Colour Reproduction

Motivates specifying color numerically (there are other reasons to do this also)

General (man in the street) observation--color reproduction sort of works.
Specifying Colour
Test Light

Three standard lights

Match?
Test Light

Three standard lights

Match?
Trichromacy

Experimental fact about people (with “normal” colour vision)---matching works (for reasonable lights), provided that we are sometimes allowed negative values.

Our “knob” positions correspond to (X,Y,Z) in the standard colorimetry system.

Technical detail: (X,Y,Z) are actually arranged to be positive by a linear transformation, but these “knob” positions cannot correspond to any physical light.
Specifying Colour

(50,150,75)

(50,150,75)
Specifying Colour

We don’t want to do a matching experiment every time we want to use a new color!
Grassman’s Contribution

Colour matching is linear
Test Light

Three standard lights

Match
Test Light

Three standard lights

Match (with twice as much)
Matching is Linear (Part 1)

C1 is matched with (X1,Y1,Z1)

C = a*C1

C is matched with a * (X1, Y1, Z1)
Match with \((X_1, Y_1, Z_1)\)
Test Light (C2)  Three standard lights

Match with (X2, Y2, Z2)
Match with?

Test Light

Three standard lights

Match with?
Test Light

Three standard lights

Match with \((X_1+X_2, Y_1+Y_2, Z_1+Z_2)\)
Matching is Linear (formal)

\[ C = a \cdot C_1 + b \cdot C_2 \]

\( C_1 \) is matched with \((X_1, Y_1, Z_1)\)
\( C_2 \) is matched with \((X_2, Y_2, Z_2)\)

\( C \) is matched by
\[ a \cdot (X_1, Y_1, Z_1) + b \cdot (X_2, Y_2, Z_2) \]
Specifying Color

On my monitor it’s
(R,G,B) = (75,150,100)
Specifying Colour

But what is (R,G,B)?
Specifying Colour

R matches \((X_r, Y_r, Z_r)\)
G matches \((X_g, Y_g, Z_g)\)
B matches \((X_b, Y_b, Z_b)\)
Specifying Colour

Then by

\((R,G,B) = (75,150,100)\)

you mean \((X,Y,Z)\),

where .....
\[ X = 75 \cdot X_r + 150 \cdot X_g + 100 \cdot X_b \]

\[ Y = 75 \cdot Y_r + 150 \cdot Y_g + 100 \cdot Y_b \]

\[ Z = 75 \cdot Z_r + 150 \cdot Z_g + 100 \cdot Z_b \]

(No need to match--just compute!)
Specifying Colour

…, now that we have specified the colour, I can print it!
\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
= 
\begin{pmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{pmatrix}
\begin{pmatrix}
75 \\
100 \\
150
\end{pmatrix}
\]
\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
= 
\begin{pmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{pmatrix}
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
\]
| X | Y | Z | = | M | R | G | B |
Colour Reproduction
(Monitors & Projectors)

Find \((R,G,B)\)

\[
\begin{array}{c|c|c}
X & Y & Z \\
\end{array}
\]

apple
\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= M
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
apple
\[
\begin{bmatrix}
\text{R} \\
\text{G} \\
\text{B}
\end{bmatrix}
= \text{apple}^{-1}
\begin{bmatrix}
\text{X} \\
\text{Y} \\
\text{Z}
\end{bmatrix}
\text{apple}
\]
Possible problems?
Qualitative features of CIE $x, y$

- Linearity implies that colors obtainable by mixing lights with colors A, B lie on line segment with endpoints at A and B
- Monochromatic colours (spectral colors) run along the “Spectral Locus”
- Dominant wavelength = Spectral color that can be mixed with white to match
Qualitative features of CIE $x, y$

- Purity = (distance from C to spectral locus)/(distance from white to spectral locus)
- Wavelength and purity can be used to specify color.
- Complementary colors = colors that can be mixed to get white