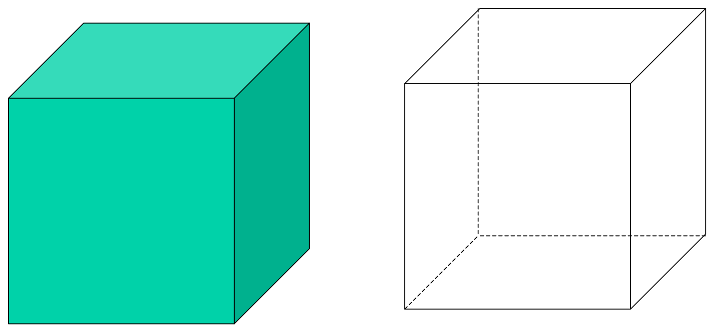
Drawing in 2D*



*That's all there really is!

Displaying lines

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  - lines have integer vertices
  - lines all lie within the displayable region of the frame buffer
- Other algorithms will take care of these issues.

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  - lines have integer vertices
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- Other algorithms will take care of these issues.
- Consider lines of the form y=m x + c, where 0<m<1
- Other cases follow by symmetry
- (Boundary cases, e.g. m=0, m=1 also work in what follows, but are often considered separately, because they can be done very quickly as special cases).

Displaying lines

- Variety of naive (poor) algorithms:
  - step x, compute new y at each step by equation, rounding
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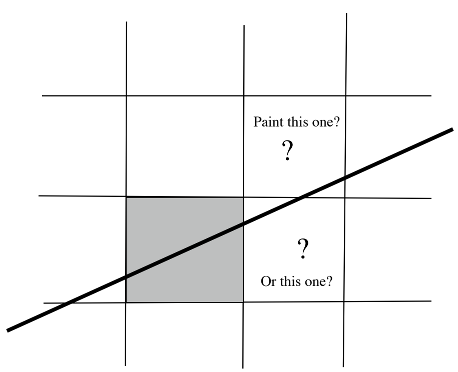
Displaying lines

- Variety of naive (poor) algorithms:
  - step x, compute new y at each step by equation, rounding
  - step x, compute new y at each step by adding m to old y, rounding
- What if we don't assume m<1?
  - our lines can have holes in them!

Bresenham's algorithm [H&B, pp 95-99]

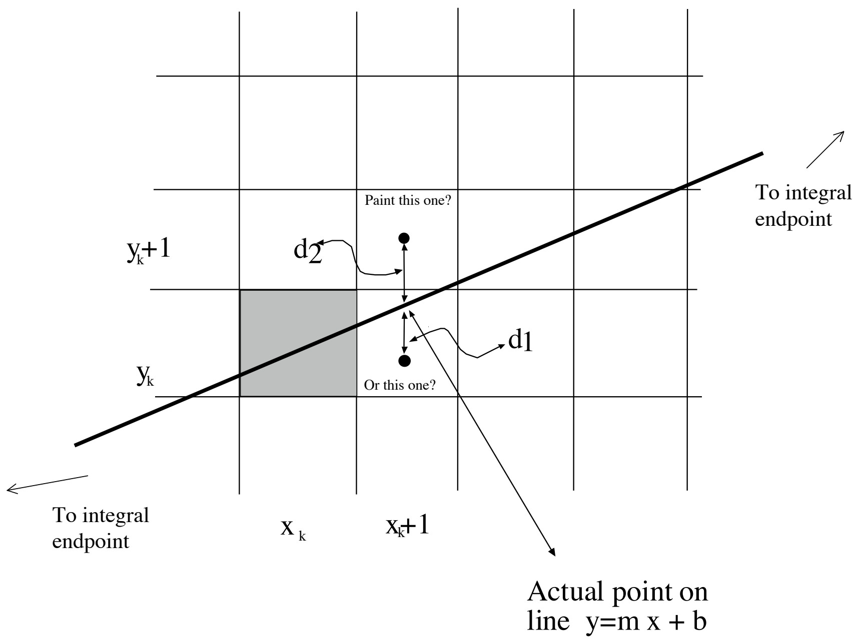
- Plot the pixel whose y-value is closest to the line



Bresenham's algorithm [H&B, pp 95-99]

- Plot the pixel whose y-value is closest to the line

- Given (x_k, y_k), must **choose** from either (x_k+1, y_k+1) or (x_k+1, y_k)---recall we are working on case 0<m<1
- We can derive a "decision parameter" for this choice that is easy to update and cheap to compute (no floating point operations if endpoints are integral).
- "decision parameter" == "determiner"

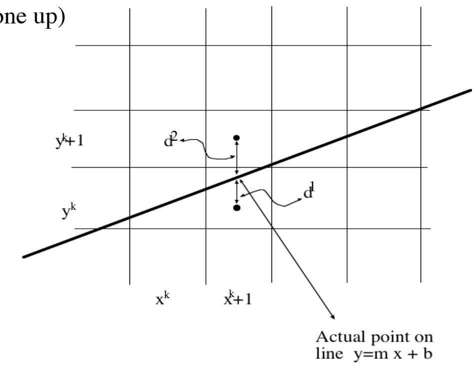


Bresenham's algorithm

- Decision parameter is $d_1 - d_2$

  $d_1 - d_2 < 0$ => plot at y_k (same level as previous)

  otherwise => plot at y_k+1 (one up)



(Current point is, (x_k, y_k) line goes through (x_k+1, y))

$$d_1 = y - y_k \quad and \quad d_2 = (y_k+1) - y$$

So $$d_1 - d_2 = (y - y_k) - ((y_k+1) - y)$$

Plugging in $y = m(x_k+1) + b$

Gives:

$$d_1 - d_2 = 2m(x_k+1) - 2y_k + 2b - 1$$

Avoiding Floating Point

From the previous slide

$$d_1 - d_2 = 2m(x_k+1) - 2y_k + 2b - 1$$

Recall that,

$$m = (y_{end} - y_{start})/(x_{end} - x_{start}) = dy/dx$$

So, for integral endpoints we can avoid division (and floating point ops) if we scale by a factor of dx. Use determiner P_k.

$$\begin{aligned} p_k &= (d_1 - d_2)dx \\ &= (2m(x_k+1) - 2y_k + 2b - 1)dx \\ &= 2(x_k+1)dy - 2y_k(dx) + 2b(dx) - dx \\ &= 2(x_k)dy - 2y_k(dx) + 2(dy) + 2b(dx) - dx \\ &= 2(x_k)dy - 2y_k(dx) + \text{constant} \end{aligned}$$

(No division)

Incremental Update

From previous slide

$$p_k = 2(x_k)dy - 2y_k(dx) + \text{constant}$$

Finally, express the next determiner in terms of the previous, and in terms of the decision on the next y.

$$\begin{aligned} p_{k+1} &= 2(x_k + 1)dy - 2y_{k+1}(dx) + \text{constant} \\ &= p_k + 2dy - 2(y_{k+1} - y_k) \end{aligned}$$

Either 1 or 0 depending on decision on y

Bresenham algorithm

- p_{k+1} = p_k + 2 dy - 2 dx $(y_{k+1}-y_k)$
- Exercise*: check that p_0= 2 dy - dx
- Algorithm (for the case that 0<m<1):
  - x=x_start, y=y_start, p=2 dy - dx, **mark** (x, y)
  - until x=x_end
    - x=x+1
    - p>0 ? y=y+1, **mark** (x, y), p=p+2 dy - 2 dx
    - else y=y, **mark** (x, y), p=p+2 dy
- Some calculations can be done once and cached.

*Hint: For p_0, (x_k,y_k) is on the line. Use formula for the line and plug into expression for p_k from two slides back: $2(x_k)dy - 2y_k(dx) + 2(dy) + 2b(dx) - dx$

Issues

- End points may not be integral due to clipping (or other reasons)
- Brightness is a function of slope.
- Discretization problems "aliasing" (related to previous point).

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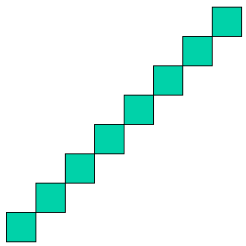
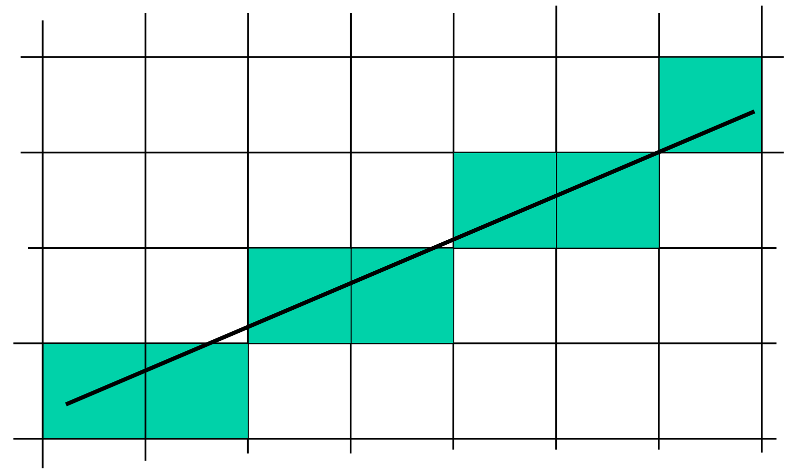
Line drawing--simple line (Bresenham) brightness issues



8 pixels per 8*sqrt(2) length

8 pixels for 8 length
(Brighter)

Line drawing--discretization artifacts (often called aliasing)



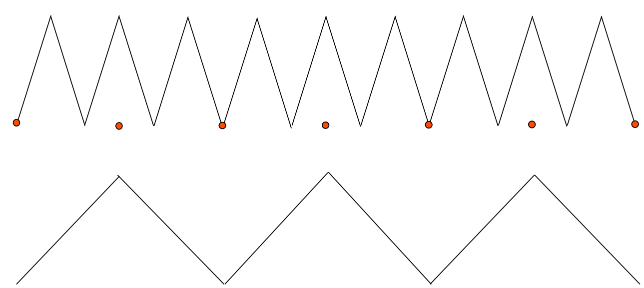
Aliasing

[H&B, pp 214-221]

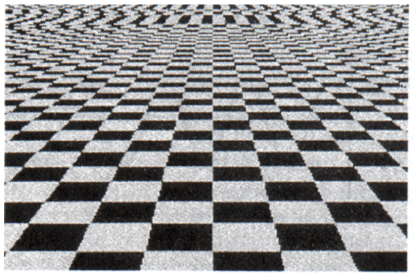
- We are using discrete binary squares to represent perfect mathematical entities
- To get a value for that that square we used a “sample” at a particular discrete location.
- The sample is somewhat arbitrary due to the choice of discretization, leading to the jagged edges.
- Insuffіcent samples mean that higher frequency parts of the signal can “alias” (masquerade as) lower frequency information.

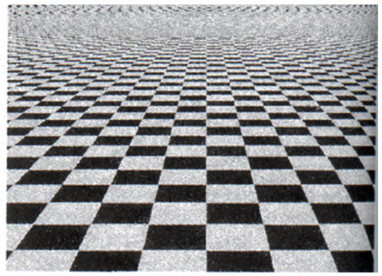
Aliasing

[H&B, figuure 4-46]



Aliasing





from Watt and Policarpo, The Computer Image

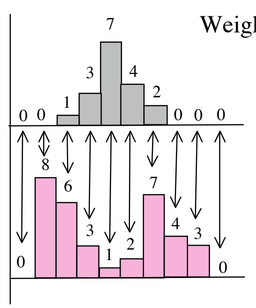
Aliasing (cont)

- Points and lines as discussed so far have no width. To make them visible we concocted a way to sample them based on which discrete cell was closer
- General approach to reducing aliasing is to exploit ability to draw levels of gray between black and white.
- Example--give the line some width; brightness is proportional to area that pixel shares with line
- A more principled approach (which subsumes the above) is to “filter” before sampling.

Linear Filters (background)

- General process: Form new image whose pixels are a **weighted sum** of original pixel values, using the same set of weights at each point.

Weights (kernel of filter)

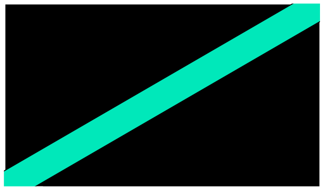


Multiply lined up pairs of numbers and then sum up to get weighted average at the filter location. Then shift the filter and do the same to get the next value.

Signal

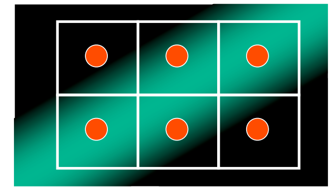
Aliasing via filtering and then sampling

- A filter can be thought of as a weighted average. The weights are given by the filter function. (Examples to come).
- **Conceptually**, we smooth (convolve) the object to be drawn by applying the filter to the mathematical representation.
- This blurs the object, widens the area it occupies
- Now we “sample” the blurred image--i.e., report the value of the blurred function at the (x,y) of interest, and then fill the square with that brightness.
- (**Technically** we only need to compute the blur at the sampling locations)

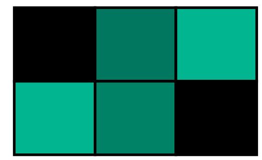


Line with width

Blurred



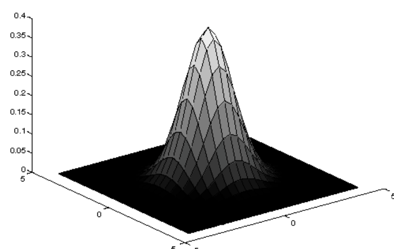
Sample



Paint with
sample value

Aliasing via filtering and then sampling

- Ideal filter is usually Gaussian
- Easier and much faster to approximate Gaussian with a cone



Anti-aliasing via filtering and then sampling

Technically we "convolve" the function representing the primitive g(x,y) with the filter, h (ζ, η)

$$g \otimes h = \iint g(x-\xi, y-\eta)\, h(\xi,\eta)\, d\xi\, d\eta$$

Exact expression is optional