Example: Dielectrics

- Examples: Paints, plastics
- Reasonably well approximated by a specular part and a Lambertian body part.

Non conductive matrix with scattering particles of the order of the wavelength of light---note: the same general process explains why the sky is blue.

Trichromaticity

Empirical fact--colors can be approximately described/matched by three quantities (assuming normal color vision).

Need to reconcile this observation with the spectral characterization of light.

Colour Reproduction

Motivates specifying color numerically (there are other reasons to do this also)

General (man in the street) observation--color reproduction sort of works.

Color receptors

"Red" cone
"Green" cone
"Blue" cone

Principle of univariance: cones give the same kind of response, in different amounts, to different wavelengths. Output of cone is obtained by summing over wavelengths.

Responses measured in a variety of ways

\[ \text{Response of } k\text{'th cone} = \int \rho_k(\lambda)E(\lambda)d\lambda \]
Specifying Colour

Test Light Three standard lights

Match?

Trichromacy

Experimental fact about people (with “normal” colour vision)---matching works (for reasonable lights), provided that we are sometimes allowed negative values.

Our “knob” positions correspond to $(X,Y,Z)$ in the standard colorimetry system.

Technical detail: $(X,Y,Z)$ are actually arranged to be positive by a linear transformation, but these “knob” positions cannot correspond to any physical light.
Specifying Colour

We don’t want to do a matching experiment every time we want to use a new color!

Grassman’s Contribution

Colour matching is linear
Matching is Linear (Part 1)

$C_1$ is matched with $(X_1, Y_1, Z_1)$

$C = a \cdot C_1$

$C$ is matched with $a \cdot (X_1, Y_1, Z_1)$
Matching is Linear (formal)

\[ C = a \cdot C_1 + b \cdot C_2 \]

\[ C_1 \text{ is matched with } (X_1, Y_1, Z_1) \]
\[ C_2 \text{ is matched with } (X_2, Y_2, Z_2) \]

\[ C \text{ is matched by } a \cdot (X_1, Y_1, Z_1) + b \cdot (X_2, Y_2, Z_2) \]
Specifying Colour

But what is (R,G,B)?

Specifying Colour

R matches \((X_r, Y_r, Z_r)\)
G matches \((X_g, Y_g, Z_g)\)
B matches \((X_b, Y_b, Z_b)\)

Specifying Colour

Then by 
\((R,G,B)=(75,150,100)\)
you mean \((X,Y,Z)\), where …..

\[
X = 75 \times X_r + 150 \times X_g + 100 \times X_b \\
Y = 75 \times Y_r + 150 \times Y_g + 100 \times Y_b \\
Z = 75 \times Z_r + 150 \times Z_g + 100 \times Z_b
\]

(No need to match--just compute!)
Specifying Colour

…, now that we have specified the colour, I can print it!

\[
\begin{bmatrix}
X \\ Y \\ Z
\end{bmatrix} = \begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix} \begin{bmatrix}
R \\ G \\ B
\end{bmatrix}
\]

\[
\begin{bmatrix}
X \\ Y \\ Z
\end{bmatrix} = \begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
R \\ G \\ B
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_r &amp; X_g &amp; X_b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_r &amp; Y_g &amp; Y_b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z_r &amp; Z_g &amp; Z_b</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
75 \\
100 \\
150
\end{bmatrix}
\]

\[
\begin{bmatrix}
X \\ Y \\ Z
\end{bmatrix} = \begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix} \begin{bmatrix}
R \\ G \\ B
\end{bmatrix}
\]

\[
\begin{bmatrix}
X \\ Y \\ Z
\end{bmatrix} = \begin{bmatrix}
75 \\
100 \\
150
\end{bmatrix}
\]

\[
\begin{bmatrix}
R \\ G \\ B
\end{bmatrix}
\]
Colour Reproduction (Monitors & Projectors)

Find (R,G,B)

\[
\begin{align*}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{\text{apple}} &= M^{-1} \\
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{apple}}
\end{align*}
\]

Possible problems?
XYZ color space

XYZ color space is a linear transformation of the matches to standard lights.

The transformation is used to ensure that all color coordinates are positive.

This means that XYZ corresponds to matches of fictitious (physically impossible) lights.
Qualitative features of CIE x, y

- Why the funny shape?

One measurement of human cone absorption

XYZ response curves

- Linearity implies that colors obtainable by mixing lights with colors A, B lie on line segment with endpoints at A and B
- Monochromatic colours (spectral colors) run along the “Spectral Locus”