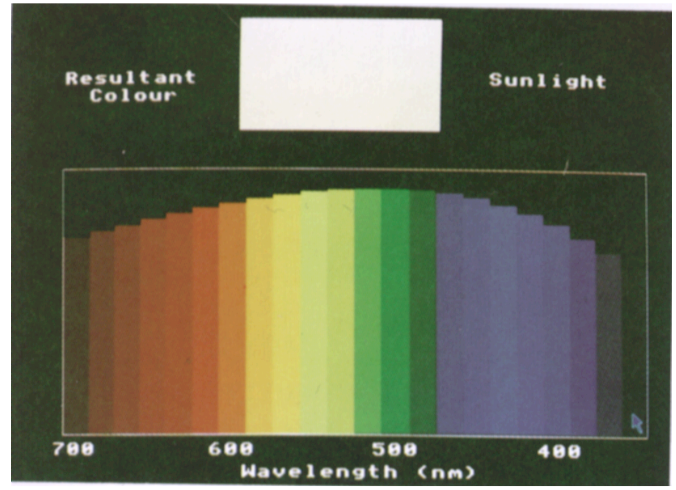
The colors of the rainbow

- Light is electromagnetic radiation, occurring at different wavelengths (or photon energies)
- The radiation around us is a mix of these
- Visible portion is about 400 to 700 nm
- Certain applications may require modeling some UV also.
- Light is specified by its spectrum recording how much power is at each wavelength.

Sunlight



Two disparate source spectra

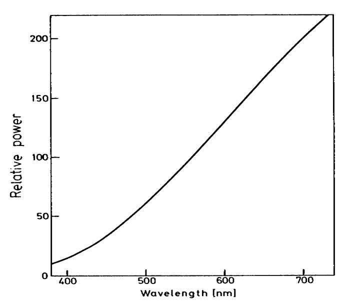


Fig. 4.1. Wavelength composition of light from a tungsten-filament lamp [typified by CIE ILL A (Sect. 4.6)]. Relative spectral power distribution curve. Color temperature: 2856 K

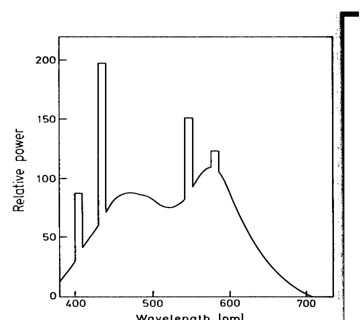
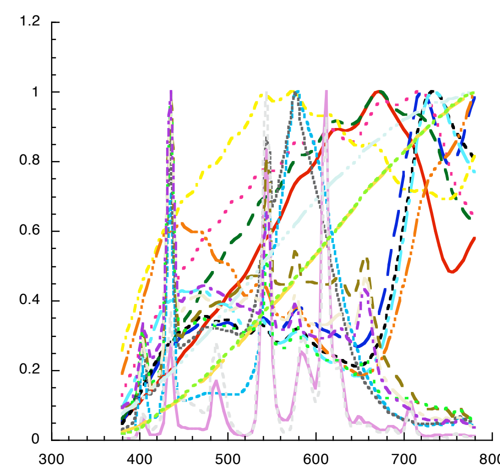
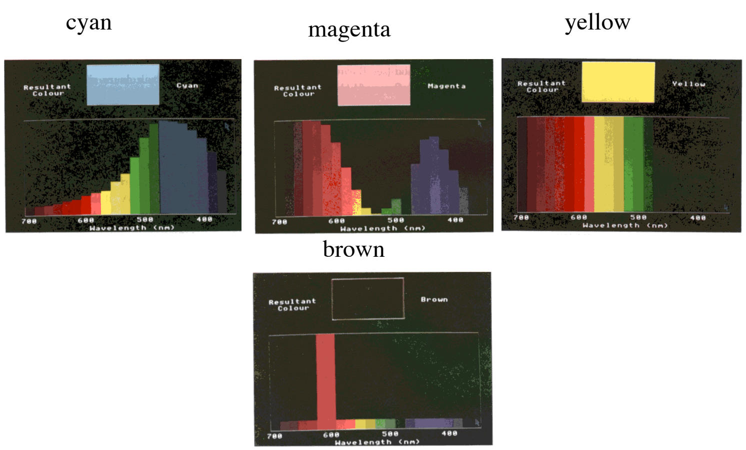


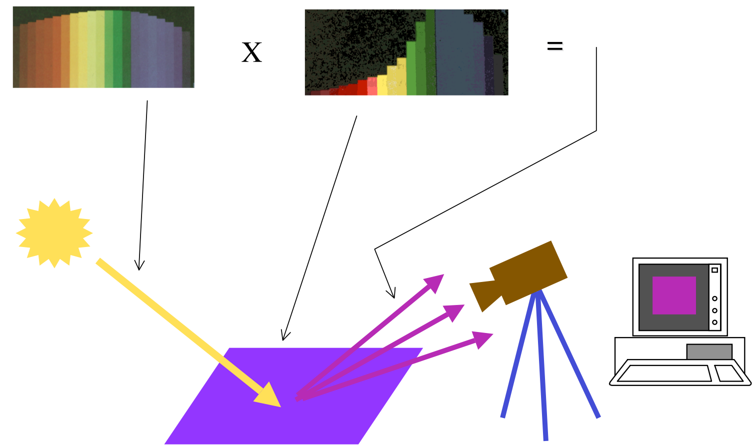
Fig. 4.2. Wavelength composition of light from a daylight fluorescent lamp. Typical relative spectral power distribution curve. Correlated color temperature: 6000 K. (Based on data of Jerome reported in [Ref. 3.14, p. 37])

Energy spectra of 20 other common lights



Absorption spectra: real pigments

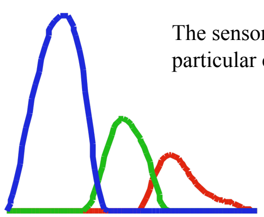




Sensors

Sensors (including those in your eyes) have a varied sensitivity over wavelength

Different variations lead to different kinds of sensor responses ("colors" in a naïve sense)

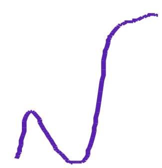


The sensor sensitivity spectra for a particular color camera (they vary a lot).

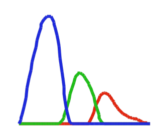
<--- wavelength --->

Image Formation (Spectral)

$$(R,G,B) = \int_{380}^{780}$$



$*$



$d\lambda$

More formally,

The response of an image capture system to a light signal $L(\lambda)$ associated with a given pixels is modeled by

$$\rho^{(k)} = \int L(\lambda) R^{(k)}(\lambda) d\lambda$$

where $R^{(k)}(\lambda)$ is the sensor response function for the k^{th} channel.

Note the usual case of three channels

$$(R, G, B) = \left(\rho^{(1)}, \rho^{(2)}, \rho^{(3)}\right)$$

Discrete Version

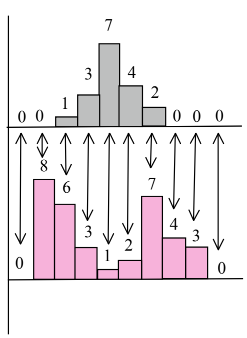
Often we represent functions by vectors. For example, a spectra might be represented by 101 samples in the range of 380 to 780 nm in steps of 4nm.

Then $L(\lambda)$ becomes the vector **L**, $R^{(k)}(\lambda)$ becomes the vector $\mathbf{R^k}$, and the response is given by a dot product:

$$\rho^{(k)} = \mathbf{L} \bullet \mathbf{R^{(k)}}$$

Sensor/light interaction example

R=(0,0,1,3,7,4,2,0,0,0)

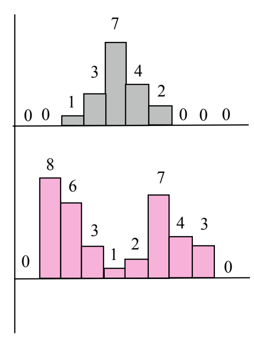


L=(0,8,6,3,1,2,7,4,3,0)

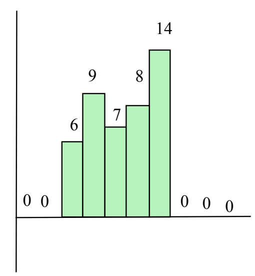
Multiply lined up pairs of numbers and then sum up

Sensor/light interaction example

R=(0,0,1,3,7,4,2,0,0,0)



L=(0,8,6,3,1,2,7,4,3,0)



L .* R=
(0*0, 0*8, 1*6, 3*3, 7*1, 4*2, 2*7, 0*4, 0*3, 0*0)
= (0, 0, 6, 9, 7, 8, 14, 0, 0, 0)

L • P = 0 + 0 + 6 + 9 + 7 + 8 + 14
= 44

Image Formation (Spectral)

- Note that by this model, light capture is linear.
- Formally this means

?

Image Formation (Spectral)

- Note that by this model, light capture is linear.
- Formally this means:

$$L_1(\lambda) -> \rho_1^{(k)} \text{ and } L_2(\lambda) -> \rho_2^{(k)}$$

- Then:

$$aL_1(\lambda) + bL_2(\lambda) -> a\rho_1^{(k)} + b\rho_2^{(k)}$$

Image Formation (Spectral)

- Note that image formation loses spectral information
- This means that two quite different spectra can map into the same color

One tricky bit

Electronic capture (e.g. "CCD") is linear, but typically the circuitry will put the sensor responses through a non-linear mapping (e.g. approximate square root).

This is because display is usually either non-linear due to physics (CRT) or by design (to be like a CRT). This is better because there is less relative noise where humans will notice it.

(A bit more on this later).

Causes of color

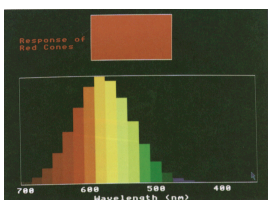
- The sensation of color is caused by the brain.
- One way to get it is through a **response** of the eye to the presence/absence of light at various wavelengths.
- Dreaming, hallucination, etc.
- Pressure on the eyelids

Trichromaticity

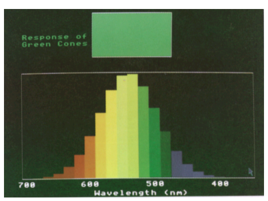
Empirical fact--colors can be approximately described/matched by three quantities (assuming normal color vision).

Need to reconcile this observation with the spectral characterization of light

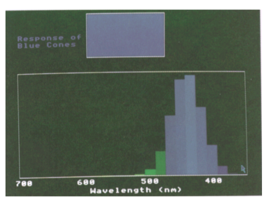
Color receptors



"Long" cone



"Medium" cone



"Short" cone

Some understanding results from an analogy with camera sensors

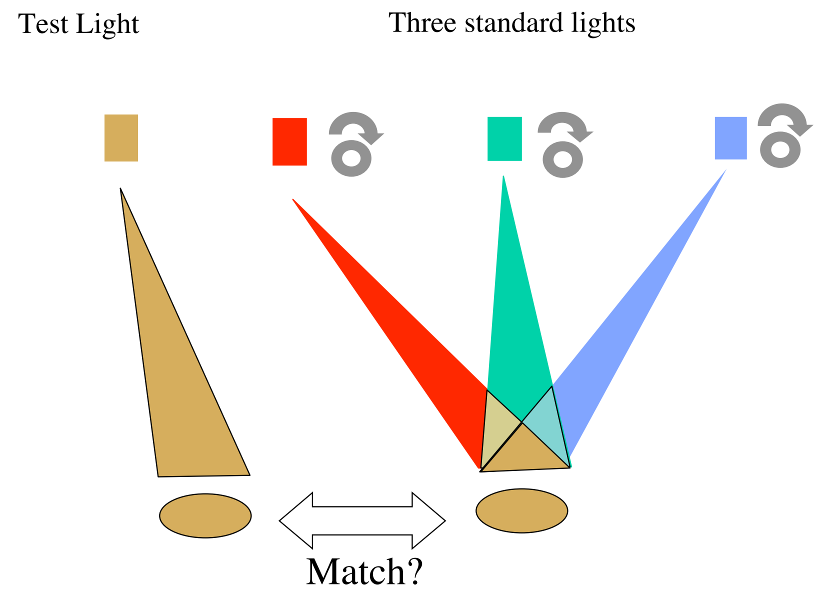
Directly determining the camera like sensitivity response is hard!

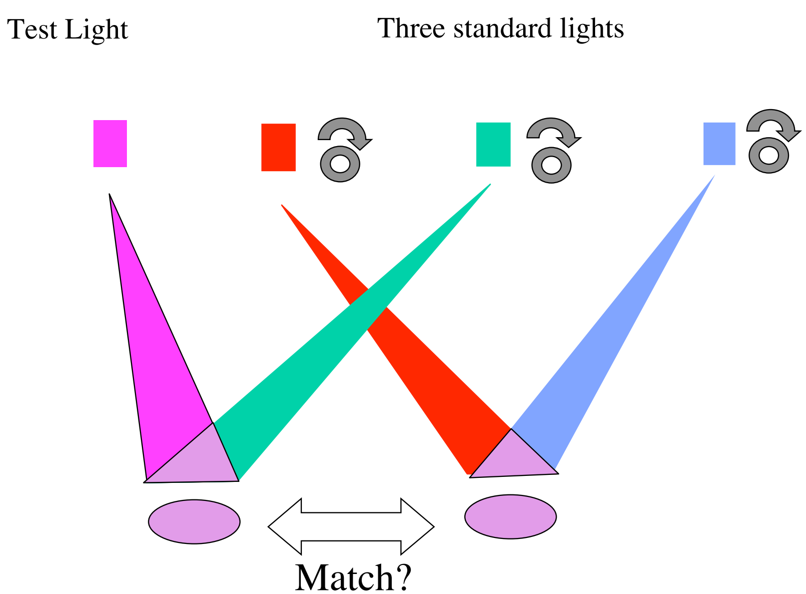
Colour Reproduction

Motivates specifying color numerically (there are other reasons to do this also)

General (man in the street) observation--color reproduction *sort of* works.

Specifying Colour





Trichromacy

Experimental fact about people (with "normal" colour vision)---matching works (for reasonable lights), provided that we are sometimes allowed negative values.

Our "knob" positions correspond to (X,Y,Z) in the standard colorimetry system.

Technical detail: (X,Y,Z) are actually arranged to be **positive** by a linear transformation, but these "knob" positions **cannot** correspond to any **physical** light.

Specifying Colour

(50,150,75)

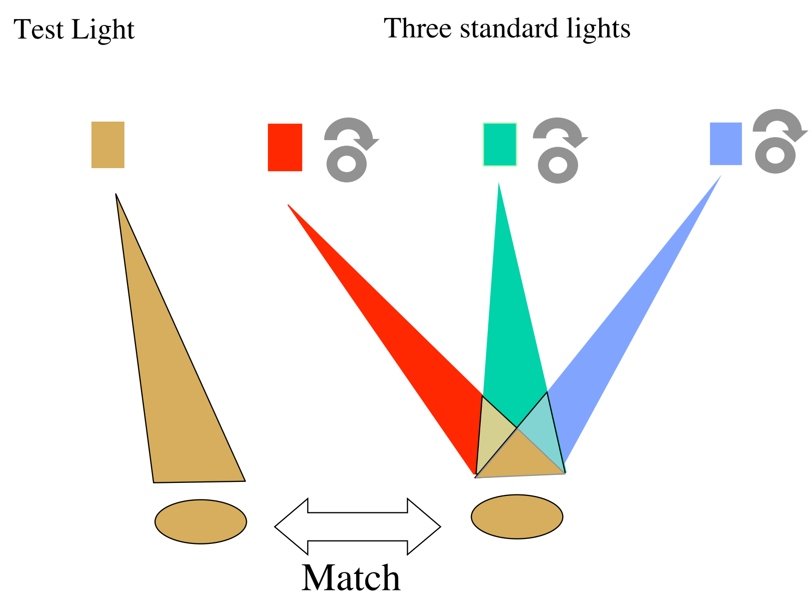
(50,150,75)

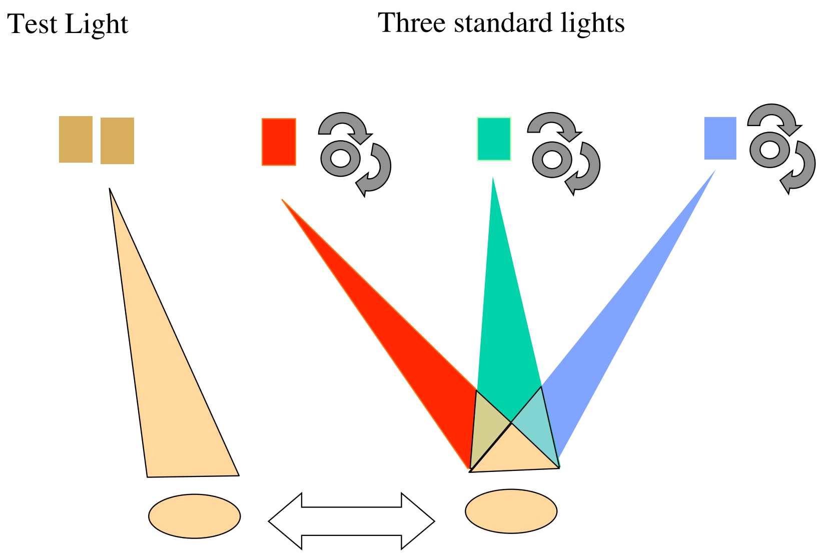
Specifying Colour

We don't want to do a matching experiment every time we want to use a new color!

Grassman's Contribution

Colour matching is linear





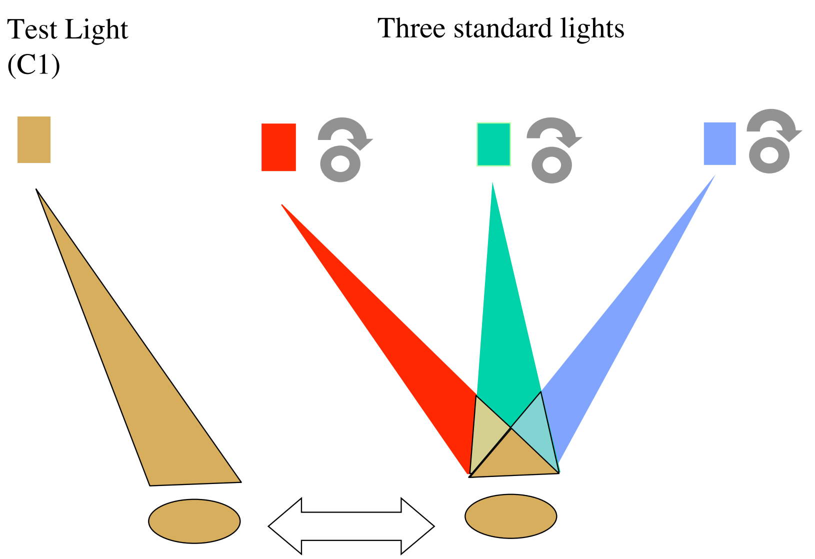
Match (with twice as much)

Matching is Linear (Part 1)

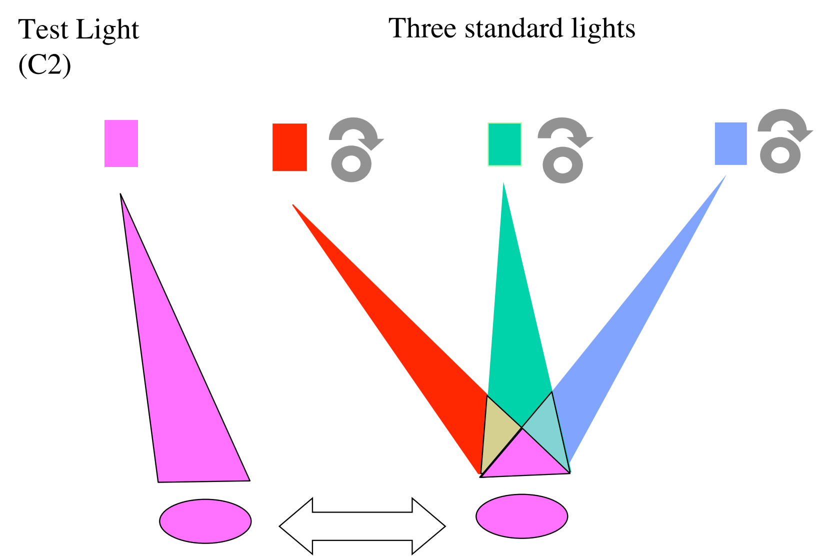
C1 is matched with (X1,Y1,Z1)

C = a*C1

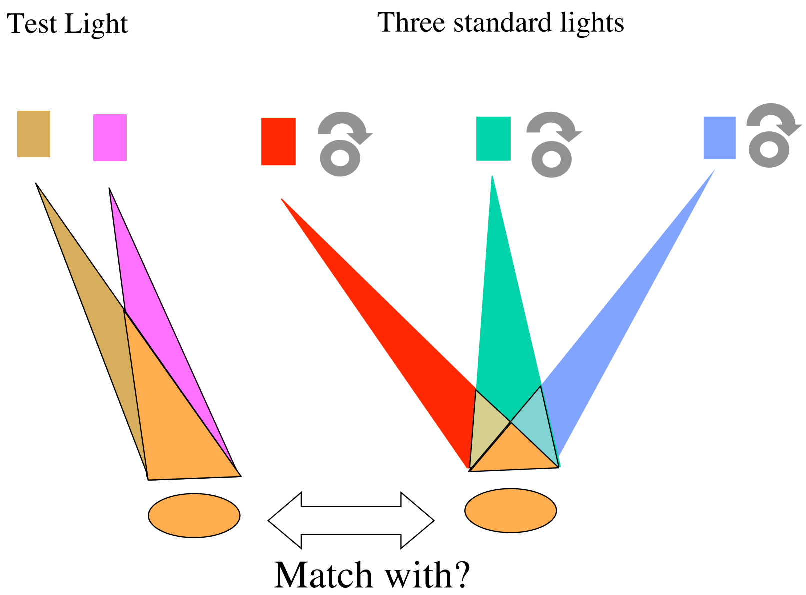
C is matched with a * (X1, Y1, Z1)

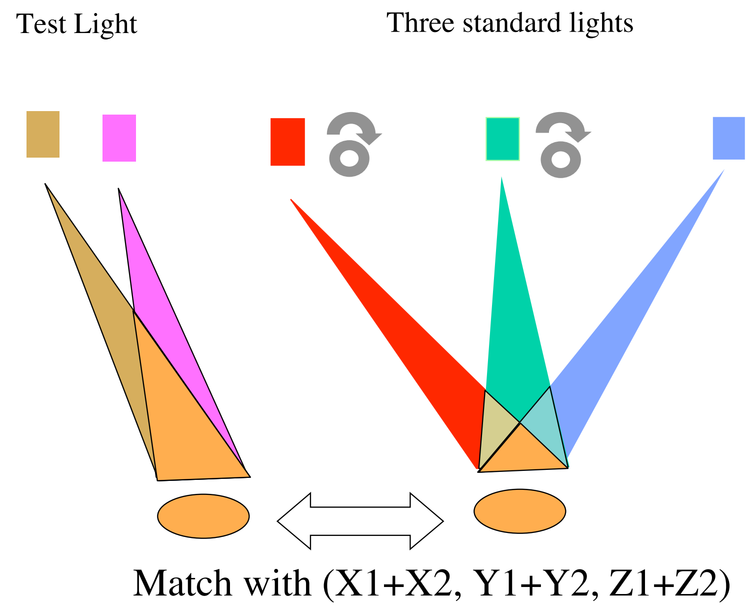


Match with (X1, Y1, Z1)



Match with (X2, Y2, Z2)





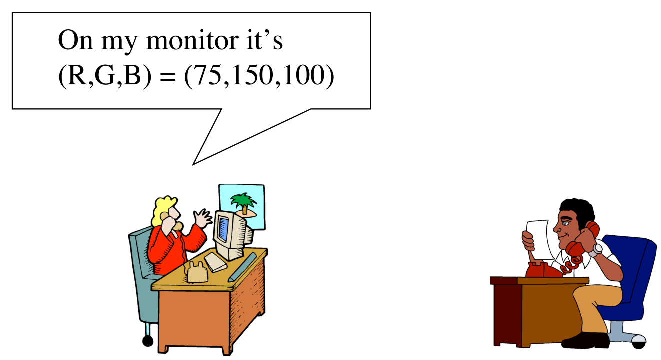
Matching is Linear (formal)

C = a*C1 + b*C2

C1 is matched with (X1,Y1,Z1)
C2 is matched with (X2,Y2,Z2)

C is matched by
a*(X1,Y1,Z1) + b*(X2,Y2,Z2)

Specifying Color



Specifying Colour

But what is (R,G,B)?

Specifying Colour

R matches (X_r, Y_r, Z_r)

G matches (X_g, Y_g, Z_g)

B matches (X_b, Y_b, Z_b)

Specifying Colour

Then by (R,G,B)=(75,150,100) you mean (X,Y,Z), where …..

$$X = 75^* X_r + 150^* X_g + 100^* X_b$$

$$Y = 75^* Y_r + 150^* Y_g + 100^* Y_b$$

$$Z = 75^* Z_r + 150^* Z_g + 100^* Z_b$$

(No need to match--just compute!)

Specifying Colour

… , now that we have
specified the colour,
I can print it!

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{vmatrix} \begin{vmatrix} 75 \\ 100 \\ 150 \end{vmatrix}$$

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{vmatrix} \begin{vmatrix} R \\ G \\ B \end{vmatrix}$$

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = M \begin{vmatrix} R \\ G \\ B \end{vmatrix}$$

Colour Reproduction
(Monitors & Projectors)

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{apple}$$

Find (R,G,B)

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{apple} = M \begin{vmatrix} R \\ G \\ B \end{vmatrix}_{apple}$$

$$\begin{vmatrix} R \\ G \\ B \end{vmatrix}_{apple} = M^{-1} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{apple}$$

$$\begin{vmatrix} R \\ G \\ B \end{vmatrix}_{apple} = M^{-1} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{apple}$$

Possible problems?

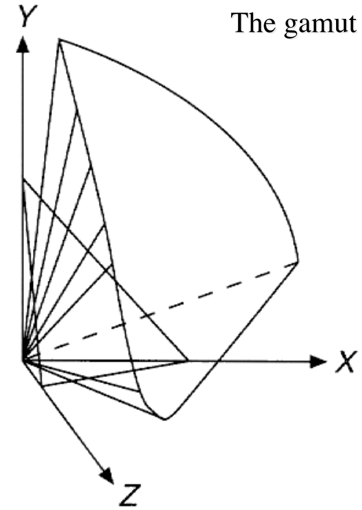
XYZ color space

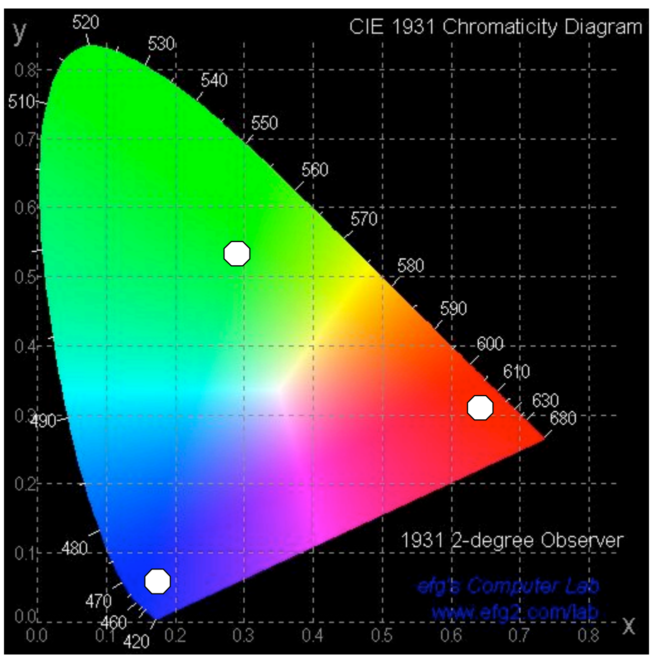
XYZ color space is a linear transformation of the matches to standard lights.

The transformation is used to ensure that all color co-ordinates are positive

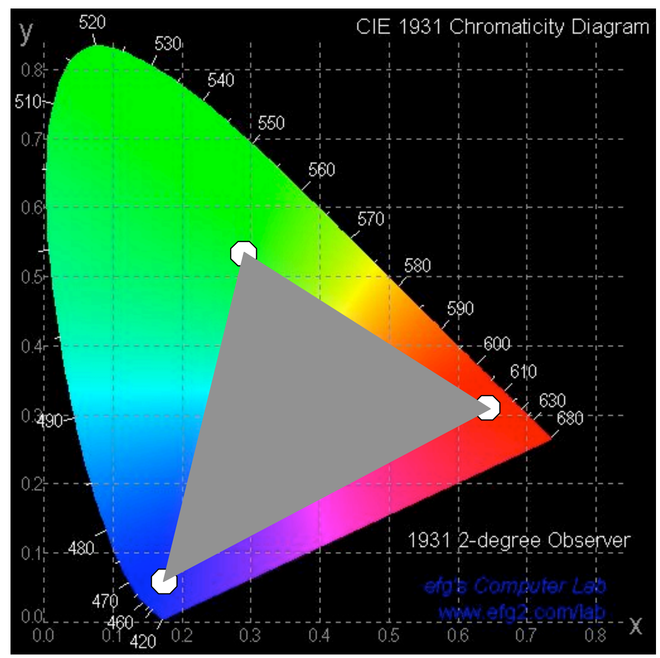
This means that XYZ corresponds to matches of fictitious (physically impossible) lights.

The gamut of all colors

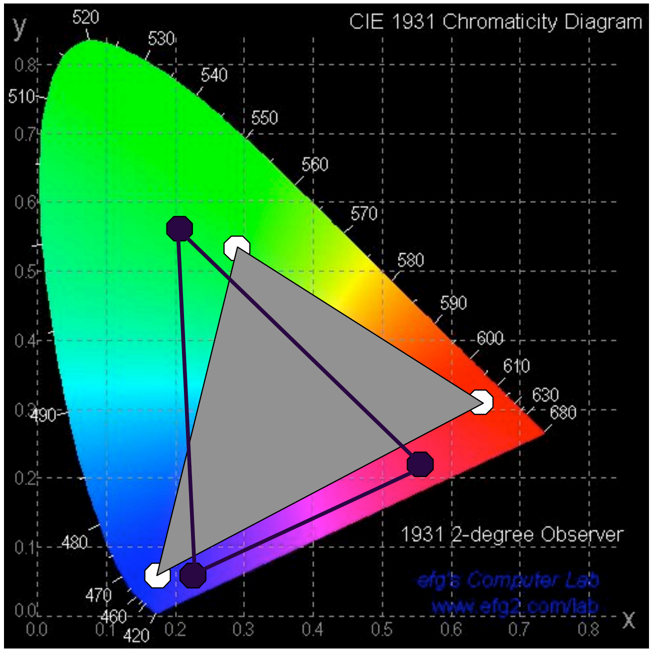




Available from efg2.com



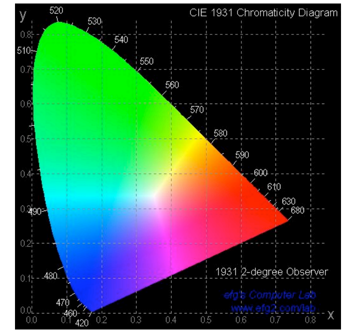
Available from efg2.com



Available from efg2.com

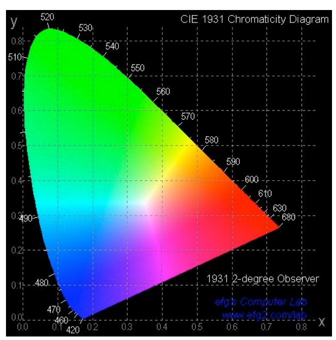
Qualitative features of CIE x, y

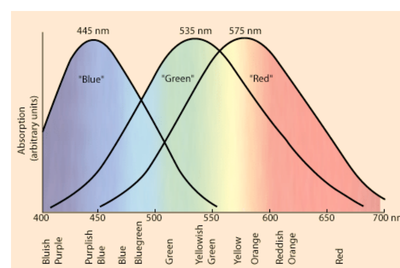
- Linearity implies that colors obtainable by mixing lights with colors A, B lie on line segment with endpoints at A and B
- Monochromatic colours (spectral colors) run along the “Spectral Locus”



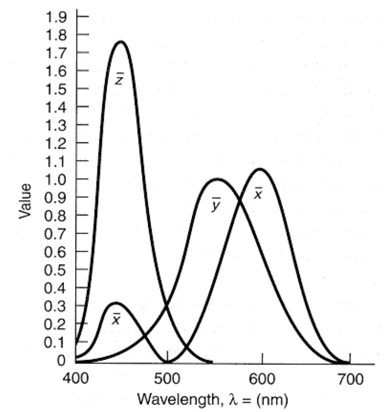
Qualitative features of CIE x, y

- Why the funny shape?





One measurement of human cone absorption



XYZ response curves

Matching is only for “aperture” color

- When color is viewed in the context of other colors numerous effects occur which complicate the characterization of color (simultaneous contrast, color constancy, etc)
- Other complications include chromatic aberration in the eye and different spatial resolution for different colors (these are linked)

Colour Reproduction

Key point--color reproduction is based on “metamerism”

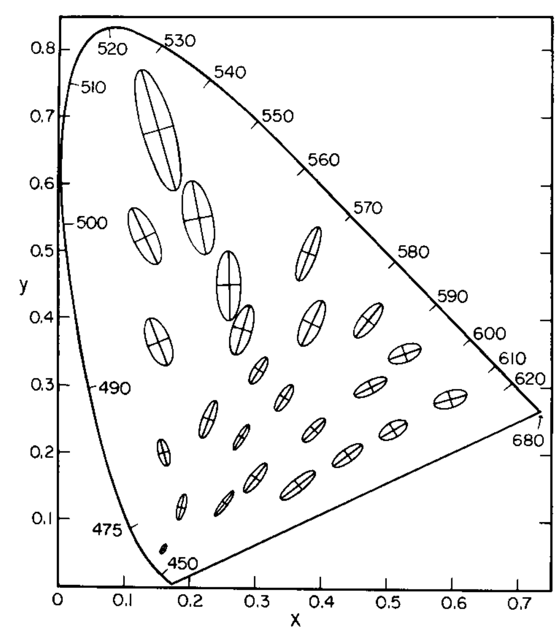
Metameric match--colors which match, despite different spectra.

Duplicating spectra would work, but for practical reasons, we duplicate the match.

For reflective surfaces, e.g prints, this means that the match can change if the illumination changes.

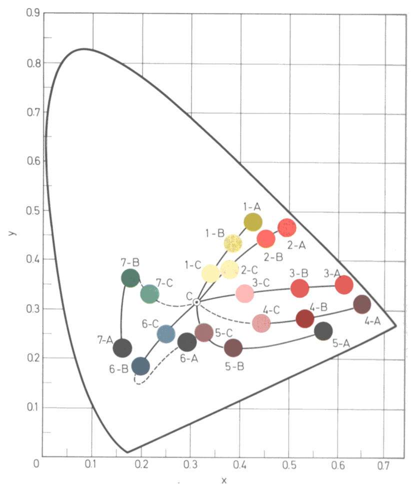
The quest for uniform colour spaces

- Definition of uniform: equal (small!) steps give the same perceived color changes.
- XYZ is not uniform!
- Uniformity only applies to small differences. There is no theory for numerically deciding if yellow is perceptually closer to green or red.



MacAdam Ellipses (scaled by a factor of 10) on CIE x, y

Mixing pigments in CIE



Color matching is linear, but combining pigments is not necessarily linear like mixing light .

Shading values for colored surfaces

- Simplest:
  - Use appropriate shading model in 3 channels, instead of one
  - Implies red albedo, green albedo, blue albedo, etc.
  - Works because the shading model is independent of wavelength.
  - Can lead to somewhat inaccurate colour reproduction in some cases - particularly coloured light on coloured surfaces
- Better
  - Use appropriate shading model at many different wavelength samples - 7 is usually enough
  - Estimate receptor response in eye using sum over wavelength
  - Set up pixel value to generate that receptor response

Monitor Gamma

A typical image encoding is **NOT** linear. Often a gamma correction is included. This leads to no end of confusion.

A “gamma” corrected image is ready to drive a CRT monitor, and has advantages that quantization (8 bits) errors are *roughly* uniformly distributed--that fact that this works is a convenient accident.

Monitor Gamma

Due to the physics involved, CRT monitor brightness is proportional to voltage^(2.5)

This is further hacked to give the “standard” gamma of 2.2

So, if an image looks good on a CRT, it is likely to be non-linear by pow(1/2.2)

LCD--more linear, but then hardware/software can be hacked to be like CRT

Confusing? Yes!