Transforming canonical frustum to box

Clipping in homogeneous coordinates

- We have a cube in \((x,y,z)\), but it is **not** a cube in homogeneous coordinates, so we must divide if we want to take advantage of this particularly nice clipping situation.
- However, dividing before clipping might be inefficient if many points are excluded, so we often clip in homogeneous coordinates.

Clipping in homogeneous coord.’s

- Write h.c.'s in caps, ordinary coords in lowercase.
- Consider case of clipping stuff where \(x>1, x<-1\)
- Rearrange clipping inequalities:

\[
\begin{align*}
\frac{X}{W} &> 1 & \Rightarrow X &> W, \\
\frac{X}{W} &< -1 & \Rightarrow X &< -W, \\
\end{align*}
\]

AND

\[
\begin{align*}
X &< W, \\
X &> -W, \\
W &> 0 \\
W &< 0
\end{align*}
\]

(So far \(W\) has been positive, but negatives occur if we further overload the use of h.c.’s)
**Clipping in homogeneous coord.’s**

- If we know that $W$ is positive (the case so far!), simply clip against region A.
- If we are using the h.c. for additional deferred division, then $W$ can be negative.
- If $W$ is negative, then we use region B. The clipping can be done by negating the point, and clipping against A, due to the nature of A and B.
- Case where object has both positive and negative $W$ is a little more complex.
- Notice that the actual clipping computations are not that different from the case in Plan A—no free lunch!

**Reminder of the last steps**

In both plans we need to project into 2D.

If we are working in the canonical view space, then we project using the standard camera model (easy) and divide.

Recall that the matrix for the standard camera model using homogeneous coordinates is:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \frac{1}{f}
\end{bmatrix}
\]

Reminder of the last steps

If we are working in homogenous coordinates, then we first divide and then projection is even easier (ignore z coordinate).

The mapping to the box—which was complete once the division was done—implicitly did the perspective projection—essentially we transformed the world so that orthographic projections holds.
Reminder of the last steps

Finally, we may need to do additional 2D transformations.

In the canonical frustum case, our (x,y) coordinates are relative to\((-f',f')\). They need to be mapped to the viewport (possibly implicitly by the graphics package).

In the canonical box case, our (x,y) coordinates are relative to\((-1,1)\). They need to be mapped to the viewport (possibly implicitly by the graphics package).

Visibility

H&B chapter 9 (similar to notes)

- Of these polygons, which are visible? (in front, etc.)

- Very large number of different algorithms known. Two main (very rough) classes:
  - Object precision: computations that decompose polygons in world coordinates
  - Image precision: computations at the pixel level

- Depth order in standard view box is same as depth order in 3D, so can work with the box.

Visibility

H&B chapter 9 (similar to notes)

- Essential issues:
  - must be capable of handling complex rendering databases.
  - in many complex worlds, few things are visible
  - efficiency - don’t render pixels many times.
  - accuracy - answer should be right, and behave well when the viewpoint moves
  - aliasing

Image Precision

- Typically simpler algorithms (e.g., Z-buffer, ray cast)

- Pseudocode (conceptual!)
  - For each pixel
    - Determine the closest surface which intersects the projector
    - Draw the pixel the appropriate color
Image Precision

- “Image precision” means that we can save time not computing precise intersections of complicated objects

- But the algorithms are subject to aliasing problems, and the sampling needs to be redone when the view changes, even if only a simple window resize

Object Precision

- The algorithms are typically more complex

- Pseudocode (conceptual)
  - For each object
    - Determine which parts are viewed without obstruction by other parts of itself or other objects
    - Draw those parts the appropriate color

Visibility - Back Face Culling

- Simple, preliminary step, to reduce the amount of work.

- Polygons from solid objects have a front face and back face

- If the viewer sees the back face, then the plane can be culled.

- Why would the viewer see the back face?
  - Because they are on the back side of the plane of the polygon.

Visibility - Back Face Culling

\[ \mathbf{n} \cdot \mathbf{v} > 0 \]

\( \mathbf{v} \) is direction from any point on the plane to the center of projection (the eye).

If \( \mathbf{n} \cdot \mathbf{v} > 0 \), then display the plane

Note that we are calculating which side of the plane the eye is on.

Question: How do we get \( \mathbf{n} \) (e.g., for the assignment)
Question: How do we get \( n \)? (e.g., for the assignment)

Answer

When you render the parallelepiped, you have to create the faces which are sequences of vertices.

To compute \( n \) from vertices, use cross product.

You need to store vertices consistently so that you can get the sign of \( n \). Consider storing them so that you can get the sign of \( n \) by RHR.

Depending on the situation you may find it easier to
1) compute \( n \) early on, and transform it using the correct formula
2) recompute it from transformed vertices.

Question: In which coordinate frames can/should we do this?

Answer

All of them. It is perhaps more natural to attempt do this in the standardized view box where perspective projection has become parallel projection.

Here, \( e=(0,0,1) \) (why?), so the test \( n.e>0 \) is especially easy (\( n_z>0 \)).

**But be careful** with the transformation which lead to \( n! \)

Also, an efficiency argument can be made for culling before division.