More General Reflection

- Many effects when light strikes a surface -- could be:
  - absorbed (could depend on incoming angle)
  - transmitted
  - reflected
  - scattered (in a variety of directions!)
- Typically assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e., they are not sources)
  - all the light leaving a point is due to that arriving at that point

More General Reflection

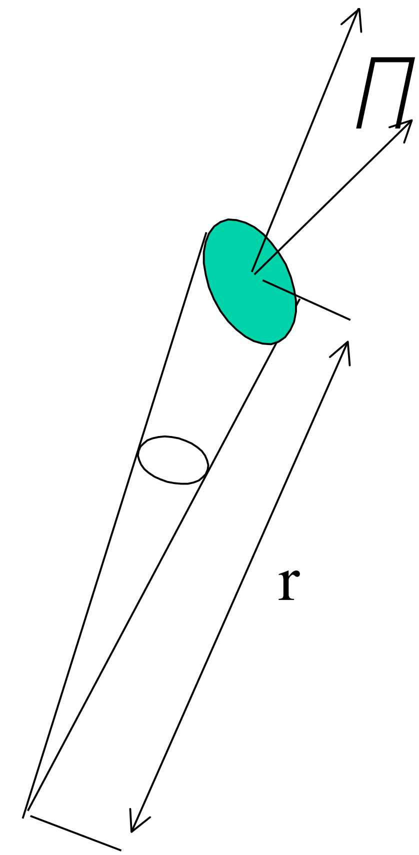
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- This is the ratio of what comes out to what came in
- What comes out <--> “radiance”
- What goes in <--> “irradiance”
- Both are characterized by two angles
- Thus BRDF is a function of four angles
- Technical discussion that follows is optional

Solid Angle

- Analogous to measuring angles radians
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\theta}{r^2}$$

- Units are steradians (sr)



Optional

Radiance

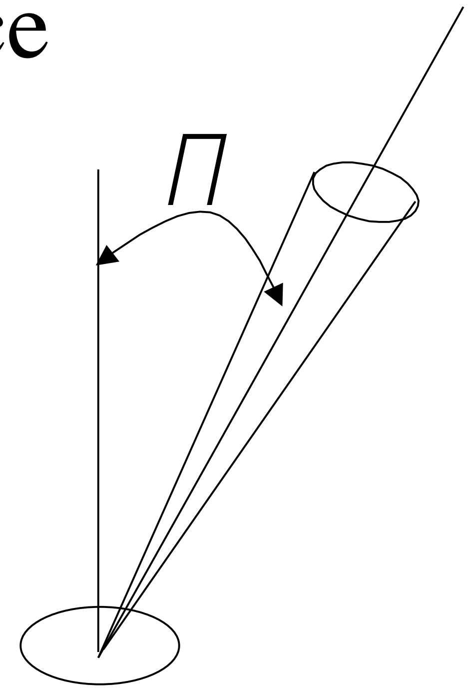
- Amount of light at a point in a particular direction
- Power emitted per unit area per unit solid angle.
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

$$L\left(\underline{x}, \square, \square\right)$$

Optional

Radiance

$$L(\underline{x},\theta,\phi) = \frac{\Delta P(\underline{x})}{\Delta\omega\,\Delta A\cos\theta}$$



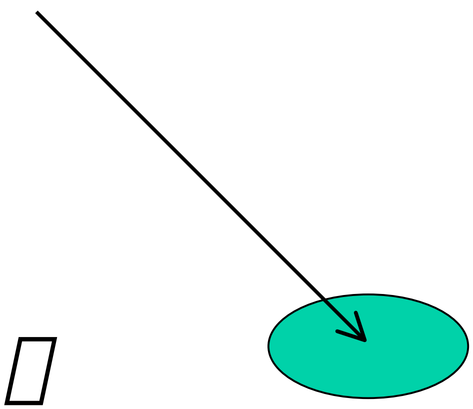
- In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p

Optional

Irradiance

- Irradiance is the amount of light (power) falling on a surface per unit area.
- Units are watts/m^2
- Generally a function of direction

$$E(\underline{x},\theta,\phi) = L(\underline{x},\theta,\phi)\ \delta\omega\ \cos\theta$$



Optional

BRDF (Bidirectional reflectance distribution function)

- The irradiance at a point from a particular angle is:

$$L_i(\underline{x}, \theta_i, \phi_i) \cos\theta_i \, d\omega$$

- The radiance leaving (reflected) in a particular outgoing direction due to the incoming angle is:

$$dL_o(\underline{x}, \theta_o, \phi_o)$$

- The BRDF is simply the ratio of the output to input.

$$\rho_{bd}(\underline{x}, \theta_o, \phi_o, \theta_i, \phi_i) = \frac{dL_o(\underline{x}, \theta_o, \phi_o)}{L_i(\underline{x}, \theta_i, \phi_i) \cos\theta_i \, d\omega}$$

Optional

BRDF (Bidirectional reflectance distribution function)

- If we know the BRDF then:

$$dL_o(\underline{x},\theta_o,\phi_o) = \rho_{bd}(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i,)L_i(\underline{x},\theta_i,\phi_i)\cos\theta_i d\omega$$

- We can integrate over incoming directions to get outgoing radiance in the direction of interest (e.g. towards the viewer).

$$\int_{\Omega}\rho_{bd}(\underline{x},\theta_o,\phi_o,\theta_i,\phi_i,)L_i(\underline{x},\theta_i,\phi_i)\cos\theta_i d\omega_i$$

Optional

BRDF

- Units are inverse steradians (sr^{-1})
- Symmetric in incoming and outgoing directions
  - Has been argued based on thermodynamics (some debate on the validity of such arguments), but regardles, it is not simply a function of the formulation---it depends on fundemental assumptions regarding the nature of the world.

Optional

BRDF

- The "distribution" part of the name is a hint that we need to integrate the function to get some light.
- To compute the brightness of a surface viewed from a given direction, we add up the contributions from all the input directions:

$$\int_{\Omega} \rho_{bd}\left(\underline{x}, \theta_o, \phi_o, \theta_i, \phi_i,\right) L_i\left(\underline{x}, \theta_i, \phi_i\right) \cos\theta_i \, d\omega_i$$

Optional

BRDF

- Note that what we have developed so far is mostly notation, definitions, and descriptions.
- Two approaches to obtaining BRDF's--measure and model.
- Measuring BRDF is painful (but there is some data available on-line and more clever ways to collect the have been proposed).
- Developing physics based approximations for the BRDF for simple classes of surfaces is complicated but possible--this is still an active research area.
- Adding color to the BRDF is easy (one more variable). The full form has additional variables for fluorescence and polarization.

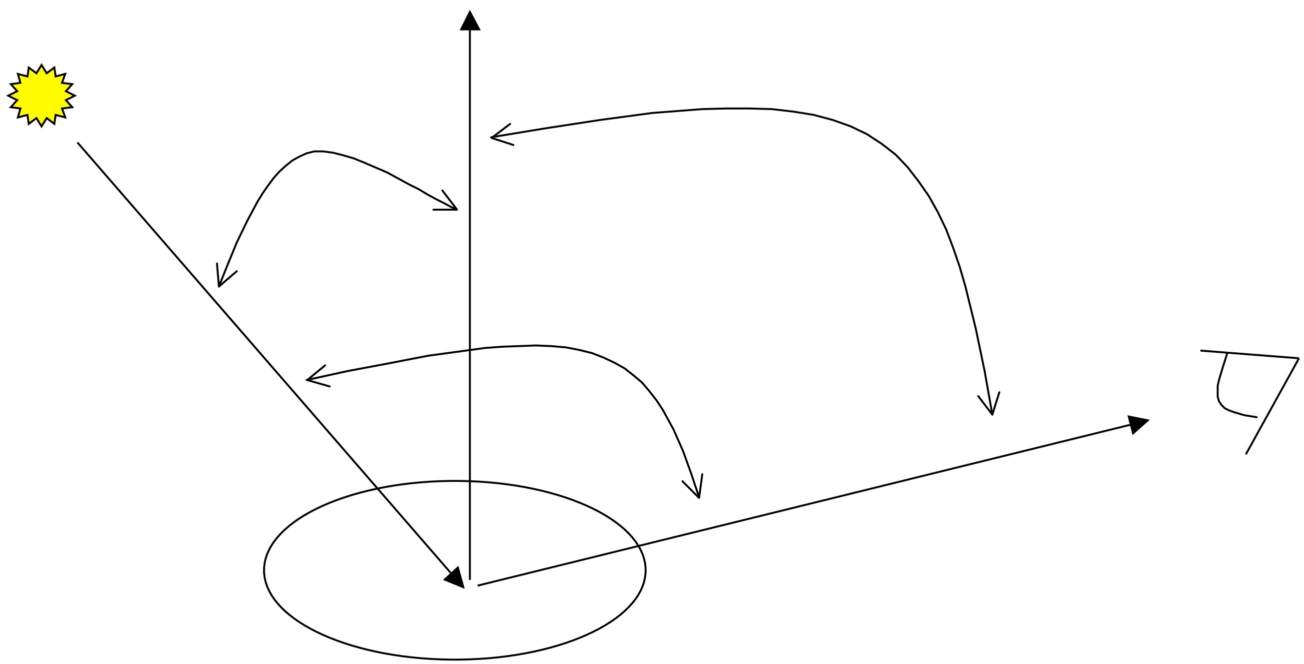
Optional

BRDF

- So why do we care about the BRDF?
  - If you have it, then you can compute the effect of any illumination distribution--a photograph only tells you the effect of one illumination distribution
  - Useful abstraction--surface reflection can be quite complex!

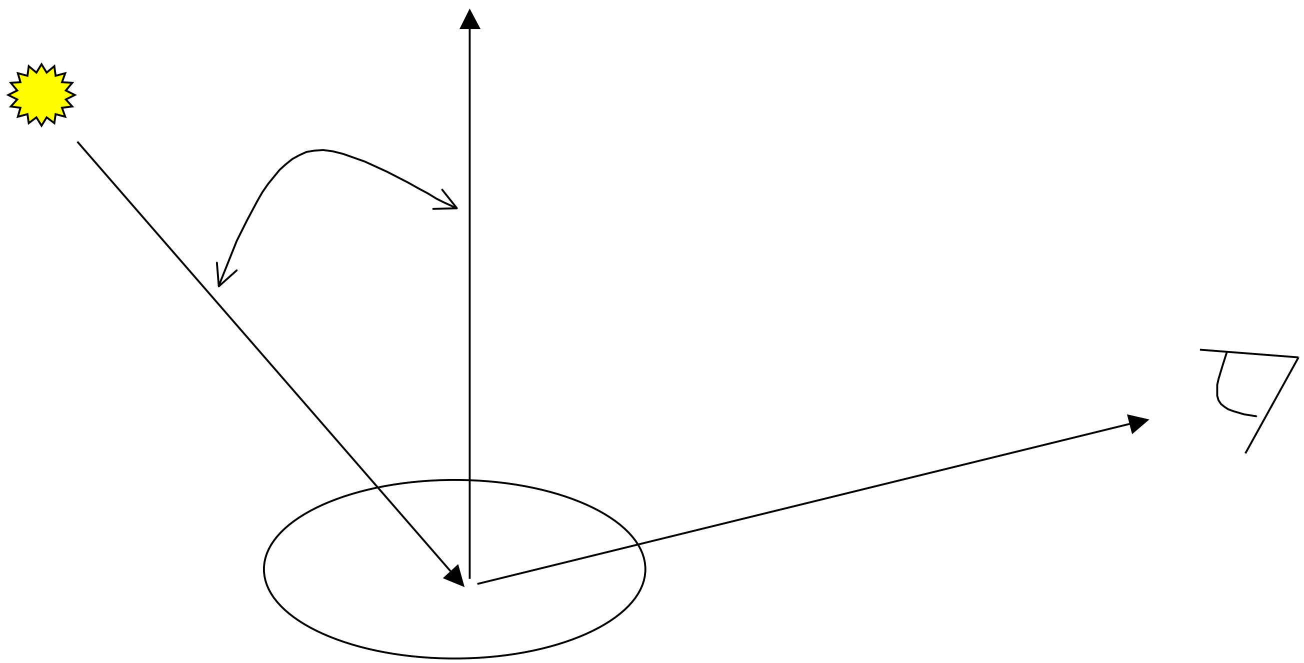
Isotropic surfaces

The BRDF for many surfaces can be well approximated as a function of 3 variables (angles), not 4. In this case, turning the surface around the normal has no effect. The surface is said to be *isotropic*.



Lambertian surfaces

- Even simpler case--the BRDF does not depend on the viewing (output) direction (e.g., Lambertian).



Lambertian surfaces and albedo

- We will refer later to “radiosity” as a unit to describe light leaving the surface taken as whole (def’n next slide)
- Recall that for a Lambertion surface, the direction that light leaves is not an issue.
- Percentage of light leaving the surface is often called diffuse reflectance, or *albedo* for a Lambertian surface.

Radiosity

- Radiosity describes light leaving the surface taken as whole
  - total power leaving a point on the surface, per unit area on the surface (Wm^{-2})

- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \theta, \phi) \cos\theta \, d\omega$$

Optional

Sources and Exitance

- Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- A source will have both
  - radiosity, because it reflects
  - exitance, because it emits

Radiosity leaving = Exitance + Radiosity due to incoming light

Standard nearby point source model (Lambertian reflection)

$$\rho_d(x)\left(\frac{N(x)\bullet S(x)}{r(x)^2}\right)$$

- N is the illuminated surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine
- r(x) is distance from surface point to source --- term occurs because source “looks smaller” as we move away--or, alternatively, its energy is spread out over a larger surface.

Standard distant point source model

- Nearby point source gets bigger if one gets closer, but the effect for far away points is negligible (e.g. the sun).
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much (and can be assumed to not vary with x---ignoring the possibility that it is ocluded), and the distance doesn't vary much either, and we can roll the constants together to get:

$$\rho_d(x)\left(N(x) \bullet S_d\right)$$