Determining the screen coordinates

Once you have (x,y) you need to map them back to the screen coordinates.

Use primes (‘) for mapped quantities and carets (^) for screen quantities.

The canonical frustum gives the screen as a square that is 2f’ by 2f’. Note that f’ is between 0 and 1 (why?)

Our window on the screen has corners: $(\hat{u}_{\min}, \hat{u}_{\max})$ and $(\hat{v}_{\min}, \hat{v}_{\max})$
(Unless we want to distort things we assume the same aspect ratio as the camera window.)

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Our screen coordinates are then:

$$\hat{x} = \left(\frac{\hat{u}_{\max} + \hat{u}_{\min}}{2}\right) + \left(\frac{x'}{2f'}\right) \bullet (\hat{u}_{\max} - \hat{u}_{\min})$$
$$\hat{y} = \left(\frac{\hat{v}_{\max} + \hat{v}_{\min}}{2}\right) + \left(\frac{y'}{2f'}\right) \bullet (\hat{v}_{\max} - \hat{v}_{\min})$$

Or, equivalently:

$$\hat{x} = \hat{u}_{\min} + \left(\frac{x' + f'}{2f'}\right) \bullet (\hat{u}_{\max} - \hat{u}_{\min})$$
$$\hat{y} = \hat{v}_{\min} + \left(\frac{y' + f'}{2f'}\right) \bullet (\hat{v}_{\max} - \hat{v}_{\min})$$

Notice that if X is the coordinate after the shear, then:

$$\left(\frac{x'}{f'}\right) = \frac{X * \left(\frac{2f}{u_{\max} - u_{\min}}\right) * \left(\frac{1}{f-B}\right) * \left(\frac{f'}{-z}\right)}{f'}$$
$$= \frac{X * \left(\frac{2f}{u_{\max} - u_{\min}}\right) * \left(\frac{1}{f-B}\right) * \left(\frac{f/(f-B)}{-Z/(f-B)}\right)}{\left(\frac{f}{f-B}\right)}$$
$$= \left(\frac{2f}{u_{\max} - u_{\min}}\right)\left(\frac{X}{-Z}\right)$$

which is what you would expect. (You do not need to use this formula, it is only for further explanation).

The first factor is the natural magnification as explained a few slides back.