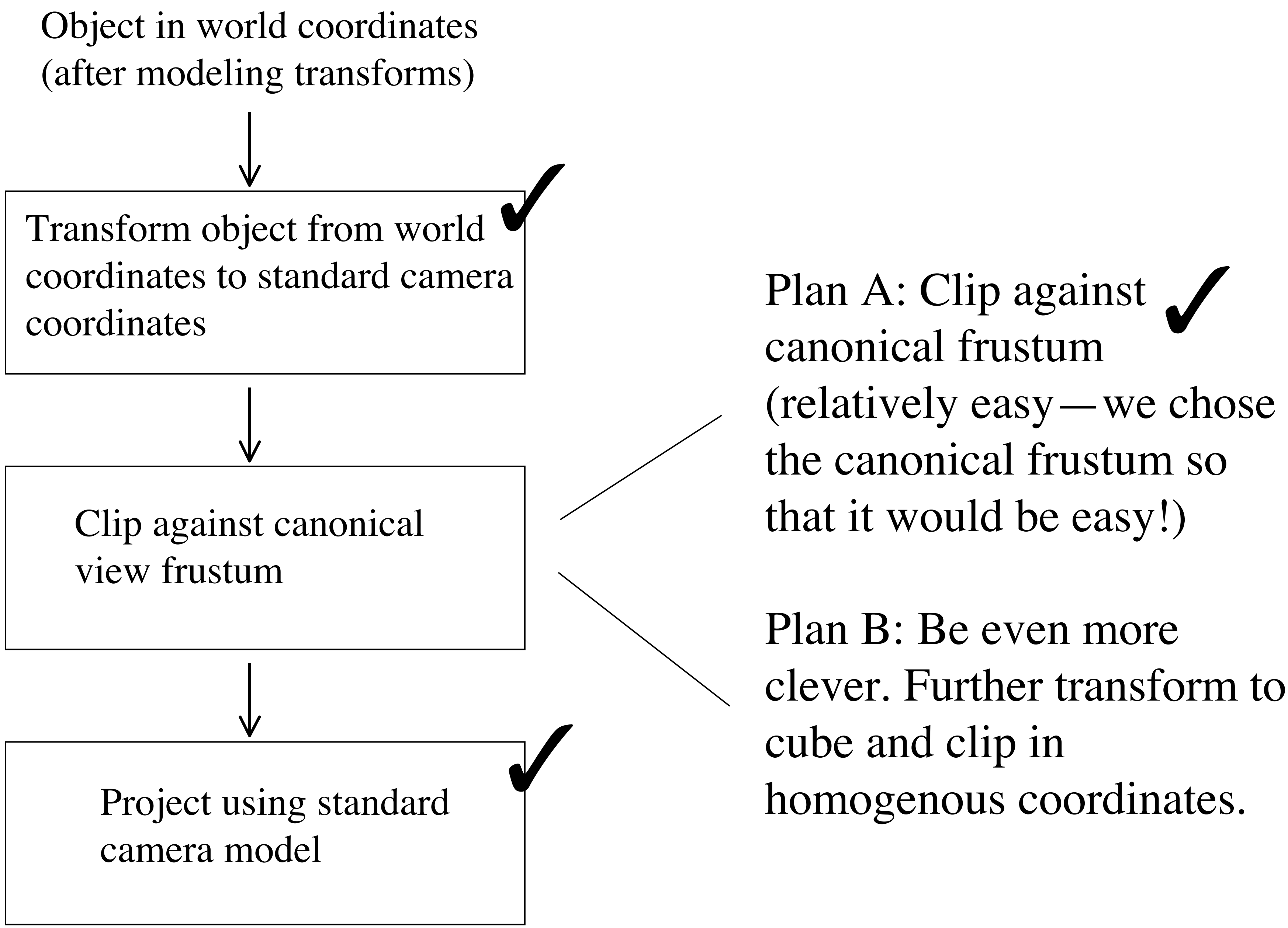
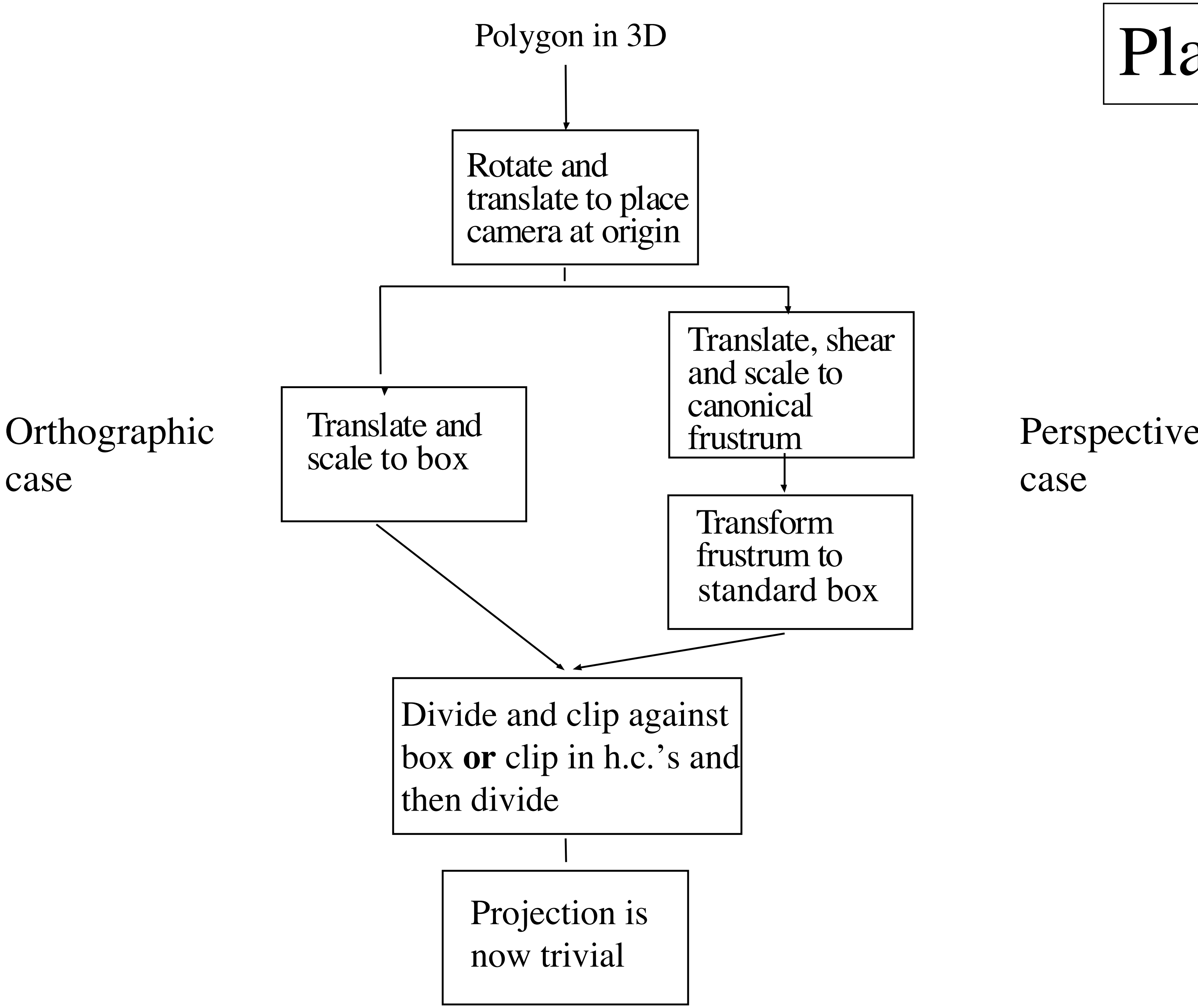

Object in world coordinates (after modeling transforms)
Transform object from world coordinates to standard camera coordinates
Clip against canonical view frustum
Project using standard camera model
Plan A: Clip against canonical frustum (relatively easy—we chose the canonical frustum so that it would be easy!)
Plan B: Be even more clever. Further transform to cube and clip in homogenous coordinates.

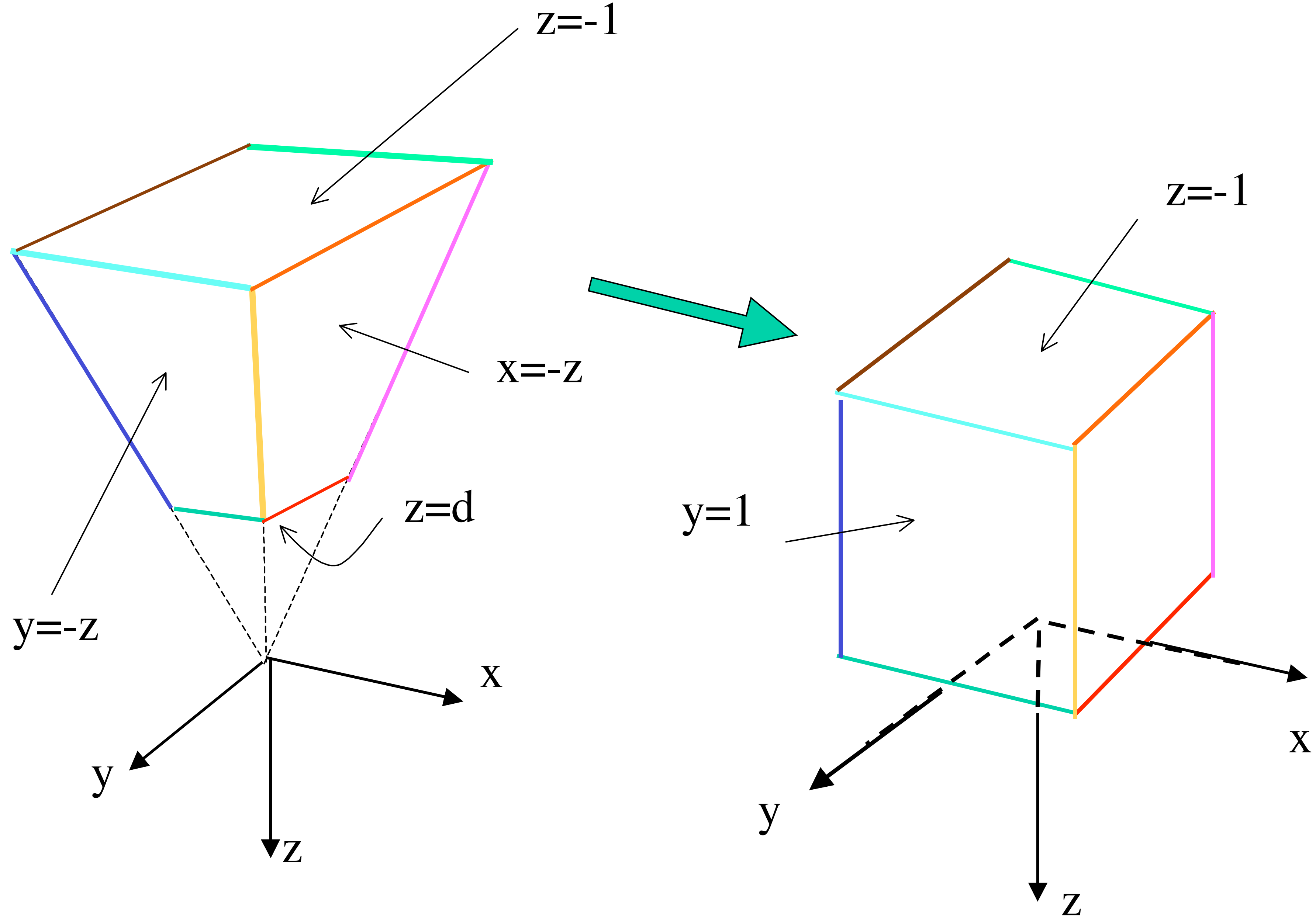
Plan B: Clipping in homogenous coords

- For any camera, can turn the view frustrum into a regular parallelepiped (box). We will use the box bounded by x = ±1, y = ±1, z = -1, and z = 0.
- Advantages
  - Simplified clipping in homogenous coordinates
  - Extends to cases where we use homogenous coordinates to represent additional information (and w could be negative).
  - Can simplify visibility algorithms.
- Approach: clever use of homogenous coordinates

Plan B


Polygon in 3D
Rotate and translate to place camera at origin
Orthographic case
Translate and scale to box
Translate, shear and scale to canonical frustrum
Perspective case
Transform frustrum to standard box
Divide and clip against box **or** clip in h.c.'s and then divide
Projection is now trivial

Transforming canonical frustum to box



Further comments on the canonical frustum

u_{min}, u_{max}, v_{min}, v_{max}, are thought of as being in the camera coordinate system ==> units are that of world coordiante system

For assignment three, you need to choose u_{min}, u_{max}, v_{min}, v_{max}, and f.

Note the reciprocal relation of u_{min}, u_{max}, v_{min}, v_{max}, and f.

For assignment three, I suggest u_{min}, u_{max}, v_{min}, v_{max}, reflect the aspect ratio of your screen window, and set f accordingly.

Further comments on the canonical frustum

Once you have screen (x,y) you need to map them back to the screen coordinates. The canonical frustum gives the screen as a square that is 2f' by 2f'.

$$\hat{x} = x \bullet \frac{u_{\max} - u_{\min}}{2f} = X \bullet \frac{f}{-Z} \bullet \frac{u_{\max} - u_{\min}}{f} = X \bullet \frac{u_{\max} - u_{\min}}{-Z}$$

$$\hat{y} = y \bullet \frac{v_{\max} - v_{\min}}{2f} = Y \bullet \frac{f}{-Z} \bullet \frac{v_{\max} - v_{\min}}{f} = Y \bullet \frac{v_{\max} - v_{\min}}{-Z}$$

Notice that f has disappeared from the RHS.

Recall that the mapping to the canonical frustum used the relation between the window size and f was used in them mapping.

Further comments homogenous coordinates

H.C.'s add a dimension.

All points that **project** onto the same point on a generalized plane (same dimension as original space) in the H.C. space are the **same**.

In this course we use the plane ???

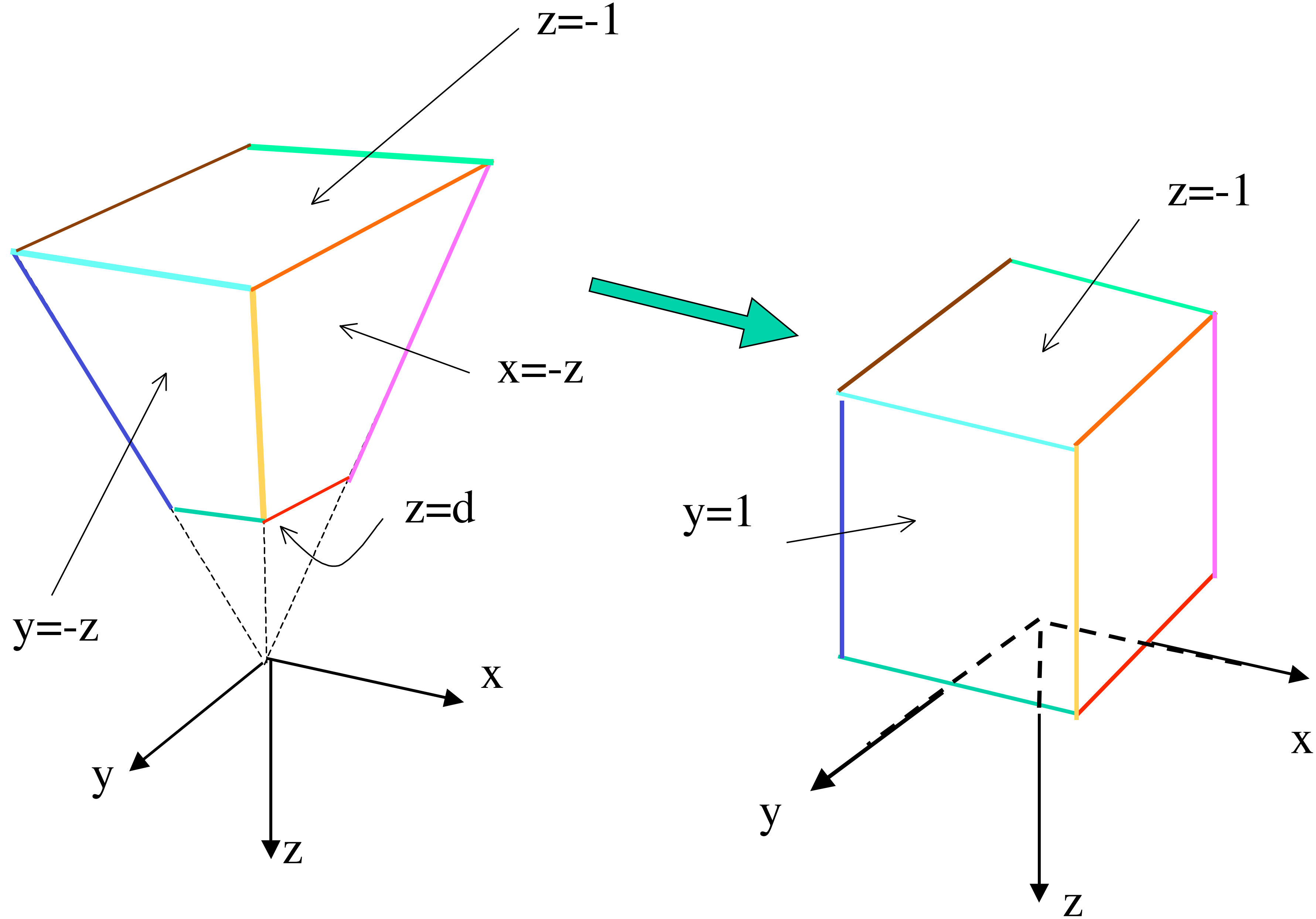
Further comments homogenous coordinates

H.C.'s add a dimension.

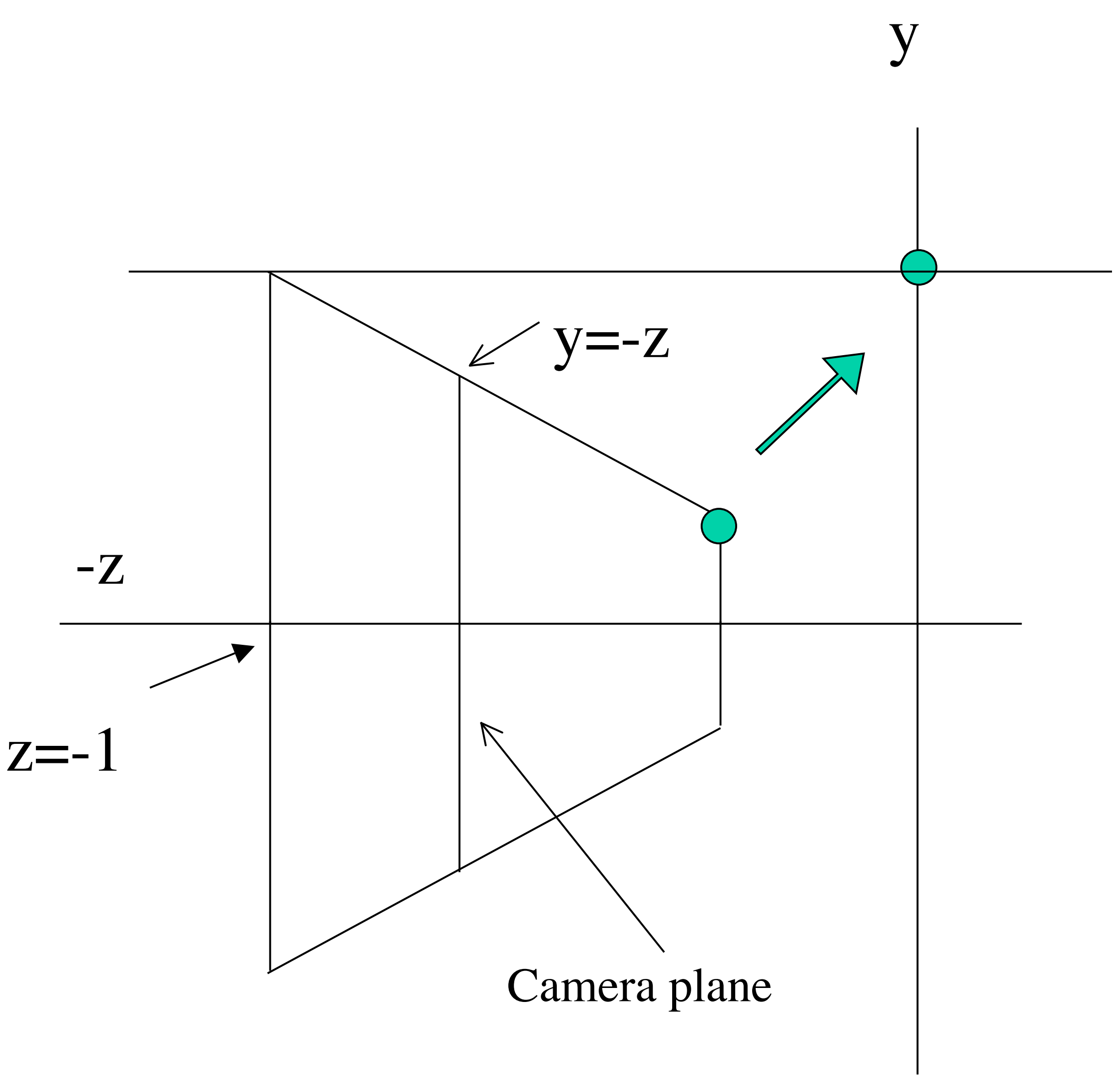
All points that **project** onto the same point on a generalized plane (same dimension as original space) in the H.C. space are the **same**.

In this course we use the plane $w = 1$

Transforming canonical frustum to box

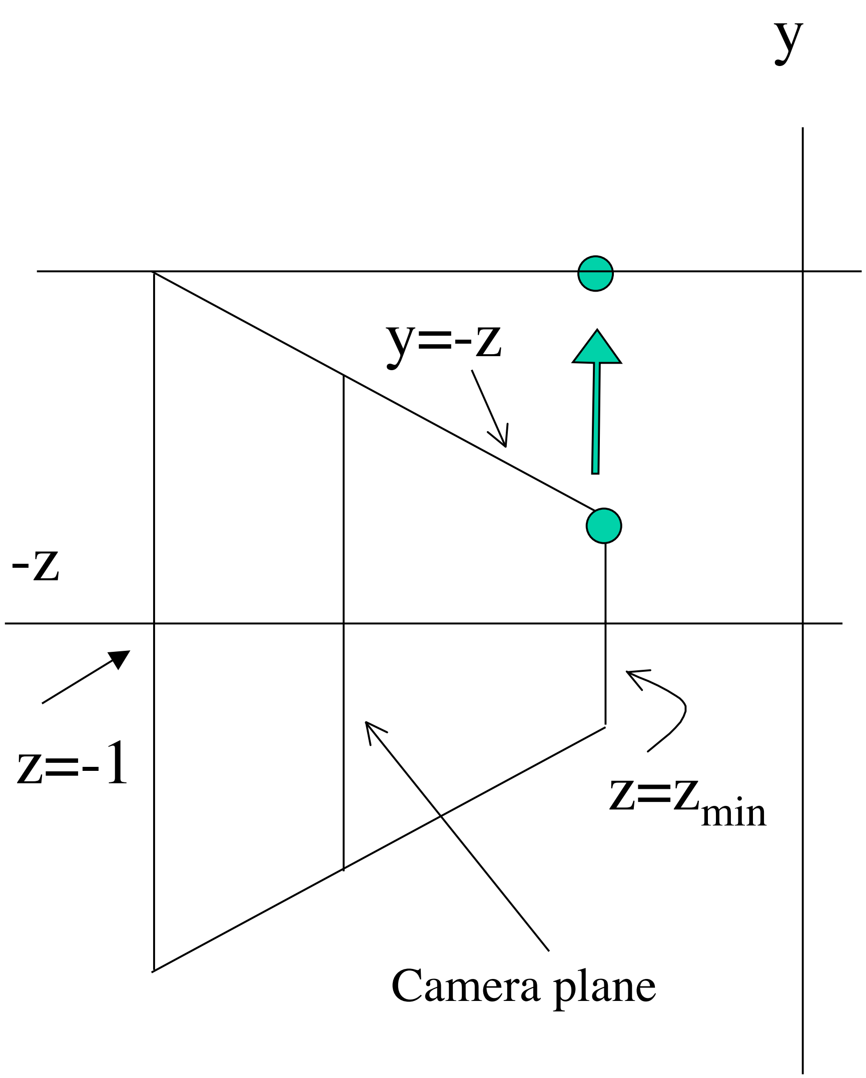


Transforming canonical frustum to box



Do this in two steps. One stretch in y (and x), and on stretch in z.

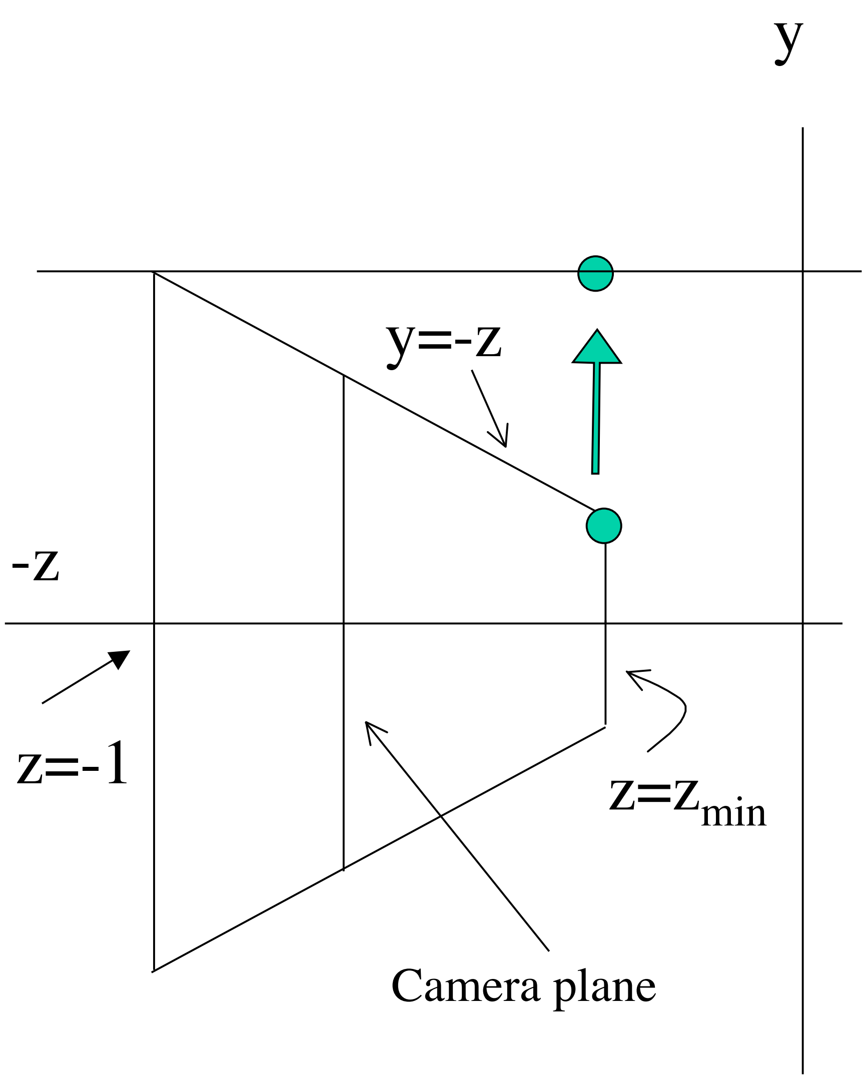
Transforming canonical frustum to box



The picture should suggest an appropriate scaling for y.

It is ?

Transforming canonical frustum to box



On top, y—> 1, so scaling is (1/y) Recall that y=-z there.

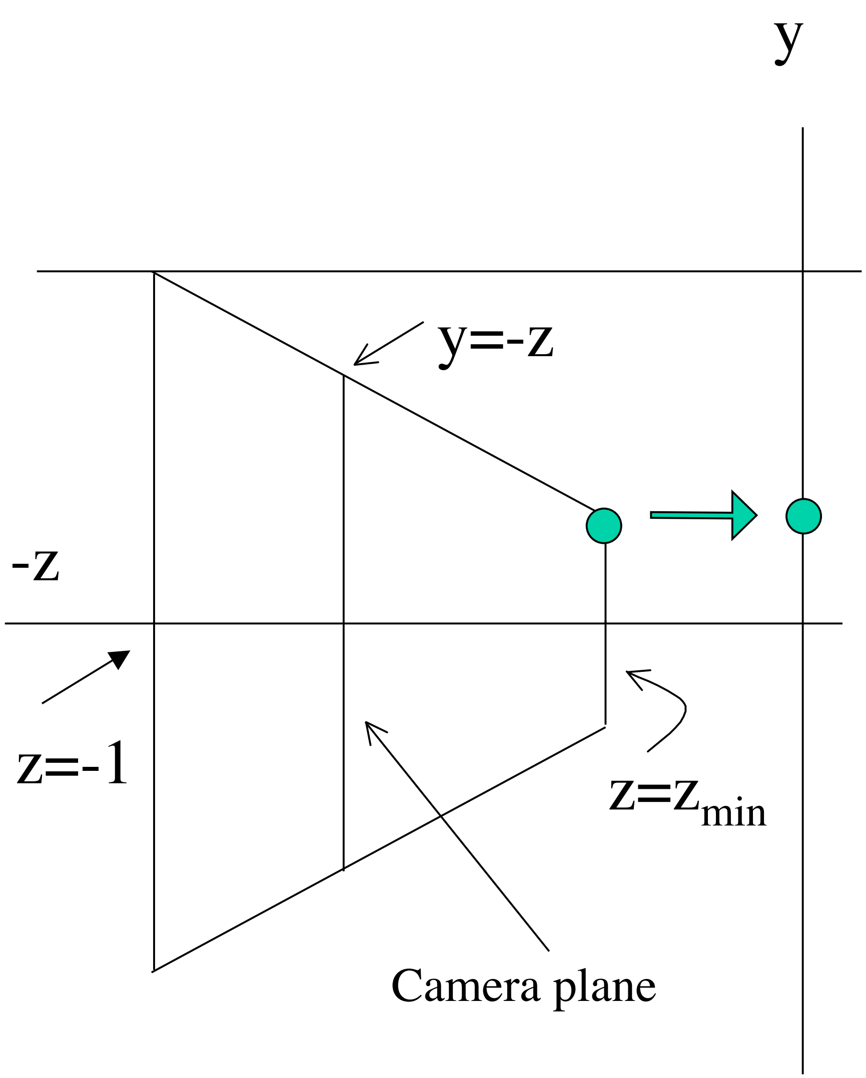
On bottom, y—> -1 so scaling is (-1/y). Recall that y=z there.

So scaling is y'= y/(-z)

Similarly, x' = x/(-z)

Transformation is **non-linear**, but in h.c., we can make w = (-z).

Transforming canonical frustum to box



For z, we translate near plane to origin. But now box is too small. Specifically it has z dimension $(1 + z_{min})$ (recall z_{min} is negative)

So we have an extra scale factor $1 / (1 + z_{min})$ and thus $z' = (z - z_{min}) / (1 + z_{min})$

But we want x and y to work nicely in h.c., with w=-z, so we use

$z' = ((z - z_{min}) / (1 + z_{min}))/(-z)$

(Thus in our box, depth transforms **non-linearly**)

In h.c.,

$x \Rightarrow x$

$y \Rightarrow y$

$z \Rightarrow (z - z_{min}) / (1 + z_{min})$

$1 \Rightarrow -z$

So, the matrix is ?

In h.c.,

x=>x

y=>y

z=>$(z - z_{min}) / (1 + z_{min})$

1=>-z

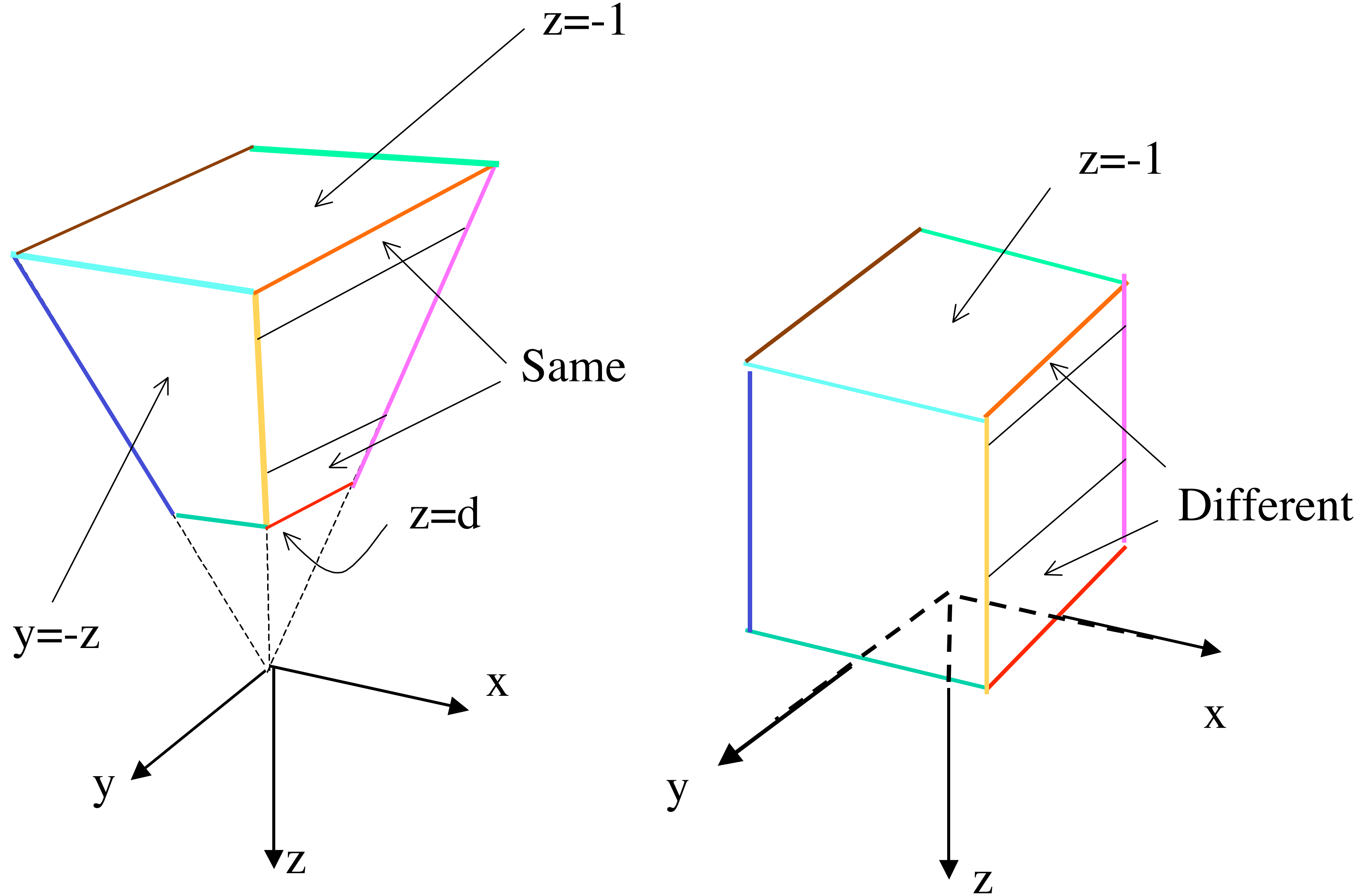
So, the matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{1+z_{min}} & \frac{-z_{min}}{1+z_{min}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Mapping to standard view volume (additional comments)

- The mapping from $[z_{min}, -1]$ to $[0,-1]$ is non-linear. (Of course, there exists a linear mapping, but not if we want everything else to work out nicely in h.c.).
- So a change in depth of $\triangle$ D at the near plane maps to a larger depth difference in screen coordinates than the same $\triangle$ D at the far plane.
- But order is preserved (important!); the function is monotonic (proof?).
- And lines are still lines (proof?) and planes are still planes (important!).

Transforming canonical frustum to box



Clipping in homogeneous coordinates

- We have a cube in (x,y,z), but it is **not** a cube in homogeneous coordinates, so we must divide if we want to take advantage of this particularly nice clipping situation.
- However, dividing before clipping might be inefficient if many points are excluded, so we often clip in homogeneous coordinates.

Clipping in homogeneous coord.'s

- Write h.c.'s in caps, ordinary coords in lowercase.
- Consider case of clipping stuff where x>1, x<-1
- Rearrange clipping inequalities:

$$\left(\frac{X}{W}\right) > 1 \qquad \left(\frac{X}{W}\right) < -1$$

becomes

$$X > W, \quad X < -W, \quad W > 0$$

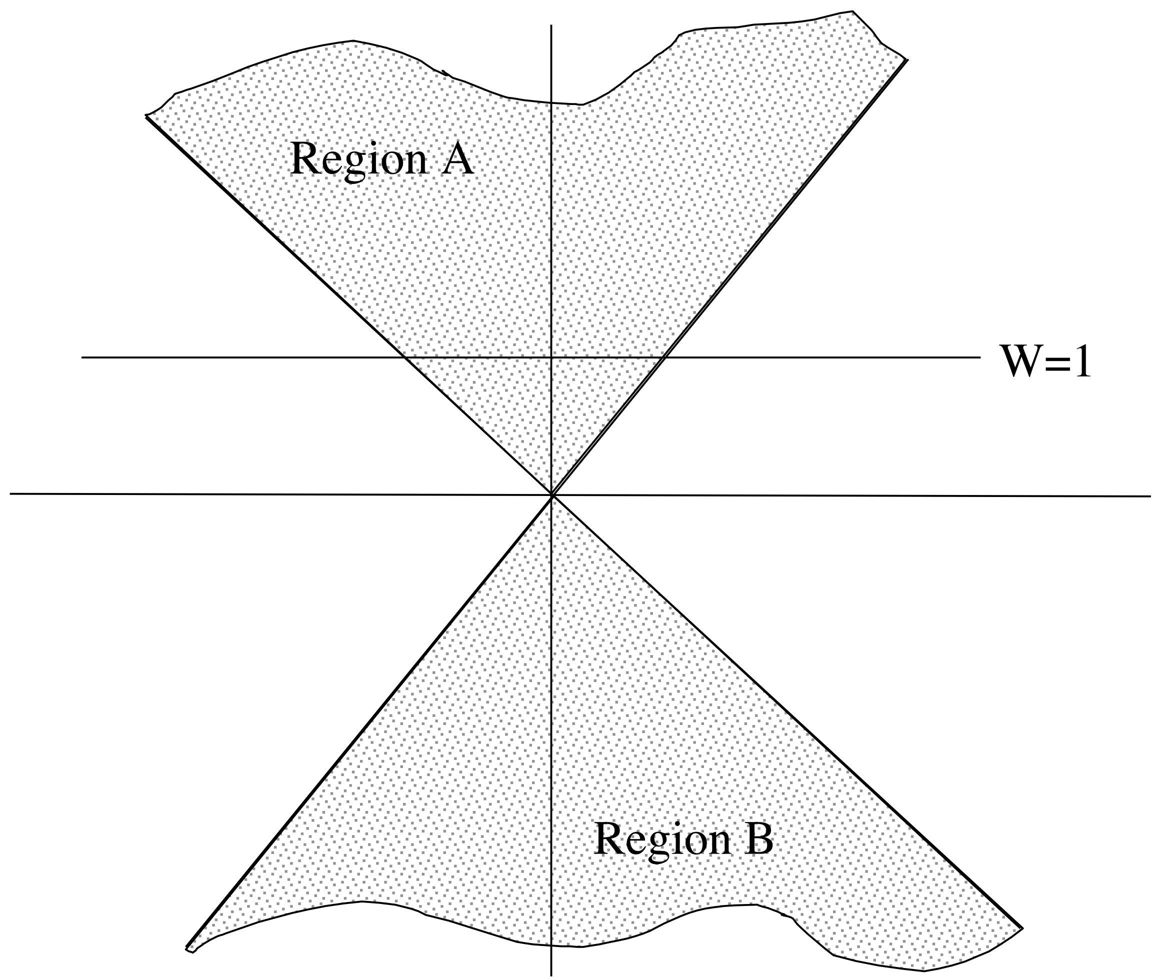
AND

$$X < W, \quad X > -W, \quad W < 0$$

(So far W is positive, but negatives occur if we further overload the use of h.c.'s)

Clipping in homogeneous coord.'s

The clipping volume in cross section



Clipping in homogeneous coord.'s

- If we know that W is positive (the case so far!), simply clip against region A
- If we are using the h.c. for additional deferred division, then W can be negative.
- If W is negative, then we use region B. The clipping can be done by negating the point, and clipping against A, due to the nature of A and B.
- Case where object has both positive and negative W is a little more complex.
- Notice that the actual clipping computations are not that different from the case in Plan A---no free lunch!