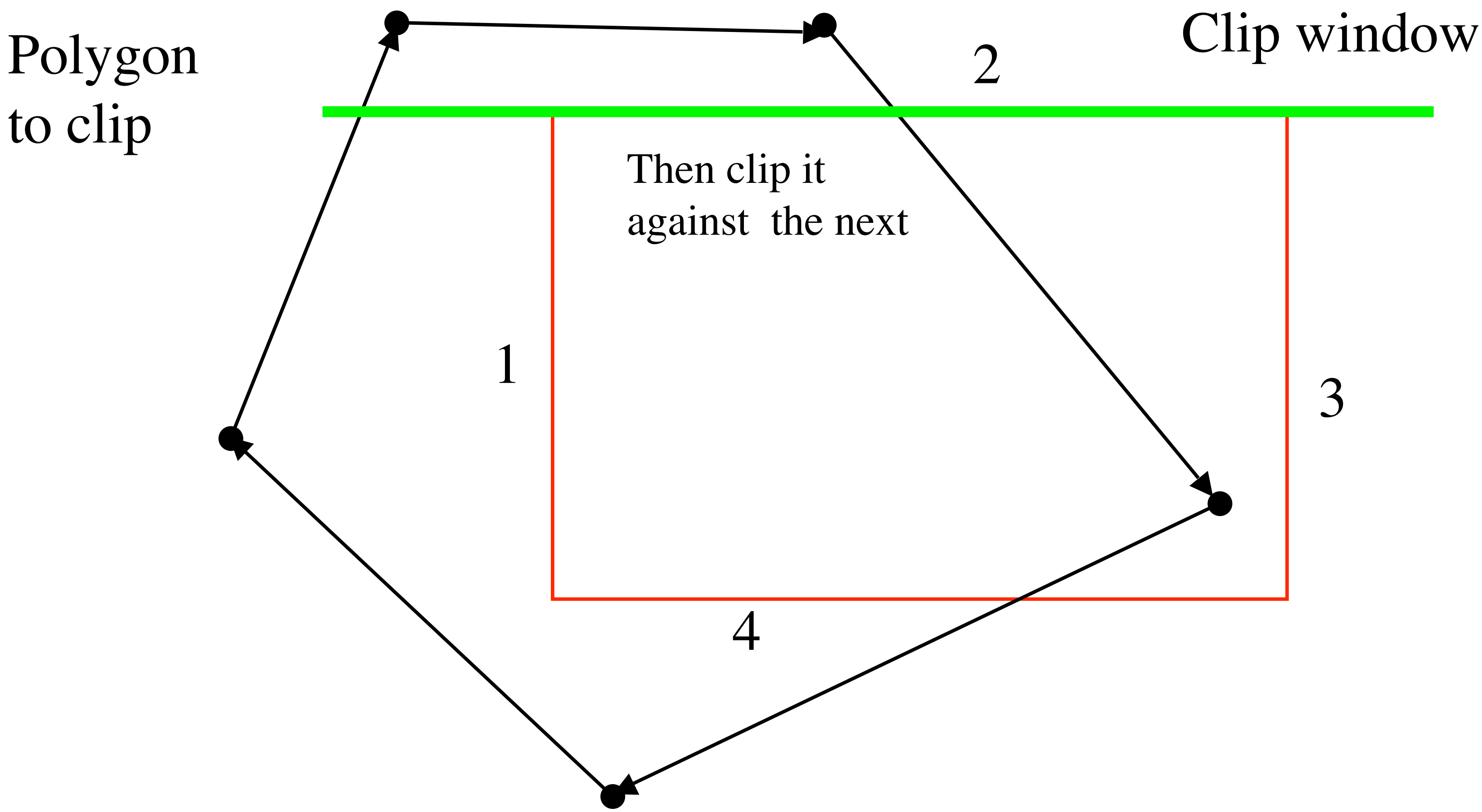
Sutherland-Hodgeman polygon clip



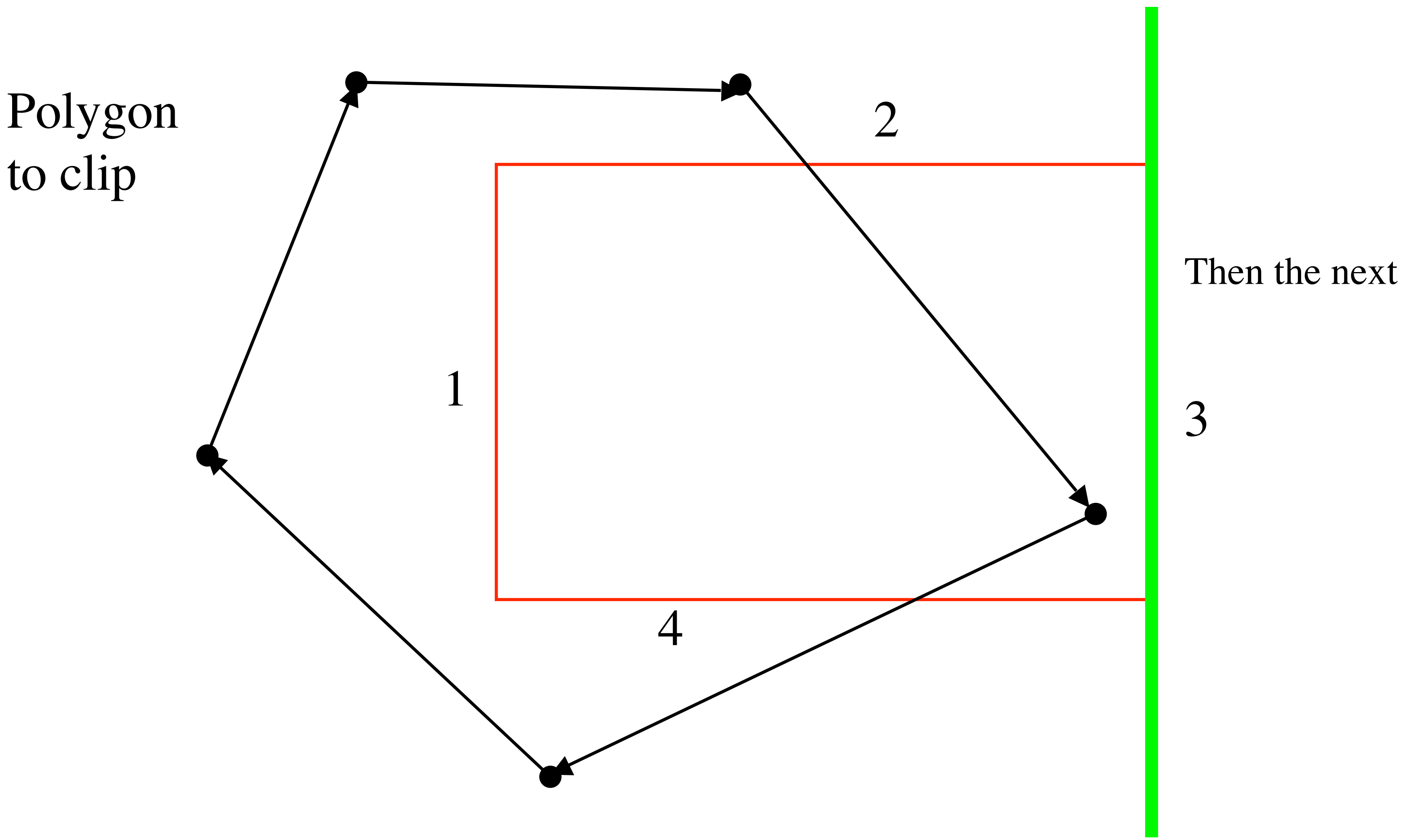
Sutherland-Hodgeman polygon clip



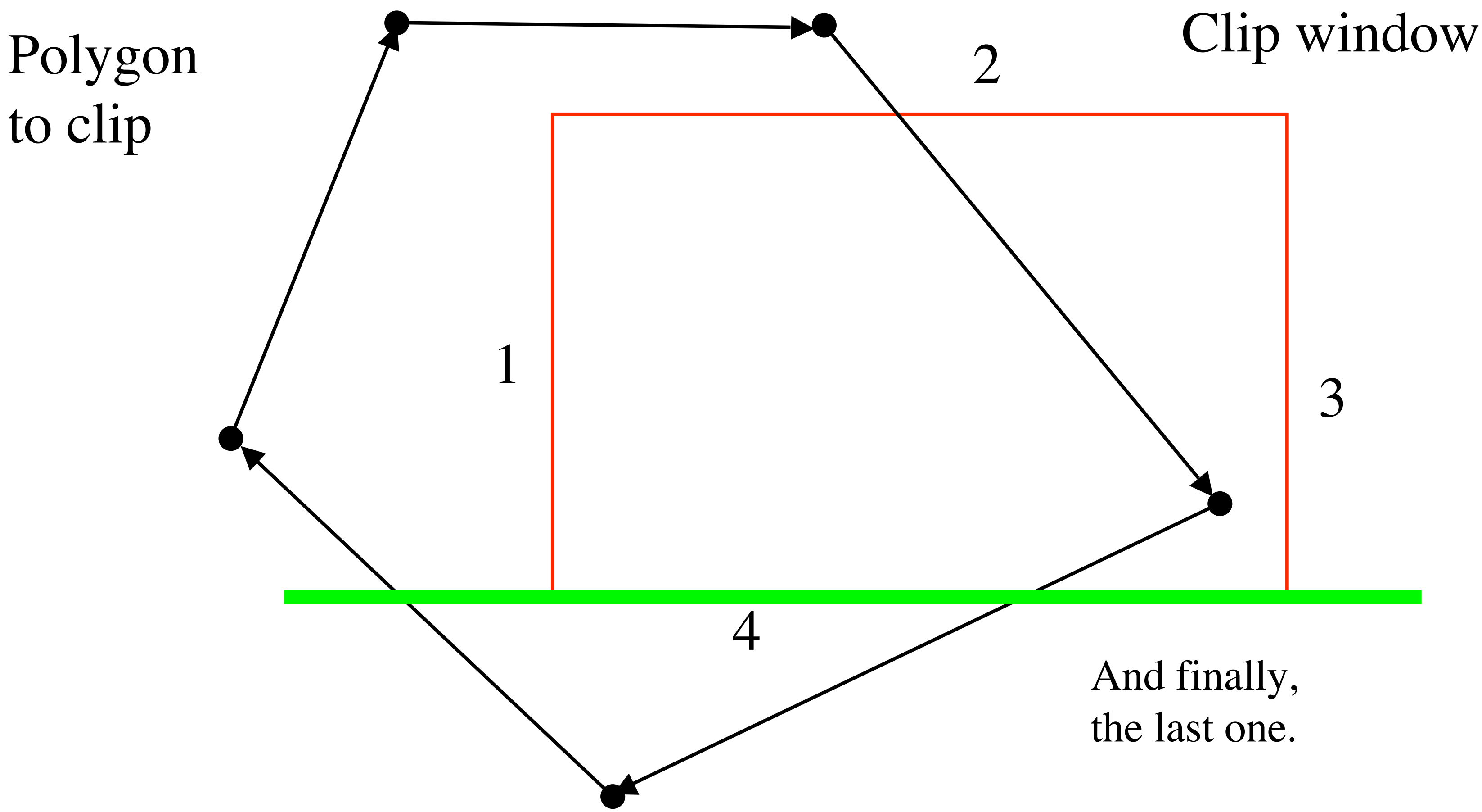
Sutherland-Hodgeman polygon clip



Sutherland-Hodgeman polygon clip



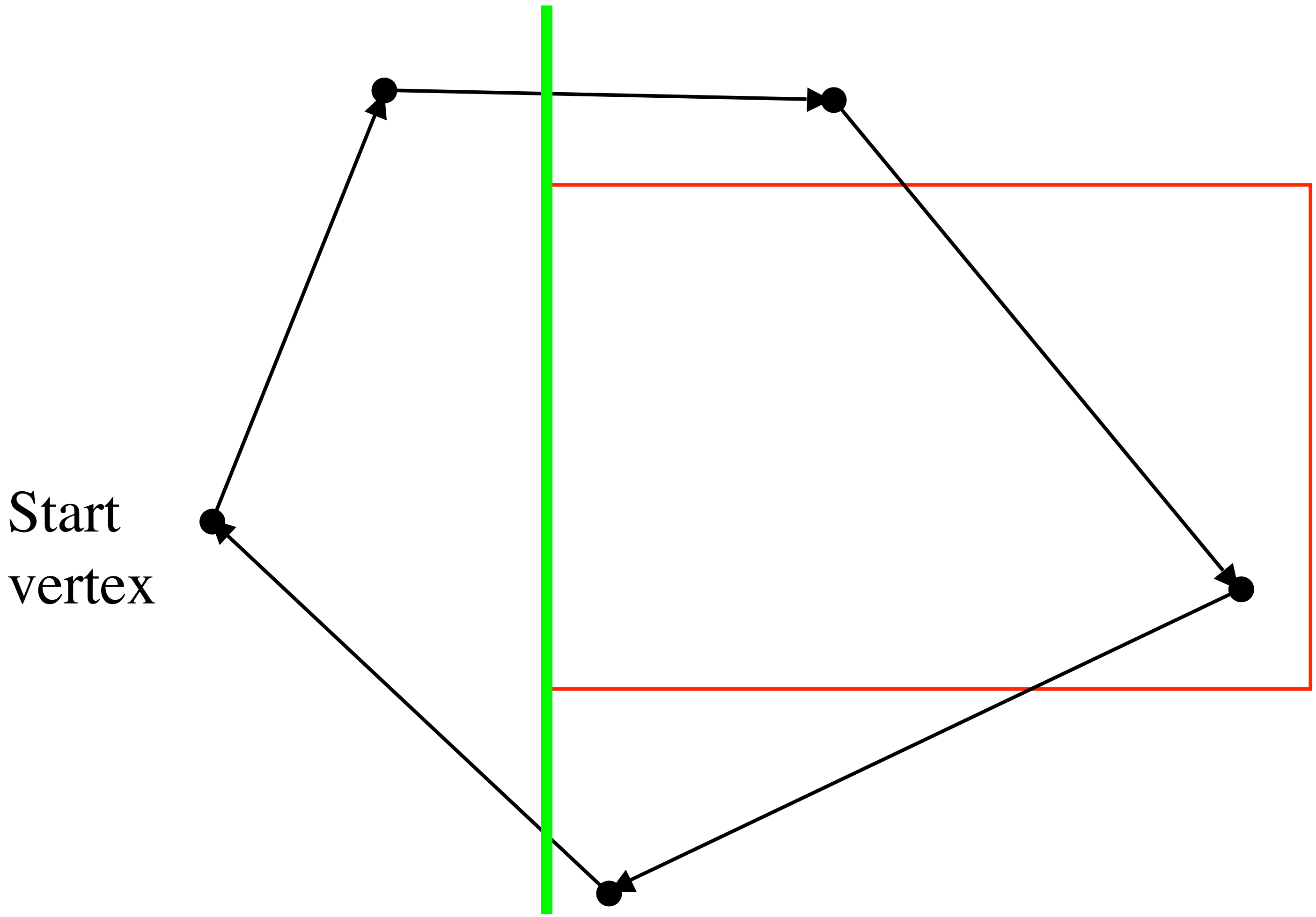
Sutherland-Hodgeman polygon clip



Clipping against current clip edge

- Polygon is a list of vertices
- Think of process as rewriting polygon, vertex by vertex
- Check start vertex
  - in - emit it
  - out - ignore it
- Walk along vertices and for each edge consider four cases and apply corresponding action.

- Four cases:
  - polygon edge crosses clip edge going from out to in
    - emit crossing, next vertex
  - polygon edge crosses clip edge going from in to out
    - emit crossing
  - polygon edge goes from out to out
    - emit nothing
  - polygon edge goes from in to in
    - emit next vertex

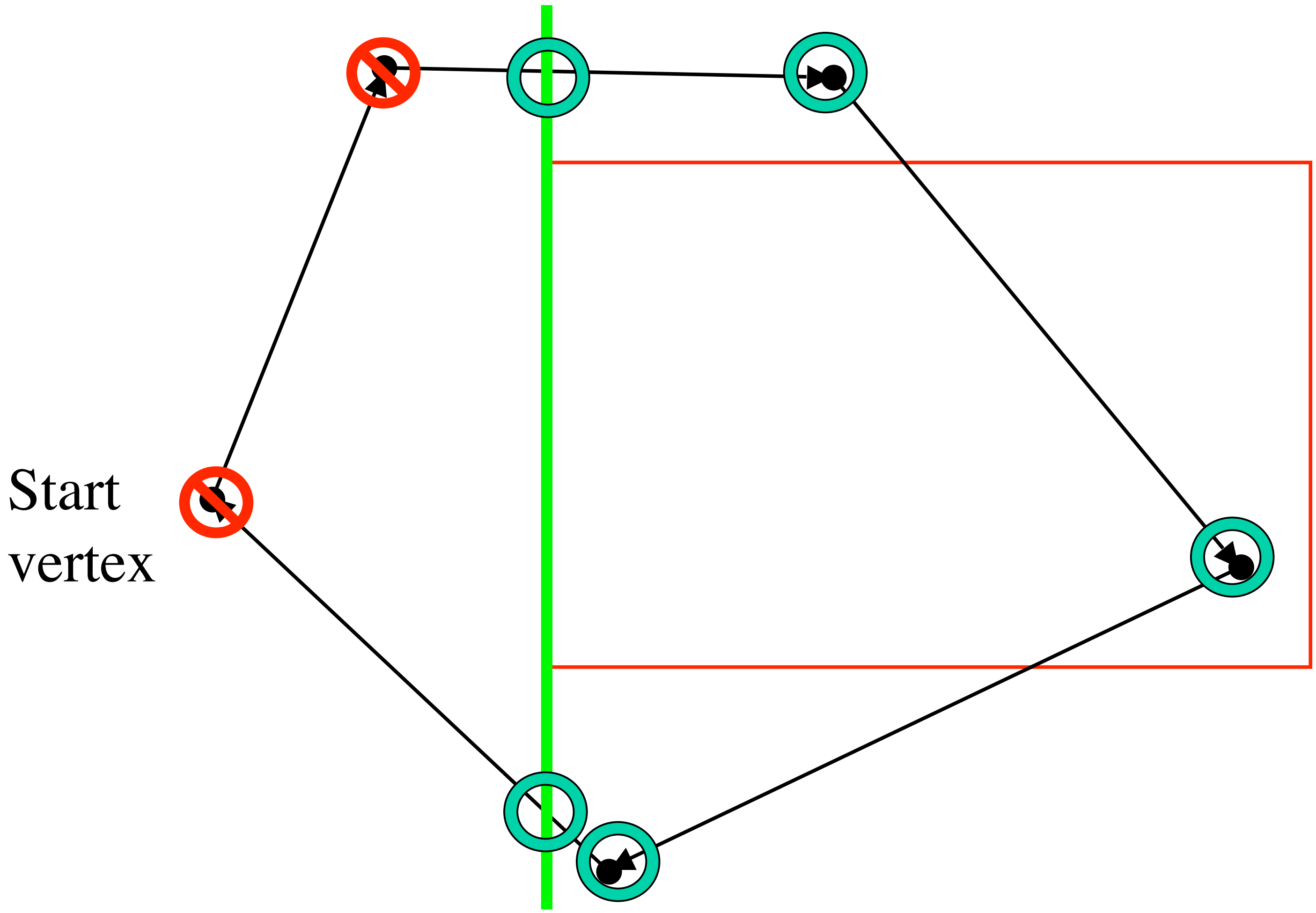


polygon edge crosses clip edge going from out to in ==> emit crossing, next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

polygon edge goes from in to in ==> emit next vertex



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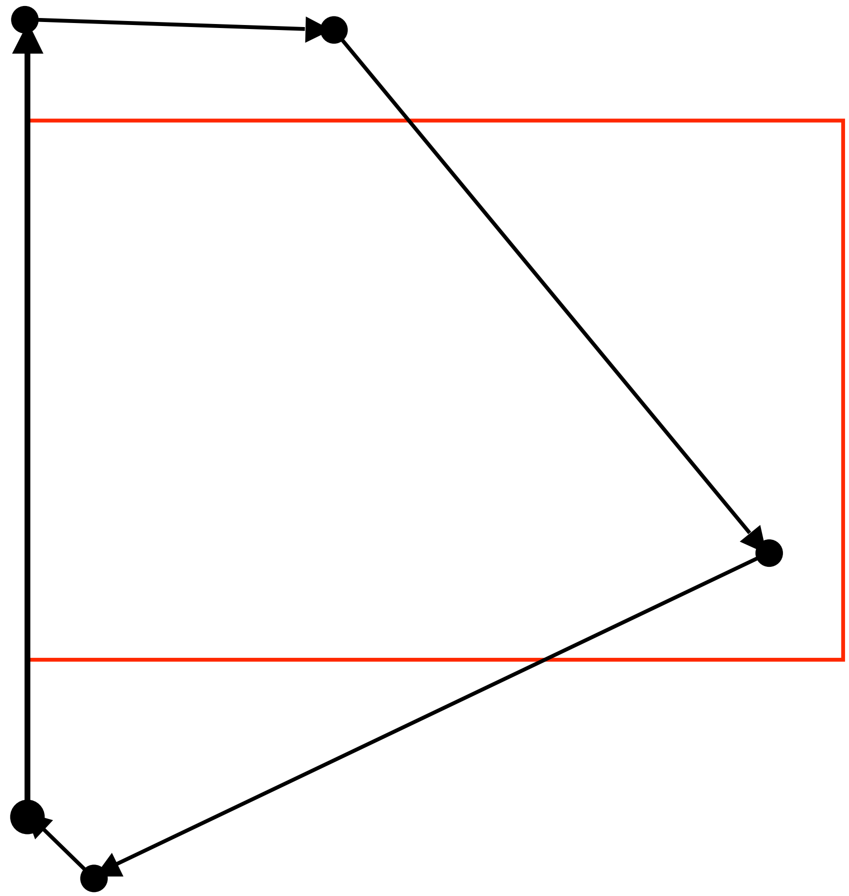
polygon edge goes from in to in ==> emit next vertex

Now have



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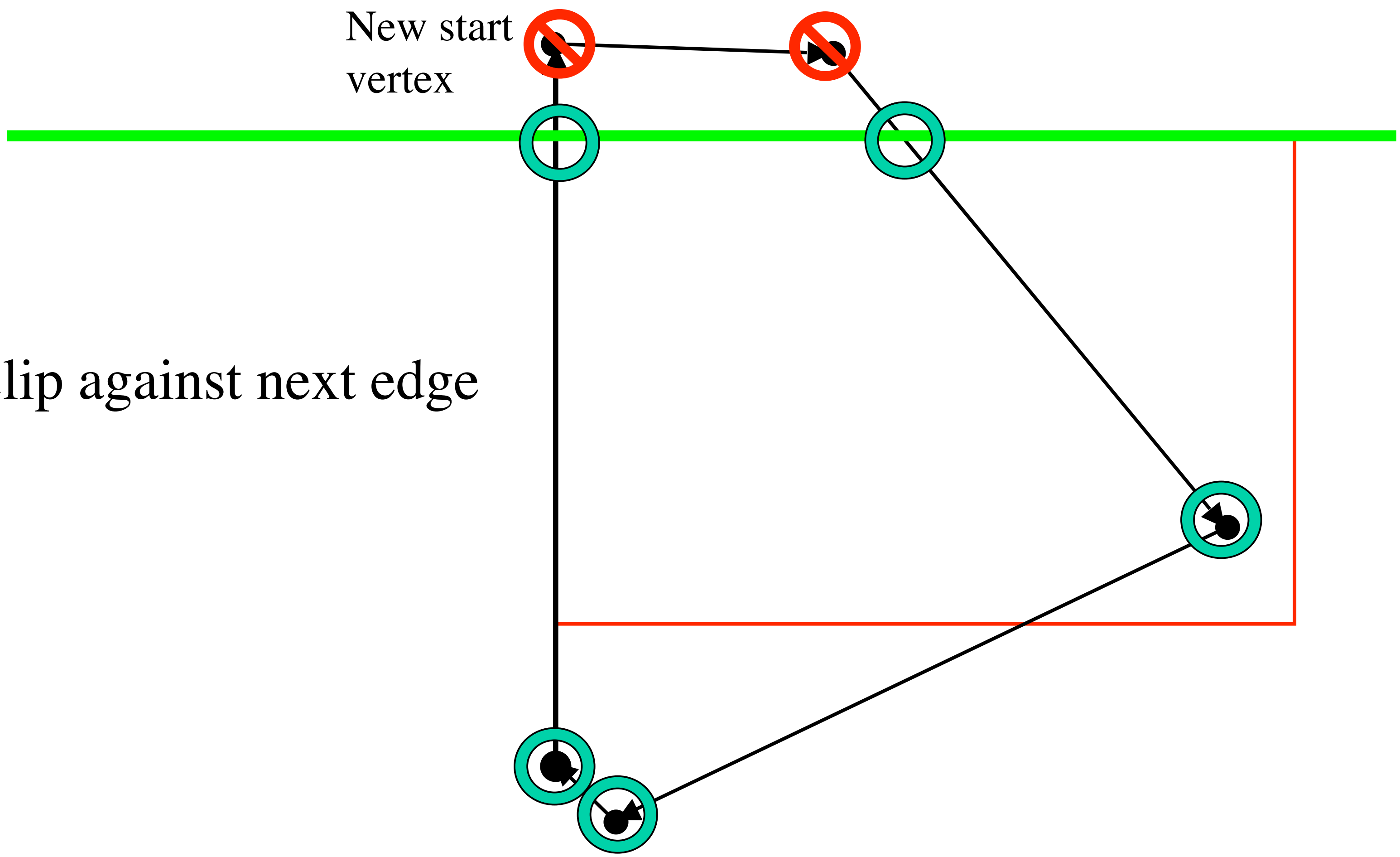


Clip against next edge

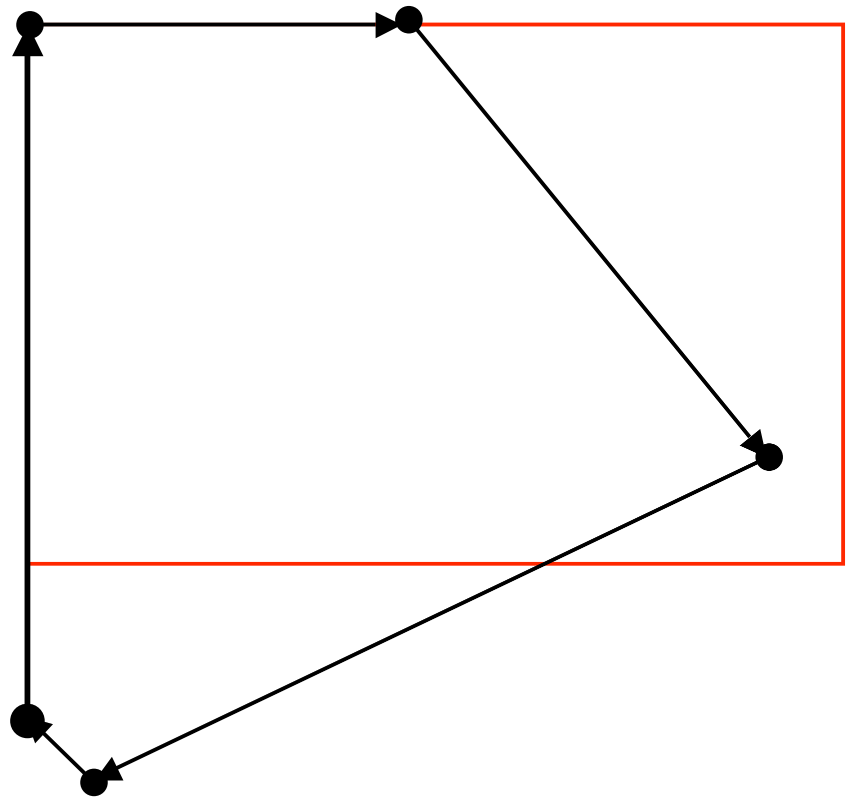
polygon edge crosses clip edge going from out to in ==> emit crossing, next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

polygon edge goes from in to in ==> emit next vertex

Now have



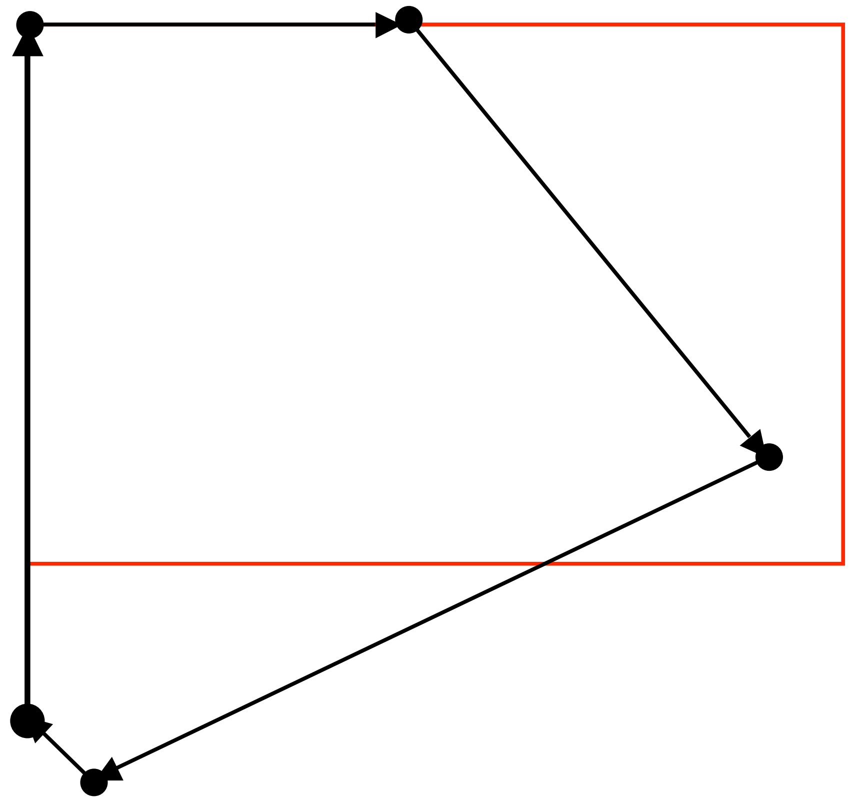
polygon edge crosses clip edge going from out to in ==> emit crossing, next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

polygon edge goes from in to in ==> emit next vertex

Clipping against next edge (right) gives



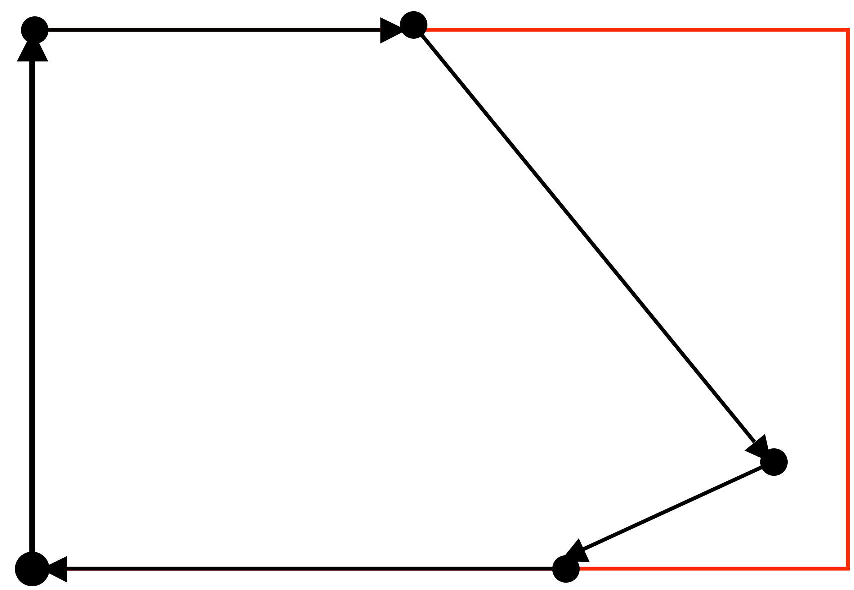
polygon edge crosses clip edge going from out to in ==> emit crossing, next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

polygon edge goes from in to in ==> emit next vertex

Clipping against final(bottom) edge gives



polygon edge crosses clip edge going from out to in ==> emit crossing, next vertex

polygon edge crosses clip edge going from in to out ==> emit crossing

polygon edge goes from out to out ==> emit nothing

polygon edge goes from in to in ==> emit next vertex

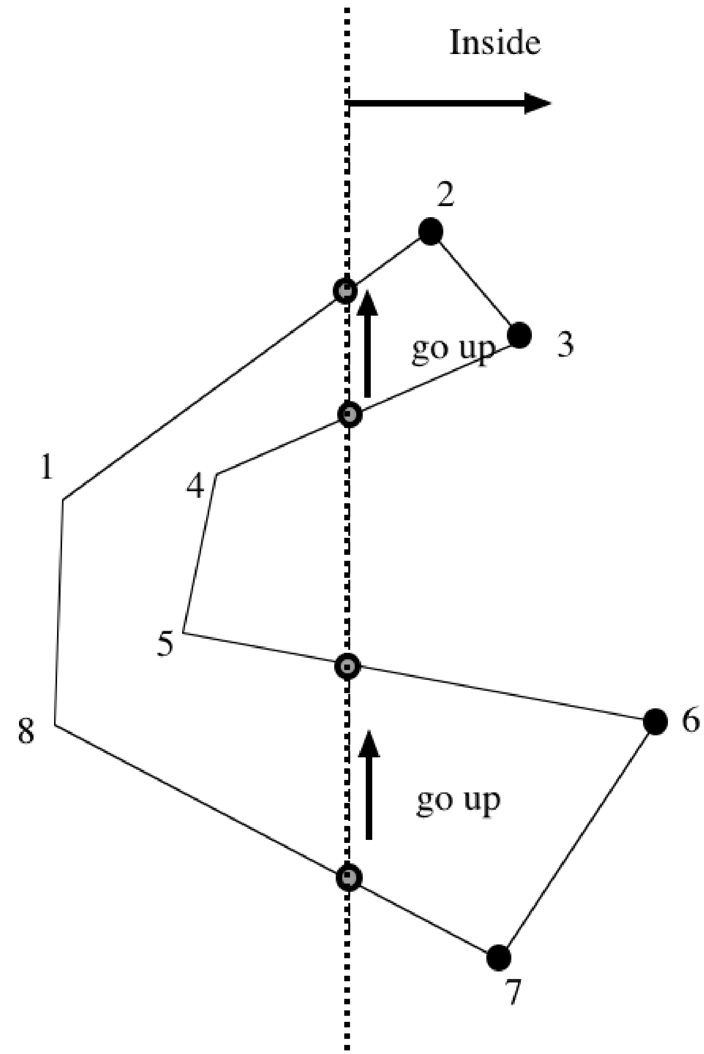
More Polygon clipping

- Notice that we can have a pipeline of clipping processes, one against each edge, each operating on the output of the previous clipper -- substantial advantage.
- Unpleasantness can result from concave polygons; in particular, polygons with empty interior.
- Can modify algorithm for concave polygons (e.g. Weiler Atherton)

Weiler Atherton

For clockwise polygon (starting outside):

- For out-to-in pair, follow usual rule
- For in-to-out pair, follow clip edge (clockwise) and then jump to next vertex (which is on the outside) and start again
- Only get a second piece if polygon is convex



Additional remarks on clipping

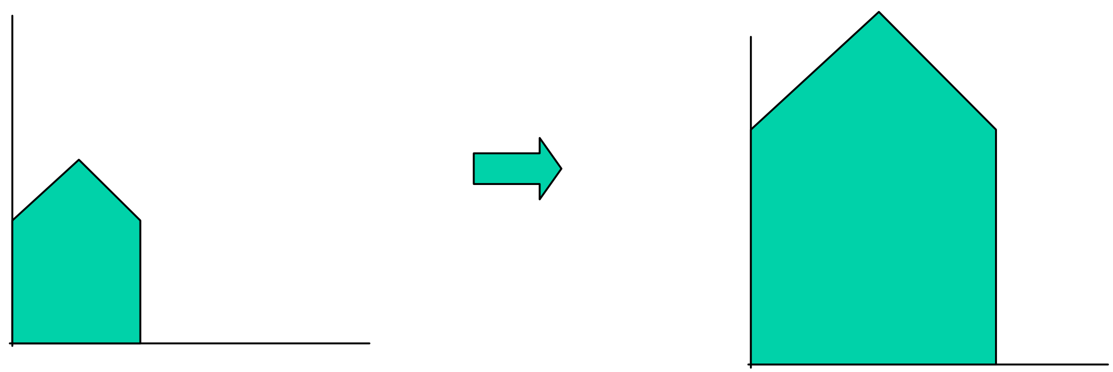
- Although everything discussed so far has been in terms of polygons/lines clipped against lines in 2D, all - except Nicholl-Lee-Nicholl - will work in 3D against convex regions without much change.
- This is because the central issue in each algorithm is the inside outside decision as a convex region is the intersection of half spaces.
- Inside-outside decisions can be made for lines in 2D, planes in 3D. e.g testing x>=0
- Hence, all (except N-L-N) can be used to clip:
  - Lines against 3D convex regions (e.g. cubes)
  - Polygons against 3D convex regions (e.g. cubes)
- NLN could work in 3D, but the number of cases increases too much to be practical.

2D Transformations

- Represent transformations by matrices
- To transform a point, represented by a vector, multiply the vector by the appropriate matrix.
- To transform lines, transform endpoints
- To transform polygons, transform vertices

2D Transformations

- Scale (stretch) by a factor of k

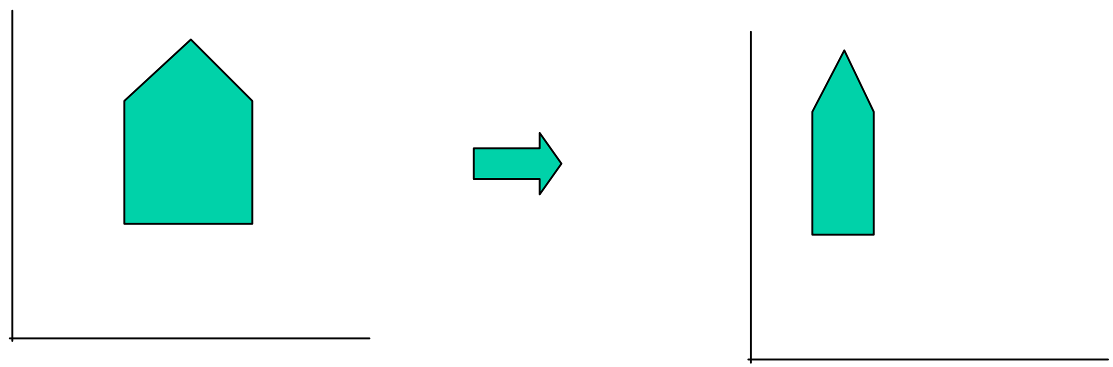


$$M = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

(k = 2 in the example)

2D Transformations

- Scale by a factor of (S_x, S_y)

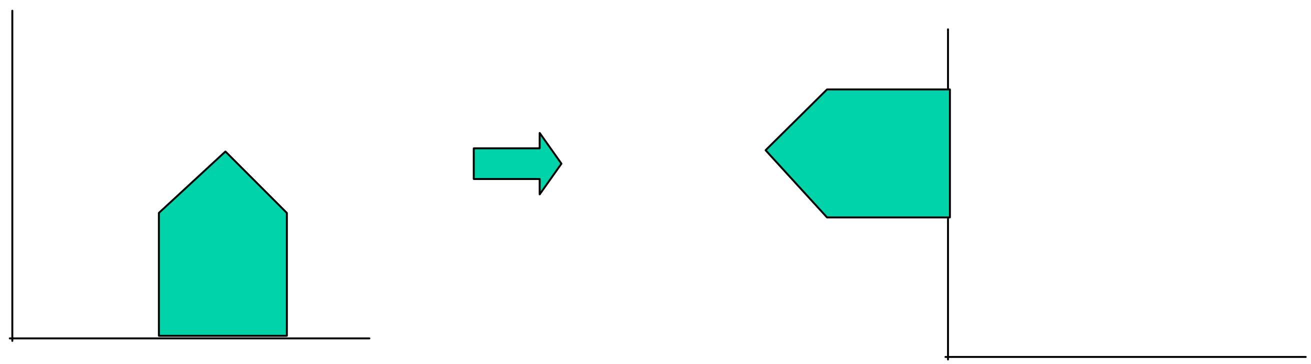


$$M = \begin{vmatrix} S_x & 0 \\ 0 & S_y \end{vmatrix}$$

(Above, $S_x = 1/2$, $S_y = 1$)

2D Transformations

- Rotate around origin by θ (Orthogonal)

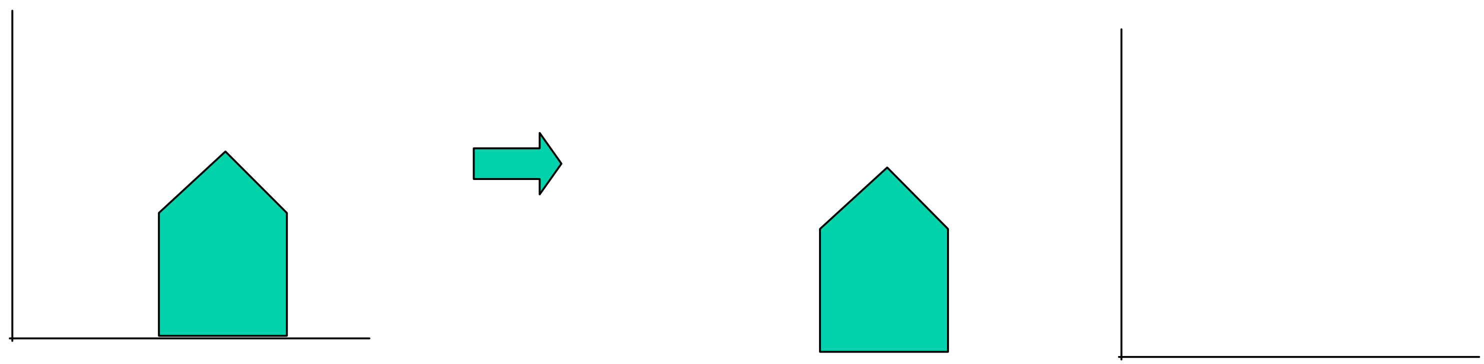


$$M = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

(Above, $\theta = 90^{o}$)

2D Transformations

- Flip over y axis (Orthogonal)

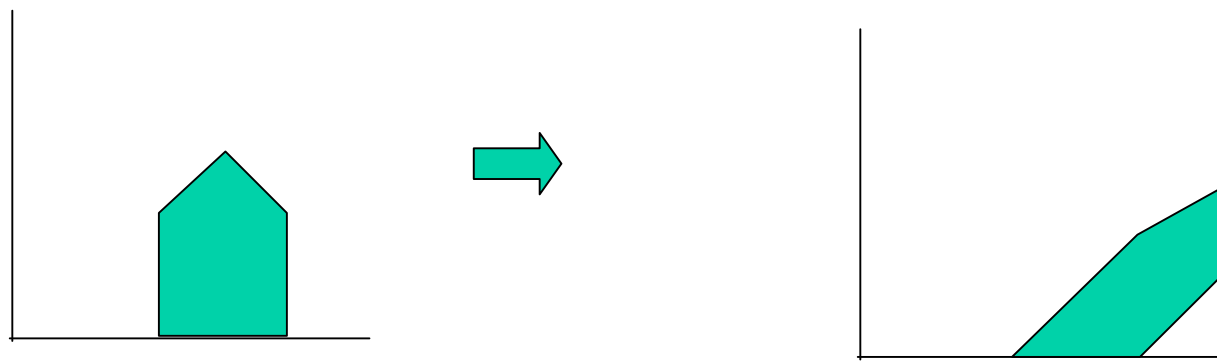


$$M = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

Flip over x axis is ?

2D Transformations

- Shear along x axis

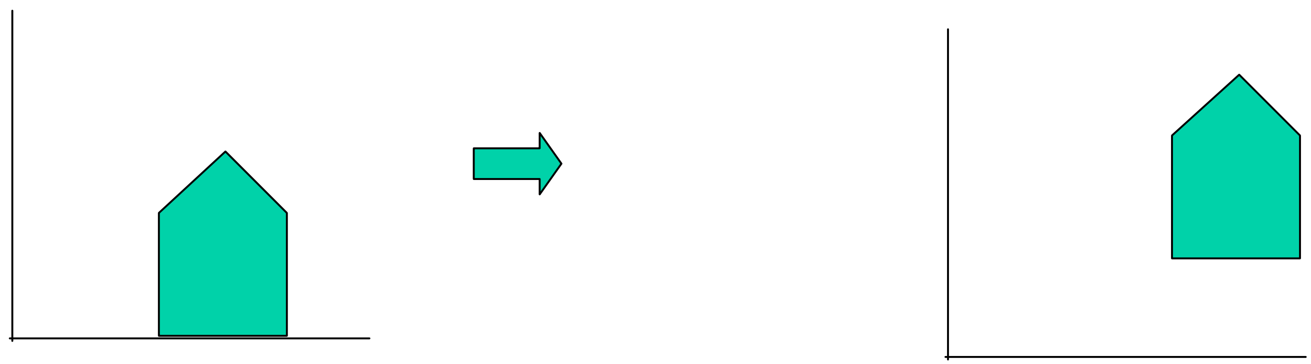


$$M = \begin{vmatrix} 1 & a \\ 0 & 1 \end{vmatrix}$$

Shear along y axis is ?

2D Transformations

- Translation $\quad (\mathbf{P}_{\text{new}} = \mathbf{P} + \mathbf{T})$



$$M = ?$$

Homogenous Coordinates

- Represent 2D points by 3D vectors
- (x,y)-->(x,y,1)
- Now a multitude of 3D points (x,y,W) represent the same 2D point, (x/W, y/W, 1)
- Represent 2D transforms with 3 by 3 matrices
- Can now do translations
- Homogenous coordinates have other uses/advantages (later)

2D Translation in H.C.

$\mathbf{P}_{new} = \mathbf{P} + \mathbf{T}$

$(x', y') = (x, y) + (t_x, t_y)$

$$\mathbf{M} = \begin{vmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{vmatrix}$$

2D Scale in H.C.

$$M = \begin{vmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

2D Rotation in H.C.

$$M = \begin{vmatrix} \cos \square & -\sin \square & 0 \\ \sin \square & \cos \square & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Composition of Transformations (§5.4)

- If we use one matrix, M_1 for one transform and another matrix, M_2 for a second transform, then the matrix for the first transform followed by the second transform is simply $M_2 M_1$
- This generalizes to any number of transforms
- Computing the combined matrix first can save lots of computation

Composition Example

- Matrix for rotation about a point, P
- Problem--we only know how to rotate about the origin.

Composition Example

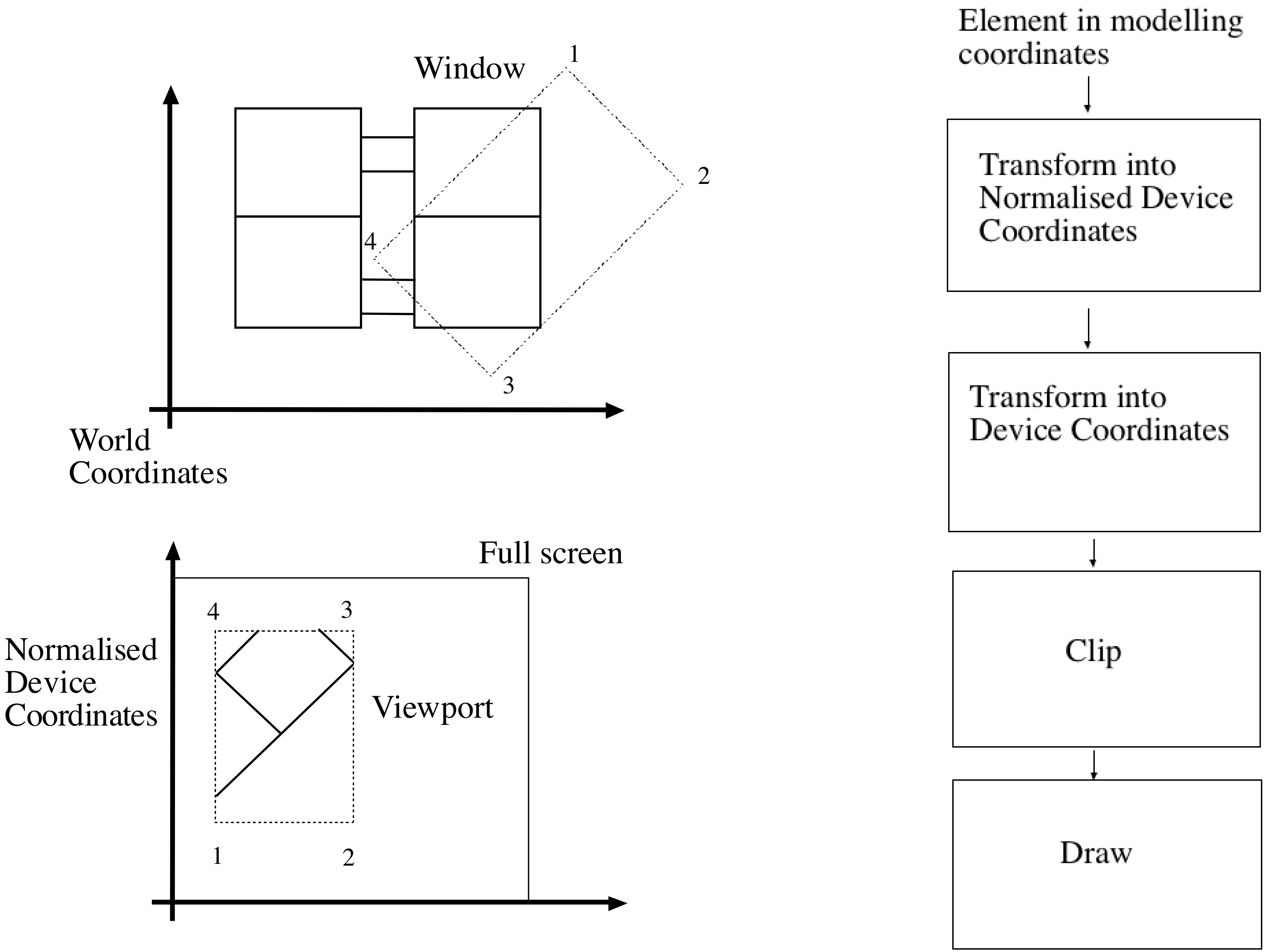
- Matrix for rotation about a point, P
- Problem--we only know how to rotate about the origin.
- Solution--translate to origin, rotate, and translate back

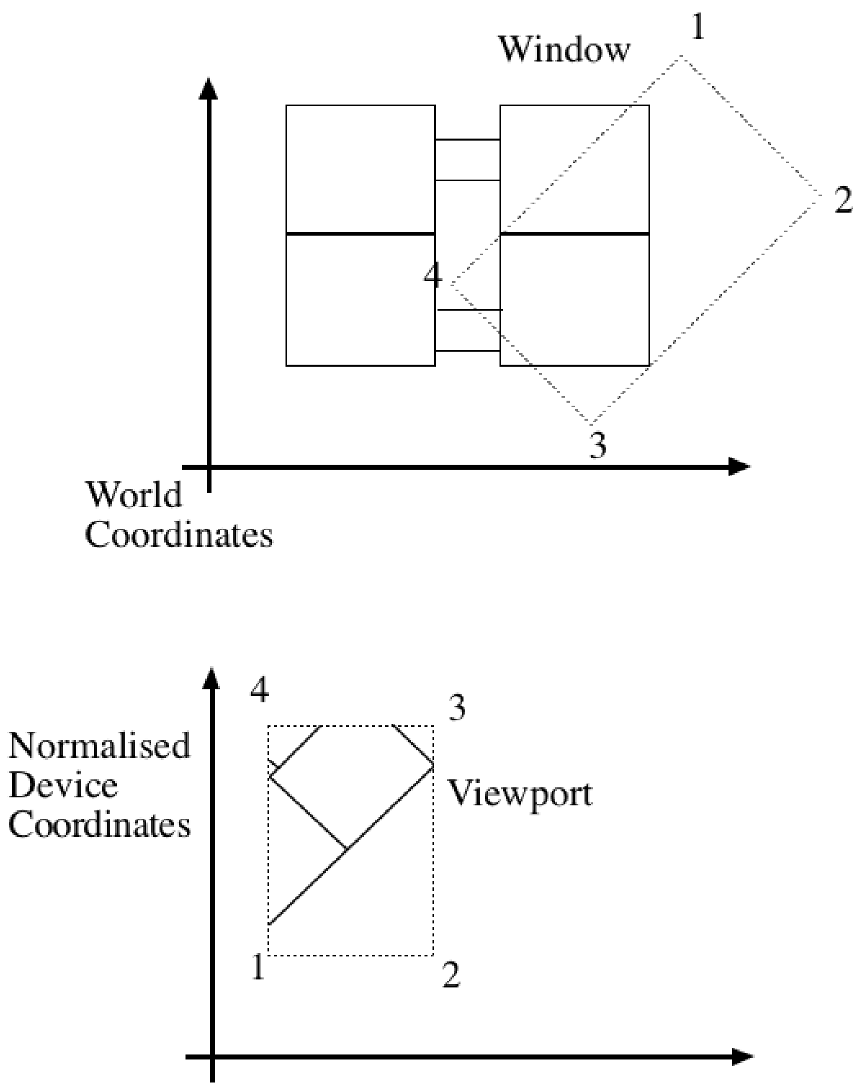
2D transformations (continued)

- The transformations discussed so far are invertable (why?). What are the inverses?

2D viewing

- Three coordinate systems are common in graphics
  - World coordinates or modeling coordinates - where the model is defined (meters, miles, etc.)
  - Normalized device coordinates; usually (0-1) in each variable.
  - Device coordinates: the actual coordinates of the pixels on the frame-buffer or the printer
- Need to construct transformations between coordinate systems
- Terminology:
  - window = region on drawing that will be displayed (rectangle)
  - viewport = region in NDC's/DC's where this rectangle is displayed (often simply entire screen).


Window
1
2
3
4
World
Coordinates
Full screen
4
3
Normalised
Device
Coordinates
Viewport
1
2
Element in modelling
coordinates
Transform into
Normalised Device
Coordinates
Transform into
Device Coordinates
Clip
Draw



- View this as a sequence of transformations in homogenous coords, then determine each element in closed form.
- Compute numerically from point correspondences.