Plan A: Clip against canonical frustum (relatively easy—we chose the canonical frustum so that it would be easy!)

Plan B: Be even more clever. Further transform to cube and clip in homogenous coordinates.

Plan B: Clipping in homogenous coords

- For any camera, can turn the view frustrum into a regular parallelepiped (box). We will use the box bounded by $x = \pm 1$, $y = \pm 1$, $z = -1$, and $z = 0$.
- Advantages
  - Simplified clipping in homogenous coordinates
  - Extends to cases where we use homogenous coordinates to represent additional information (and $w$ could be negative).
  - Can simplify visibility algorithms.
- Approach: clever use of homogenous coordinates

Transforming canonical frustum to box

Object in world coordinates (after modeling transforms)

Transform object from world coordinates to standard camera coordinates

Clip against canonical view frustum

Project using standard camera model

Polygon in 3D

Orthographic case

Rotate and translate to place camera at origin

Translate and scale to canonical frustum

Transform frustum to standard box

Divide and clip against box or clip in h.c.’s and then divide

Projection is now trivial

Perspective case

Plan B
Further comments on the canonical frustum

\( u_{\min}, u_{\max}, v_{\min}, v_{\max} \), are thought of as being in the camera coordinate system \( \Rightarrow \) units are that of world coordinate system

For assignment three, you need to choose \( u_{\min}, u_{\max}, v_{\min}, v_{\max} \) and \( f \).

Note the reciprocal relation of \( u_{\min}, u_{\max}, v_{\min}, v_{\max} \), and \( f \).

For assignment three, I suggest \( u_{\min}, u_{\max}, v_{\min}, v_{\max} \), reflect the aspect ratio of your screen window, and set \( f \) accordingly.

Further comments on the canonical frustum

Once you have screen \((x,y)\) you need to map them back to the screen coordinates. The canonical frustum gives the screen as a square that is \(2f'\) by \(2f'\).

\[
\begin{align*}
\hat{x} &= x \cdot \frac{u_{\max} - u_{\min}}{2f} = X \cdot \frac{f}{Z} \cdot \frac{u_{\max} - u_{\min}}{f} \\
\hat{y} &= y \cdot \frac{v_{\max} - v_{\min}}{2f} = Y \cdot \frac{f}{Z} \cdot \frac{v_{\max} - v_{\min}}{f}
\end{align*}
\]

Notice that \( f \) has disappeared from the RHS.

Recall that the mapping to the canonical frustum used the relation between the window size and \( f \) was used in them mapping.

Further comments homogenous coordinates

H.C.'s add a dimension.

All points that \textbf{project} onto the same point on a generalized plane (same dimension as original space) in the H.C. space are the \textbf{same}.

In this course we use the plane \( \hat{w} = 1 \).

Further comments homogenous coordinates

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Transforming canonical frustum to box

The picture should suggest an appropriate scaling for $y$. It is?

On top, $y \rightarrow 1$, so scaling is $(1/y)$
Recall that $y = z$ there.

On bottom, $y \rightarrow -1$ so scaling is $(-1/y)$. Recall that $y = -z$ there.

So scaling is $y' = y/(-z)$
Similarly, $x' = x/(-z)$

Transformation is non-linear, but in h.c., we can make $w = (-z)$. 

Do this in two steps. One stretch in $y$ (and $x$), and on stretch in $z$. 

The camera plane is at $z = -1$.

Recall that $y = z$ there.

Recall that $y = -z$ there.
For $z$, we translate near plane to origin. But now box is too small. Specifically it has $z$ dimension $(1 + z_{\text{min}})$ (recall $z_{\text{min}}$ is negative)

So we have an extra scale factor $1 / (1 + z_{\text{min}})$ and thus $z' = (z - z_{\text{min}}) / (1 + z_{\text{min}})$

But we want $x$ and $y$ to work nicely in h.c., with $w = -z$, so we use $z' = ((z - z_{\text{min}}) / (1 + z_{\text{min}})) / (-z)$

(Thus in our box, depth transforms non-linearly)

In h.c.,

$x \Rightarrow x$
$y \Rightarrow y$
$z' = (z - z_{\text{min}}) / (1 + z_{\text{min}}) / (-z)$
$1 \Rightarrow -z$

So, the matrix is

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 + z_{\text{min}} & -(1 + z_{\text{min}}) \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Mapping to standard view volume (additional comments)

- The mapping from $[z_{\text{min}}, -1]$ to $[0, -1]$ is non-linear. (Of course, there exists a linear mapping, but not if we want everything else to work out nicely in h.c.).
- So a change in depth of $\triangle D$ at the near plane maps to a larger depth difference in screen coordinates than the same $\triangle D$ at the far plane.
- But order is preserved (important!); the function is monotonic (proof?).
- And lines are still lines (proof?) and planes are still planes (important!).
Clipping in homogeneous coordinates

• We have a cube in \((x, y, z)\), but it is not a cube in homogeneous coordinates, so we must divide if we want to take advantage of this particularly nice clipping situation.
• However, dividing before clipping might be inefficient if many points are excluded, so we often clip in homogeneous coordinates.

Clipping in homogeneous coord.’s

• Write h.c.’s in caps, ordinary coords in lowercase.
• Consider case of clipping stuff where \(x > 1, x < -1\)
• Rearrange clipping inequalities:

\[
\begin{align*}
\frac{x}{w} > 1 & \quad \text{becomes} \quad x = \frac{W}{w}, \quad x > \frac{W}{w}, \quad w > 0 \\
\frac{x}{w} < 1 & \quad \text{AND} \quad x = \frac{W}{w}, \quad x < \frac{W}{w}, \quad w < 0
\end{align*}
\]

(So far \(W\) is positive, but negatives occur if we further overload the use of h.c.’s)
Clipping in homogeneous coord.’s

- If we know that $W$ is positive (the case so far!), simply clip against region A.
- If we are using the h.c. for additional deferred division, then $W$ can be negative.
- If $W$ is negative, then we use region B. The clipping can be done by negating the point, and clipping against A, due to the nature of A and B.
- Case where object has both positive and negative $W$ is a little more complex.
- Notice that the actual clipping computations are not that different from the case in Plan A---no free lunch!