Displaying lines

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  - lines have integer vertices
  - lines all lie within the displayable region of the frame buffer
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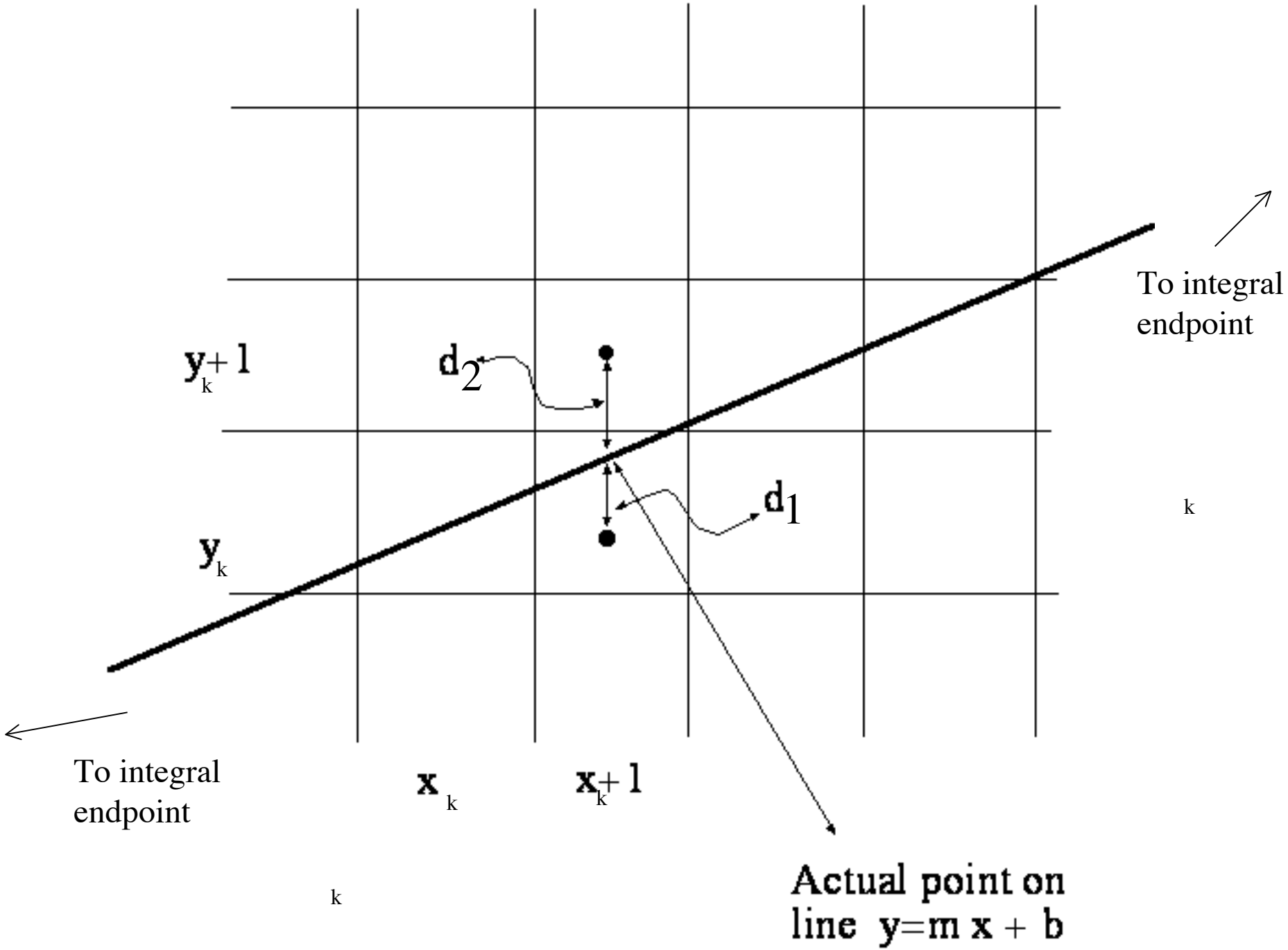
- Consider lines of the form y=m x + c, where 0<m<1
- Other cases follow by symmetry

Displaying lines

- Variety of naive (poor) algorithms:
  - step x, compute new y at each step by equation, rounding
  - step x, compute new y at each step by adding m to old y, rounding

Bresenham's algorithm

- Plot the pixel whose y-value is closest to the line
- Given (x_k, y_k), must **choose** from either (x_k+1, y_k+1) or (x_k+1, y_k)---recall we are working on case $0<m<1$
- Idea: compute value that will determine this choice that is easy to update and cheap to compute (no floating point operations if endpoints are integral).

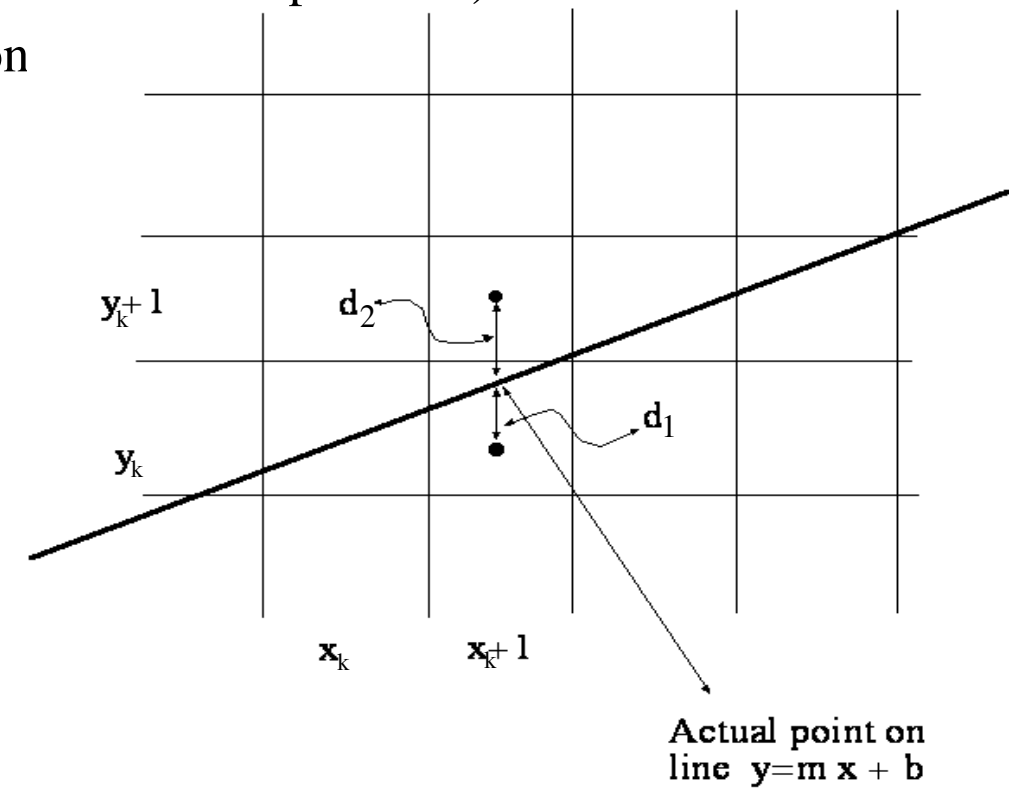

To integral endpoint
$y_k + 1$
d_2
d_1
y_k
k
To integral endpoint
x_k
$x_k + 1$
k
Actual point on line $y = m x + b$

Bresenham's algorithm

- Determiner is $d_1 - d_2$

  $d_1 - d_2 < 0 \Rightarrow$ plot at y_k (same level as previous)

  $d_1 - d_2 > 0 \Rightarrow$ plot at $y_k + 1$ (on



(Current point is, (x_k, y_k) line goes through $(x_k + 1, y)$)

$$d_1 - d_2 = (y - y_k) - ((y_k + 1) - y)$$

Plugging in $y = m(x_k + 1) + b$

Gives

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

From the previous slide)

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

Recall that,

$$m = (y_{end} - y_{start}) / (x_{end} - x_{start}) = dy / dx$$

So, for integral endpoints we can avoid floating point if we scale by a factor of dx. Use determiner P_k.

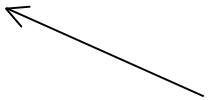
$$\begin{aligned} p_k &= (d_1 - d_2)dx \\ &= (2m(x_k + 1) - 2y_k + 2b - 1)dx \\ &= 2(x_k + 1)dy - 2y_k(dx) + 2b(dx) - dx \\ &= 2(x_k)dy - 2y_k(dx) + \text{constant} \end{aligned}$$

From previous slide

$$p_k = 2(x_k)dy - 2y_k(dx) + \text{constant}$$

Finally, express the next determiner in terms of the previous, and in terms of the decision on the next y.

$$\begin{aligned} p_{k+1} &= 2(x_k + 1)dy - 2y_{k+1}(dx) + \text{constant} \\ &= p_k + 2dy - 2(y_{k+1} - y_k) \end{aligned}$$



Either 1 or 0 depending on decision on y

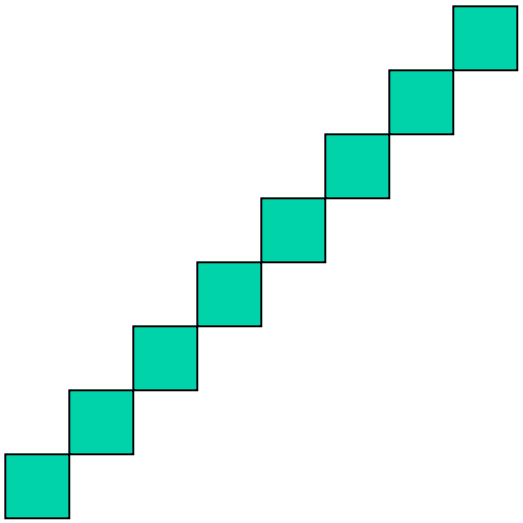
Bresenham (continued)

- $p_{k+1} = p_k + 2\,dy - 2\,dx\,(y_{k+1}-y_k)$
- Exercise: check that $p_0 = 2\,dy - dx$
- Algorithm (for 0<m<1):
  - x=x_start, y=y_start, p=2 dy - dx, **mark** (x, y)
  - until x=x+end
    - x=x+1
    - p>0 ? y=y+1, **mark** (x, y), p=p+2 dy - 2 dx
    - p<0? y=y, **mark** (x, y), p=p+2 dy
- Some calculations can be done once and cached.

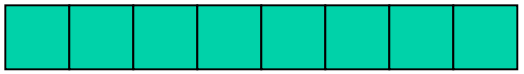
Issues

- End points may not be integral due to clipping (or other reasons)
- Brightness is a function of slope.
- Aliasing (related to previous point).

Line drawing--simple line (Bresenham) brightness issues

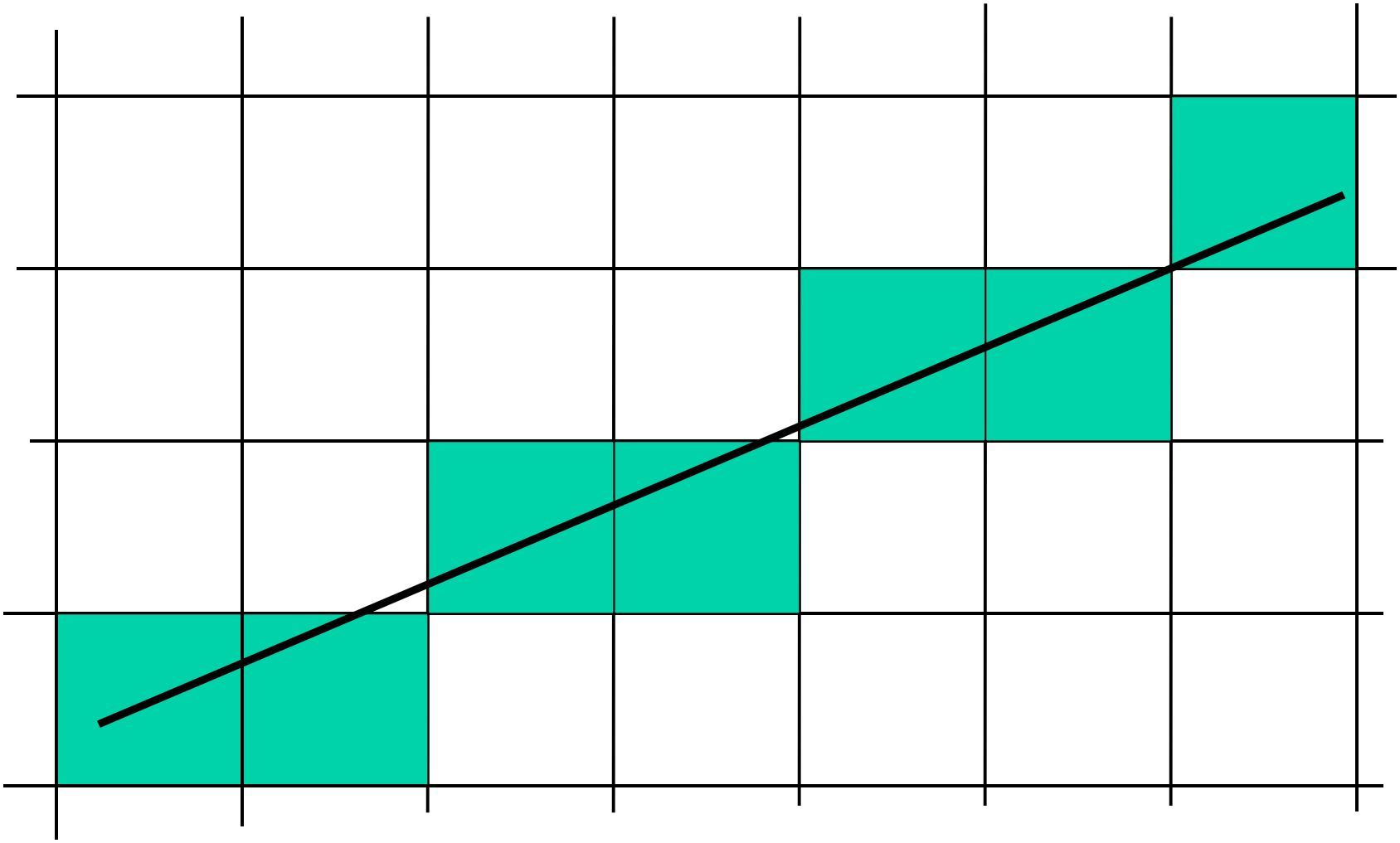


8 pixels per 8*sqrt(2) length



8 pixels for 8 length
(Brighter)

Line drawing--aliasing



Aliasing

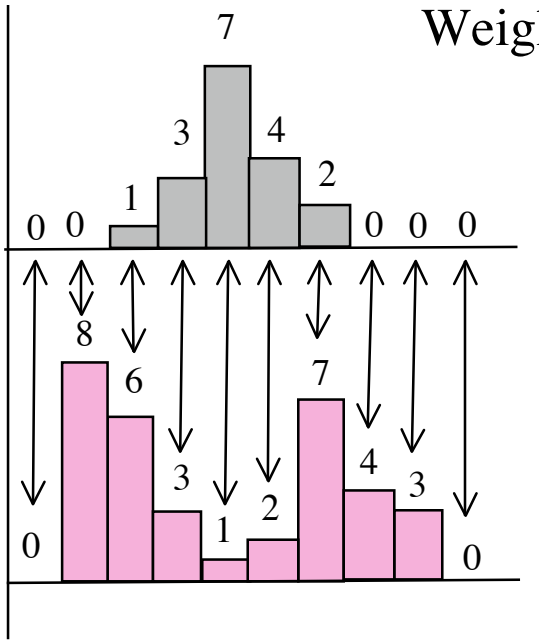
- We are using discrete binary squares to represent perfect mathematical entities
- To get a value for that that square we used a “sample” at a particular discrete location.
- The sample is somewhat arbitrary due to the choice of discretization, and reflects our discretization (leading to the jagged edges)
- Points and lines as discussed so far have no width so to have them visible we concocted a way to sample them based on which discrete cell was closer
- Solutions?

Aliasing (cont)

- General approach to reducing aliasing is to exploit ability to draw levels of gray between black and white.
- Example--give the line some width; brightness is proportional to area that pixel shares with line
- A more principled approach is to “filter” before sampling.

Linear Filters (background)

- General process: Form new image whose pixels are a **weighted sum** of original pixel values, using the same set of weights at each point.



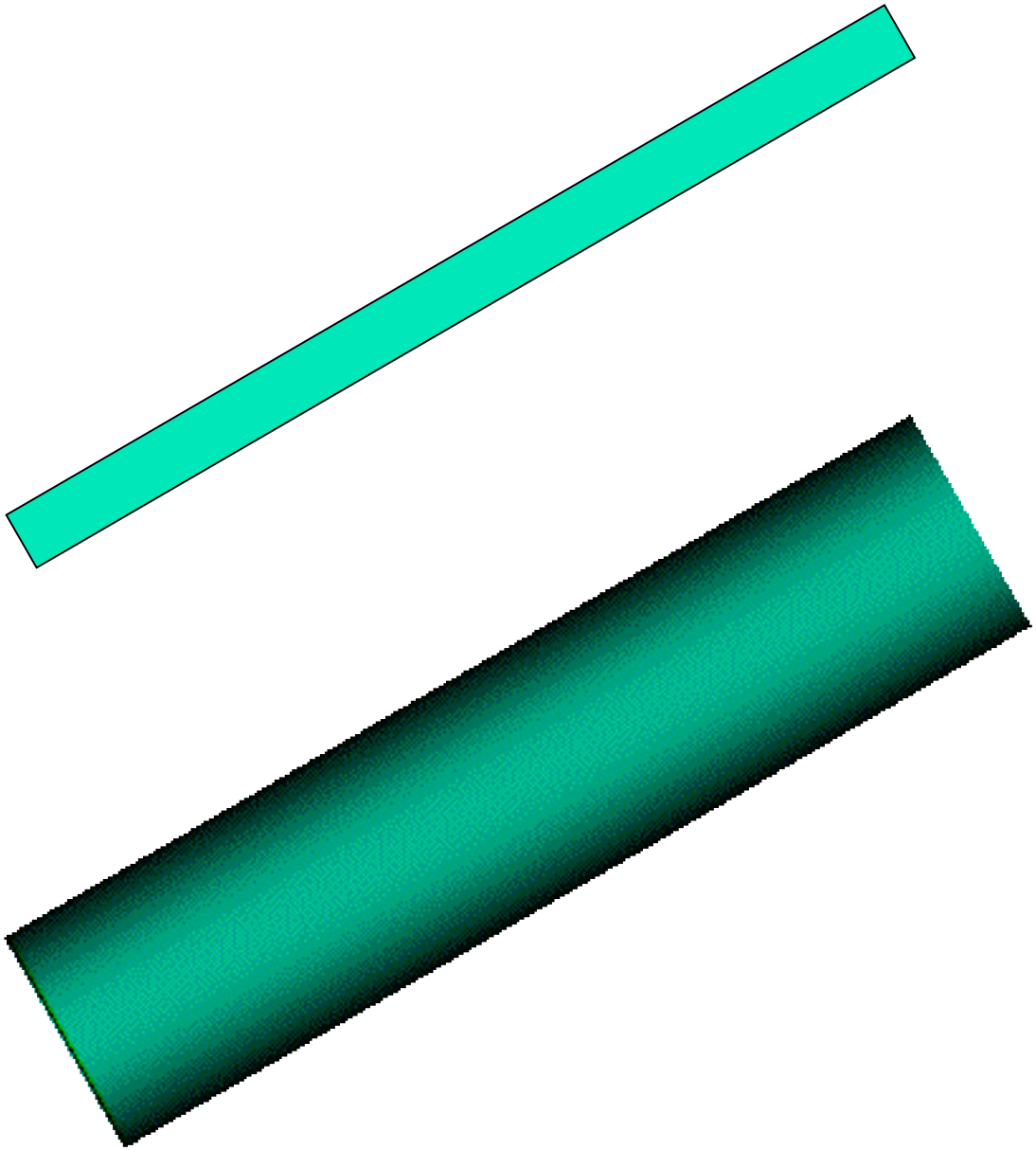
Weights (kernel of filter)

Multiply lined up pairs of numbers and then sum up to get weighted average at the filter location. Then shift the filter and do the same to get the next value.

Signal

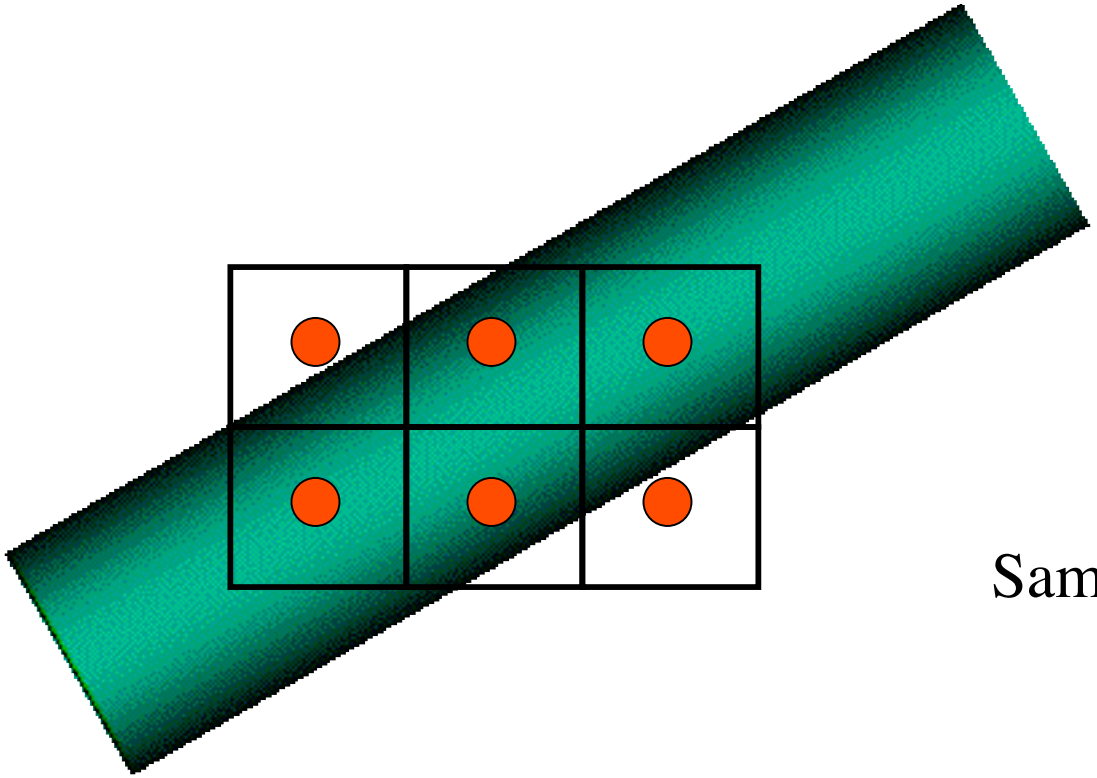
Aliasing via filtering and then sampling

- A filter can be thought of as a weighted average. The weights are given by the filter function. (Examples to come).
- **Conceptually**, we smooth (convolve) the object to be drawn by applying the filter to the mathematical representation.
- This blurs the object, widens the area it occupies
- Now we “sample” the blurred image--i.e., report the value of the blurred function at the (x,y) of interest, and then fill the square with that brightness.
- (**Technically** we only need to blur at the sampling locations)

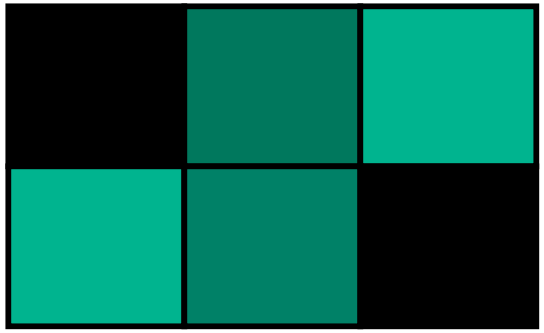


Line with width

Blurred



Sample



Paint with
sample value