Modeling

- Need to usefully represent objects in the world
- Need to provide for easy interaction
  - manual modeling
    - user would like to “fiddle” until it is right (e.g. CAD)
    - user has an idea what an object is like
  - fitting to measurements
    - laser range finder data
- Support rendering/geometric computations

Modeling tools

- Polygon meshes
- Fitting curves to points (from data)
- Fitting curves to points (user interaction)
- Generating shapes with sweeps
- Constructive solid geometry

Polygon Meshes

- Common, straightforward, often built in (e.g. torus mesh)
- Ready to render (many of the representations discussed soon are often be reduced to polygon meshes for rendering)
- Problems
  - Awkward to provide user editing
  - The number of polygons can be very large
    - Some kind of adaptive process makes sense
    - More polygons at high curvature points
    - More polygons where the object is larger
    - Extra care then needs to be taken to avoid temporal aliasing

Explicit curve representation

- Usual representation learned first
- Generally less useful in graphics, but know the term
- Explicit curve is a function of one variable. Examples
  - line, y=m*x + b
  - circle (need to glue two together) y = ±sqrt(r*r - x*x)
- Explicit surface is a function of two variables. Examples
  - plane z = m*x+ n*y + b

Implicit representation

- Also less useful for this section, but again, know the term
- An implicit curve is given by the vanishing of some functions
  – circle on the plane, x*x + y*y - r*r = 0
  – twisted cubic in space, x*y - z = 0, x*z - y*y=0, x*x - y = 0
- An implicit surface is given by the vanishing of some functions
  – sphere in space x*x + y*y + z*z - r*r = 0
  – plane a*x + b*y + c*z + d = 0

Parametric representation

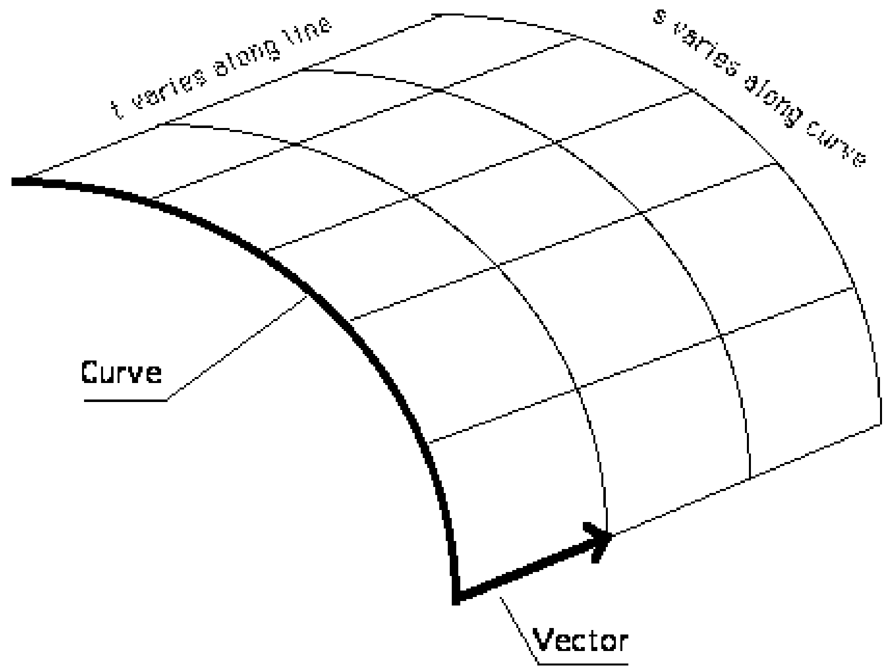
- A parametric **curve** is given as a function of one parameter. Examples:
  - circle as (cos t, sin t)
  - twisted cubic as (t, t*t, t*t*t)
- A parametric **surface** is given as a function of two parameters. Examples:
  - sphere as (cos s cos t, sin s cos t, sin t)
- Advantage - easy to compute normal, easy to render, easy to put patches together, ranges can be easy (e.g. half circle)
- Disadvantage - intersecting with rays for ray tracing can be hard

Generating Surfaces

- We can construct surfaces from curves in a variety of user intuitive ways
  - Extruded surfaces
  - Generalized cones
  - Surfaces of revolution
  - Sweeping (generalized cylinders)

Extruded surfaces

- Geometrical model - Pasta machine
- Take curve and “extrude” surface along vector
- Many human artifacts have this form - rolled steel, etc.



Parametric formula?

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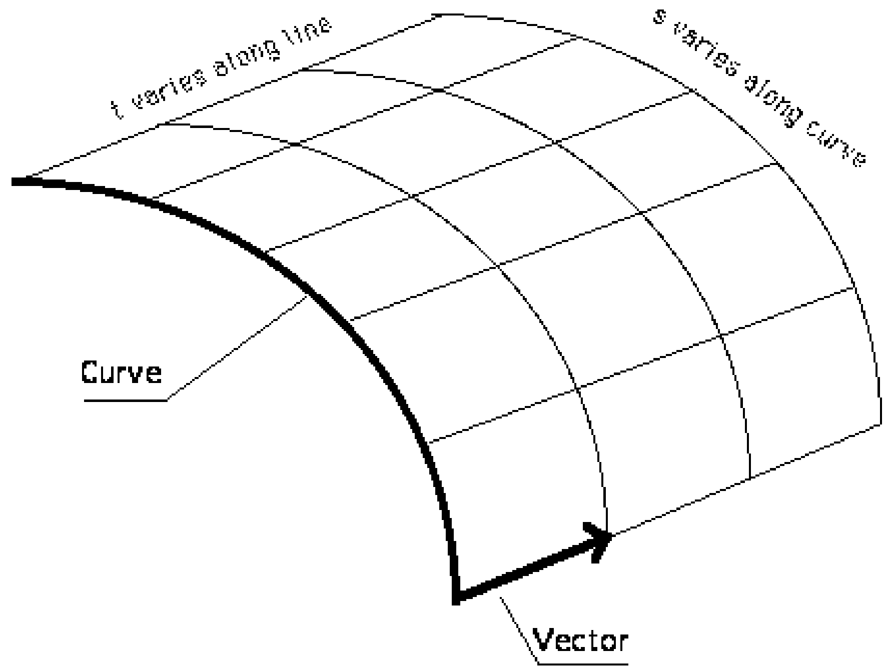
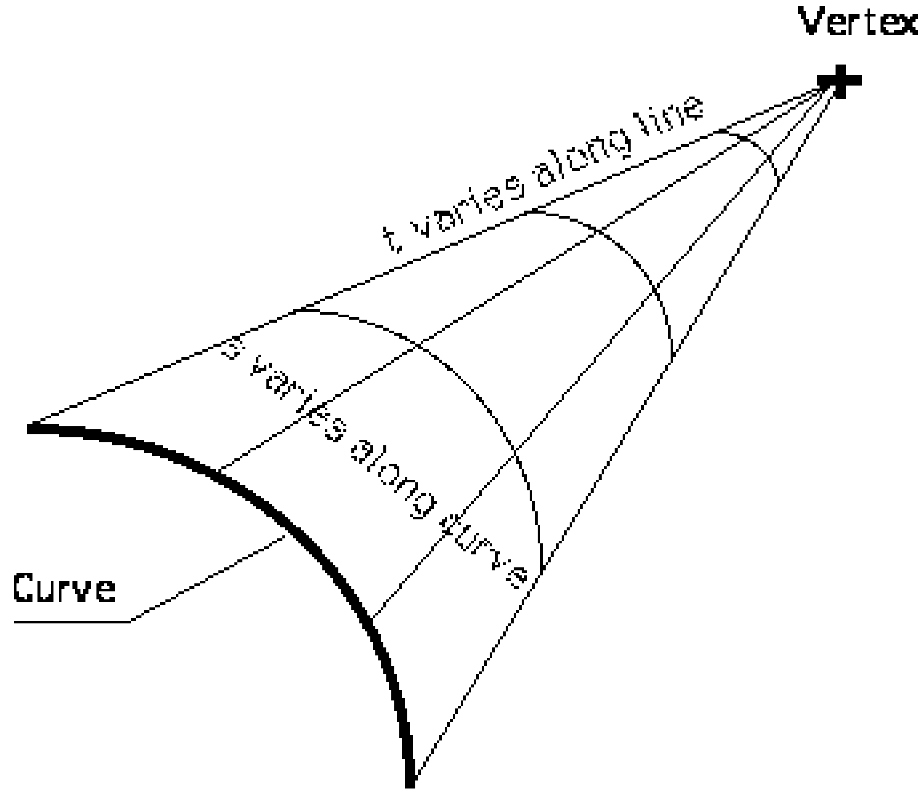


$$(x(s,t), y(s,t), z(s,t)) = (x_c(s), y_c(s), z_c(s)) + t(v_0, v_1, v_2)$$

Cones

- From every point on a curve, construct a line segment through a single fixed point in space - the vertex
- Curve can be space or plane curve, but shouldn't pass through the vertex



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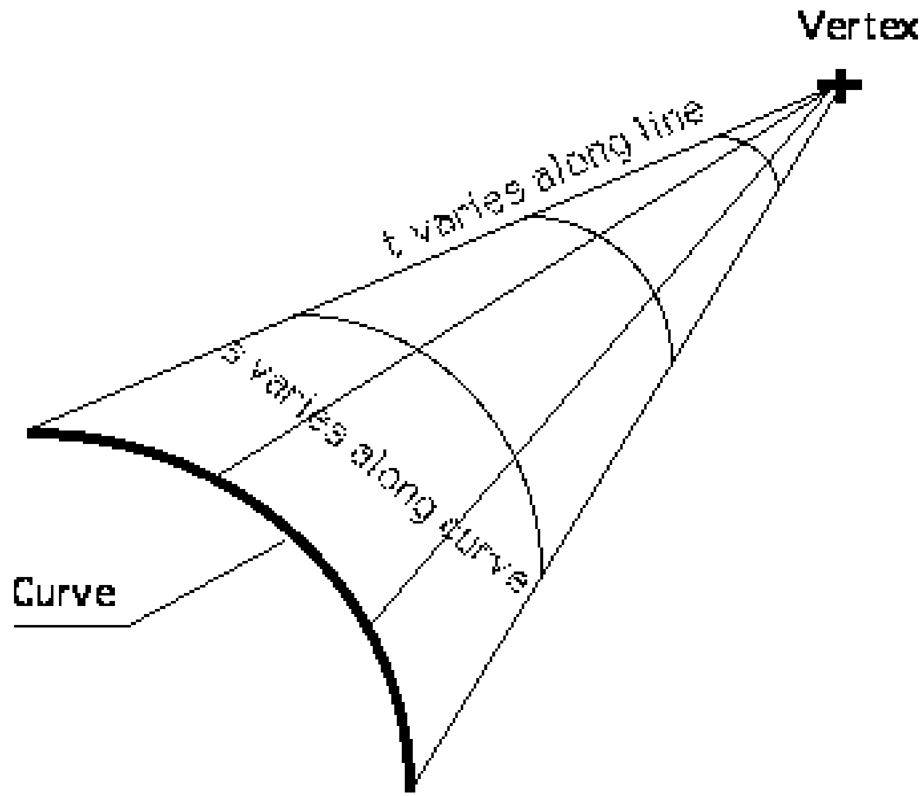
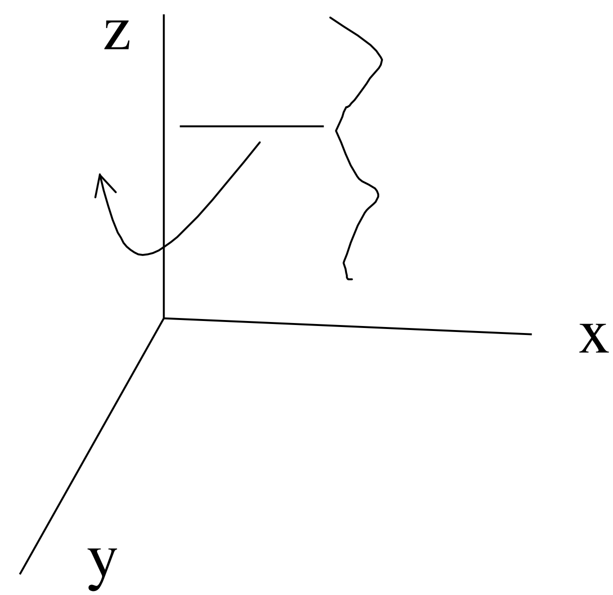


$$(x(s,t), y(s,t), z(s,t)) = (1 - t)(x_c(s), y_c(s), z_c(s)) + t(v_0, v_1, v_2)$$

Surfaces of revolution

- Plane curve + axis
- "spin" plane curve around axis to get surface
- Choice of plane is arbitrary, choice of axis affects surface
- In the example to the right, curve is on x-z plane, axis is z axis.
- So curve is $(x_c(s), z_c(s))$

Parametric formula?

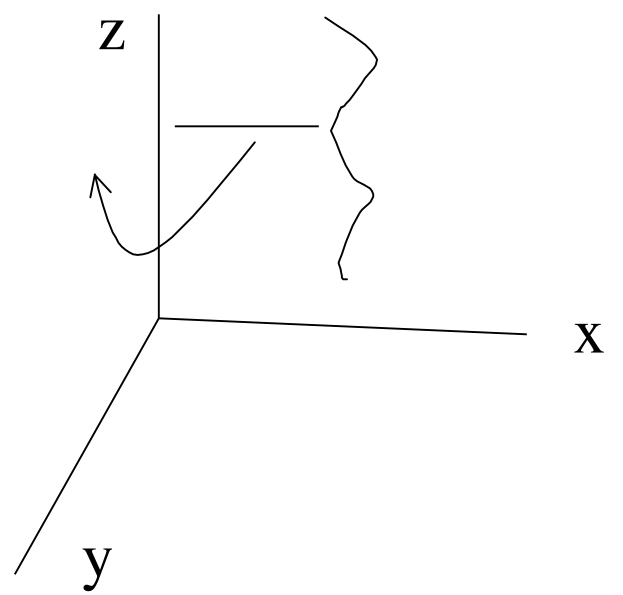


Surfaces of revolution

- Plane curve + axis
- "spin" plane curve around axis to get surface
- Choice of plane is arbitrary, choice of axis affects surface
- In the example to the right, curve is on x-z plane, axis is z axis. (Think of $x_c(s)$ as a radius)

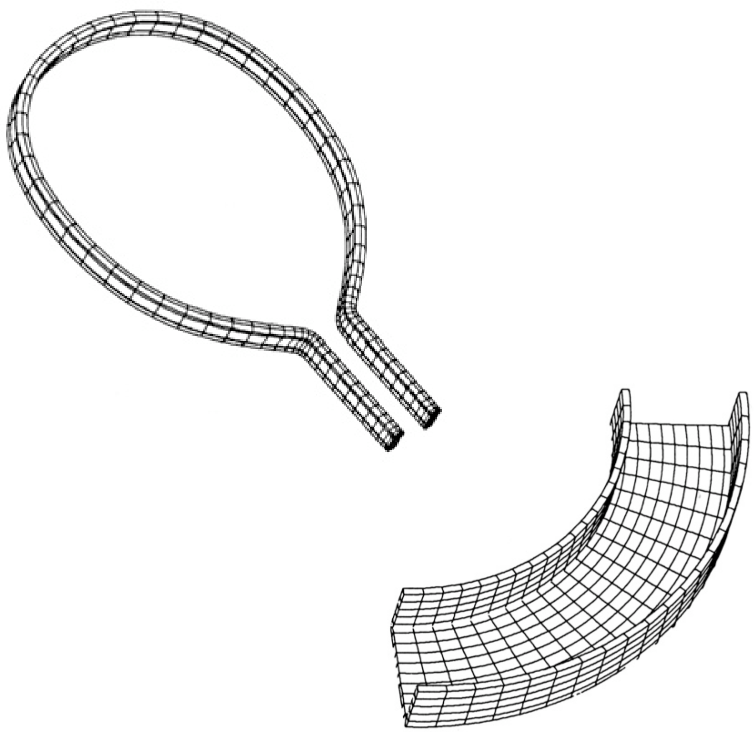
$$(x(s,t), y(s,t), z(s,t)) =$$

$$(x_c(s)\cos(t), x_c(s)\sin(t), z_c(s))$$



Optional

Sweeps/Generalized Cylinders



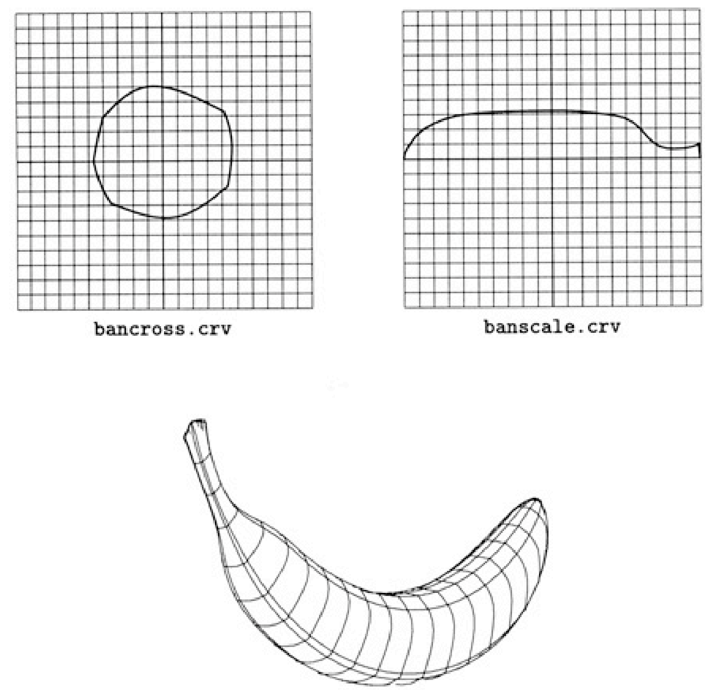
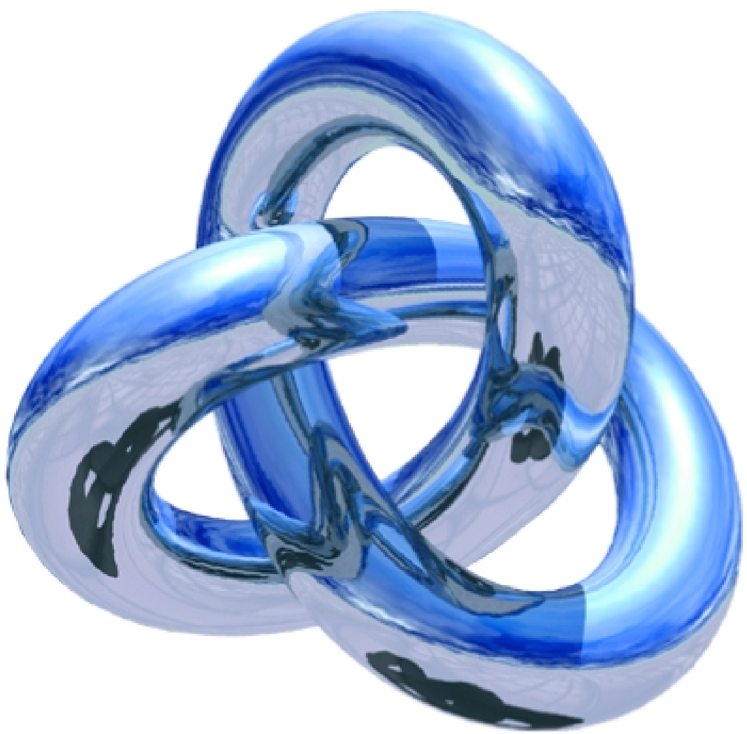


Figure 3.8: Banana example. A banana is represented by an affine transformation surface. The cross section is scaled, translated along z from -1 to 1, and rotated around the y axis.

[Synder 92, via CMU course page]

Optional

Sweeps/Generalized Cylinders



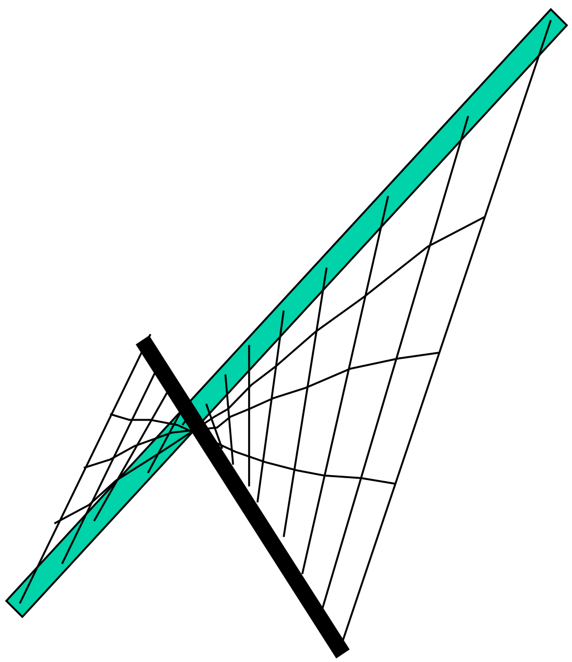
MetaCreations, via CMU course page

Ruled surfaces -1

- Popular, because it's easy to build a curved surface out of straight segments - e.g. pavilions, etc.
- Take two space curves, and join corresponding points - same s - with line segment.
- Even if space curves are lines, the surface is usually curved.

Ruled Surfaces - 2

Easy to explain,
hard to draw!



Ruled surfaces -3

Parameterized form

$$(x(s,t), y(s,t), z(s,t)) =$$
$$(1 - t)(x_1(s), y_1(s), z_1(s)) +$$
$$t(x_2(s), y_2(s), z_2(s))$$

Normals

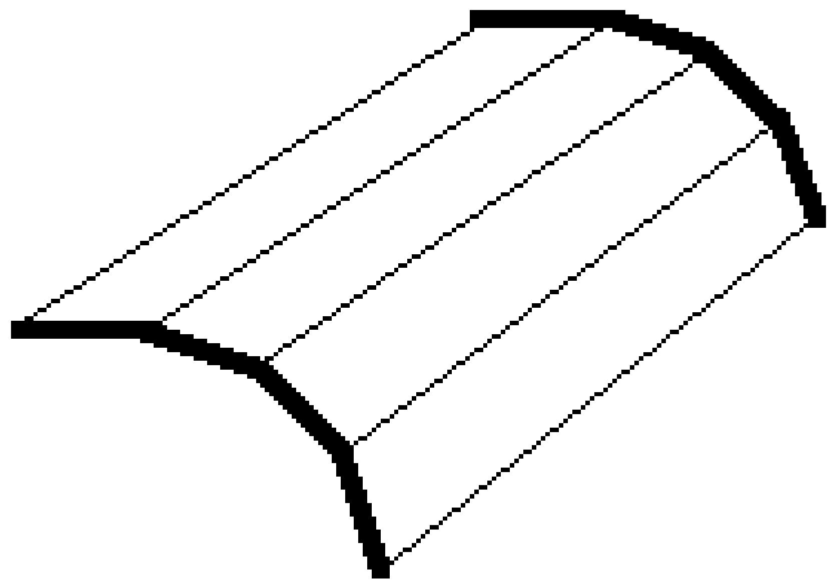
- Normal is cross product of tangent in t direction and s direction.

$$\left[\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right] \times \left[\frac{\partial x}{\partial s}, \frac{\partial y}{\partial s}, \frac{\partial z}{\partial s}\right]$$

- Examples
  - Cylinder: normal is cross-product of curve tangent and direction vector
  - Surface of revolution: take curve normal and spin round axis

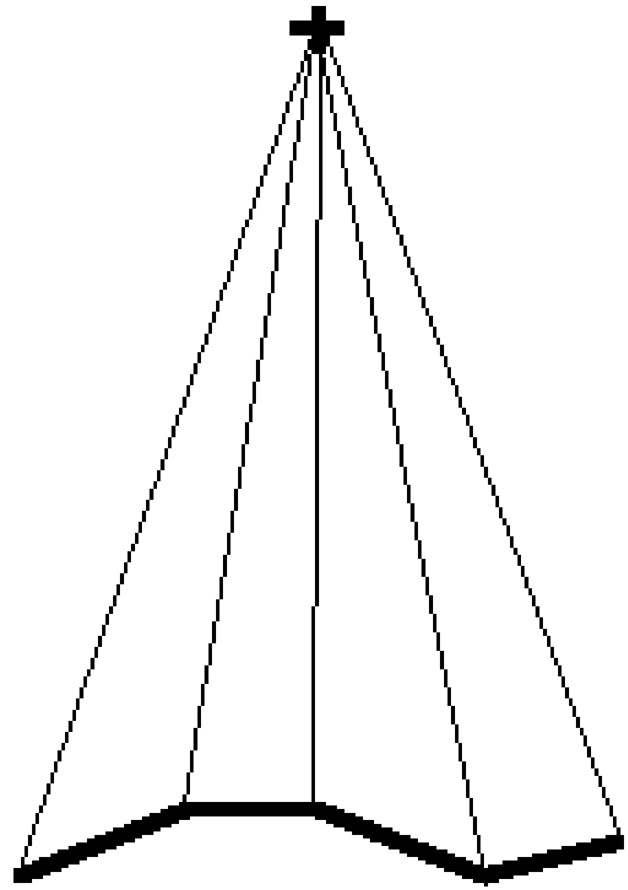
Rendering

- Cylinders: small steps along curve, straight segments along t generate polygons; exact normal is known.



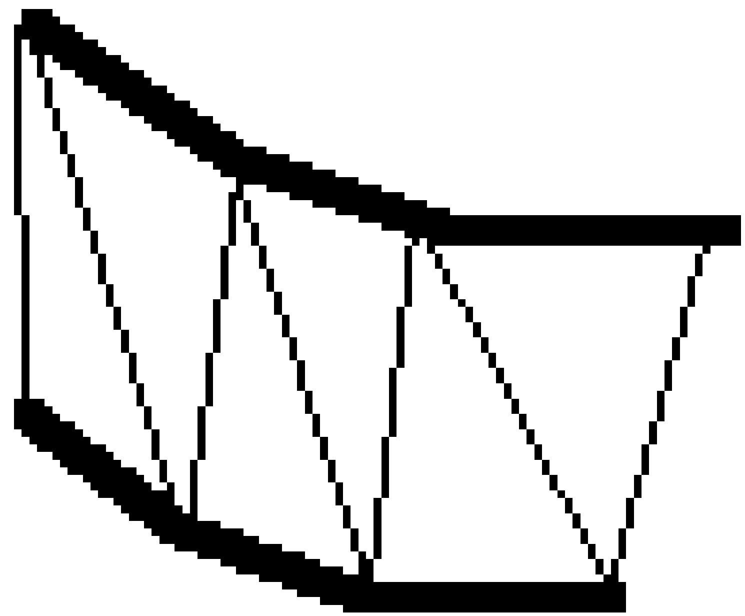
Rendering

- Cone: small steps in s generate straight edges, join with vertex to get triangles, normals known exactly except at vertex.



Rendering

- Surface of revolution: small steps in s generate strips, small steps in t along the strip generate edges; join up to form triangles. Normals known exactly.



Rendering

- Ruled surface: steps in s generate polygons, join opposite sides to make triangles - otherwise “non planar polygons” result. Normals known exactly.