The colors of the rainbow

- Light is electromagnetic radiation, occurring at different wavelengths (or photon energies)
- The radiation around us is a mix of these
- Visible portion is about 400 to 700 nm
- Certain applications may require modeling some UV also.
- Light is specified by its spectrum recording how much power is at each wavelength.

Sunlight

Two disparate source spectra

**Fig. 4.1.** Wavelength composition of light from a tungsten-filament lamp (typified by CIE ILL A [Sect. 4.6]). Relative spectral power distribution curve. Color temperature: 2850 K.

**Fig. 4.2.** Wavelength composition of light from a daylight fluorescent lamp. Typical relative spectral power distribution curve. Correlated color temperature: 6500 K. (Based on data of Jerome reported in [Ref. 3.14, p. 37])

Energy spectra of 20 other common lights
Absorption spectra: real pigments

cyan  magenta  yellow  brown

Sensors (including those in your eyes) have a varied sensitivity over wavelength. Different variations lead to different kinds of sensor responses ("colors" in a naïve sense).

The sensor sensitivity spectra for a particular color camera (they vary a lot).

Image Formation (Spectral)

\[(R,G,B) = \int_{380}^{780} \text{sensor sensitivity} \ast \text{spectral density} \, d\lambda\]
More formally,

The response of an image capture system to a light signal \( L(\lambda) \) associated with a given pixels is modeled by

\[
\rho^{(k)} = \int L(\lambda) R^{(k)}(\lambda) d\lambda
\]

where \( R^{(k)}(\lambda) \) is the sensor response function for the \( k \)th channel.

Note the usual case of three channels

\[
(R, G, B) = (\rho^{(1)}, \rho^{(2)}, \rho^{(3)})
\]

### Discrete Version

Often we represent functions by vectors. For example, a spectra might be represented by 101 samples in the range of 380 to 780 nm in steps of 4nm.

Then \( L(\lambda) \) becomes the vector \( \mathbf{L} \), \( R^{(k)}(\lambda) \) becomes the vector \( \mathbf{R}^k \), and the response is given by a dot product:

\[
\rho^{(k)} = \mathbf{L} \cdot \mathbf{R}^k
\]

#### Sensor/light interaction example

\[
\mathbf{R} = (0, 0, 1, 3, 7, 4, 2, 0, 0, 0)
\]

\[
\mathbf{L} = (0, 8, 6, 3, 1, 2, 7, 4, 3, 0)
\]

Multiply lined up pairs of numbers and then sum up

\[
\mathbf{L} \cdot \mathbf{R} = (0 \times 0, 0 \times 8, 1 \times 6, 3 \times 3, 7 \times 1, 4 \times 2, 2 \times 7, 0 \times 4, 0 \times 3, 0 \times 0) = (0, 0, 6, 9, 7, 8, 14, 0, 0, 0)
\]

\[
\mathbf{L} \cdot \mathbf{P} = 0 + 0 + 6 + 9 + 7 + 8 + 14 = 44
\]
Image Formation (Spectral)

- Note that by this model, light capture is linear.
- Formally this means:

\[ L_1(\lambda) \rightarrow \rho_1^{(k)} \text{ and } L_2(\lambda) \rightarrow \rho_2^{(k)} \]

Then:

\[ aL_1(\lambda) + bL_2(\lambda) \rightarrow a\rho_1^{(k)} + b\rho_2^{(k)} \]

One tricky bit

Electronic capture (e.g. “CCD”) is linear, but typically the circuitry will put the sensor responses through a non-linear mapping (e.g. approximate square root).

This is because display is usually either non-linear due to physics (CRT) or by design (to be like a CRT). This is better because there is less relative noise where humans will notice it.

(A bit more on this later).

Image Formation (Spectral)

- Note that by this model, light capture is linear.
- Formally this means:

\[ L_1(\lambda) \rightarrow \rho_1^{(k)} \text{ and } L_2(\lambda) \rightarrow \rho_2^{(k)} \]

- This means that image formation loses spectral information.
- This means that two quite different spectra can map into the same color.
## Causes of color

- The sensation of color is caused by the brain.
- One way to get it is through a **response** of the eye to the presence/absence of light at various wavelengths.
- Dreaming, hallucination, etc.
- Pressure on the eyelids

### Color receptors

<table>
<thead>
<tr>
<th>“Long” cone</th>
<th>“Medium” cone</th>
<th>“Short” cone</th>
</tr>
</thead>
</table>

Some understanding results from an analogy with camera sensors

Directly determining the camera like sensitivity response is hard!

## Trichromaticity

Empirical fact--colors can be approximately described/matched by three quantities (assuming normal color vision).

Need to reconcile this observation with the spectral characterization of light

## Colour Reproduction

Motivates specifying color numerically (there are other reasons to do this also)

General (man in the street) observation--color reproduction *sort of* works.
Specifying Colour

Test Light

Three standard lights

Match?

Test Light

Three standard lights

Match?

Trichromacy

Experimental fact about people (with “normal” colour vision)---matching works (for reasonable lights), provided that we are sometimes allowed negative values.

Our “knob” positions correspond to (X,Y,Z) in the standard colorimetry system.

Technical detail: (X,Y,Z) are actually arranged to be 

**positive** by a linear transformation, but these “knob” positions **cannot** correspond to any **physical** light.
Specifying Colour

We don’t want to do a matching experiment every time we want to use a new color!

Grassman’s Contribution

Colour matching is linear
Matching is Linear (Part 1)

C1 is matched with \((X_1, Y_1, Z_1)\)

\[ C = a \cdot C_1 \]

\( C \) is matched with \( a \cdot (X_1, Y_1, Z_1) \)
Matching is Linear (formal)

\[ C = a \cdot C_1 + b \cdot C_2 \]

\( C_1 \) is matched with \((X_1,Y_1,Z_1)\)
\( C_2 \) is matched with \((X_2,Y_2,Z_2)\)

\( C \) is matched by
\[ a \cdot (X_1,Y_1,Z_1) + b \cdot (X_2,Y_2,Z_2) \]

Specifying Color

On my monitor it’s\((R,G,B) = (75,150,100)\)
Specifying Colour

But what is (R,G,B)?

Specifying Colour

R matches \((X_r, Y_r, Z_r)\)
G matches \((X_g, Y_g, Z_g)\)
B matches \((X_b, Y_b, Z_b)\)

Specifying Colour

Then by
\((R,G,B)= (75,150,100)\)
you mean \((X,Y,Z)\), where …..

Specifying Colour

\[
\begin{align*}
X &= 75 \times X_r + 150 \times X_g + 100 \times X_b \\
Y &= 75 \times Y_r + 150 \times Y_g + 100 \times Y_b \\
Z &= 75 \times Z_r + 150 \times Z_g + 100 \times Z_b
\end{align*}
\]

(No need to match--just compute!)
Specifying Colour

…, now that we have specified the colour, I can print it!

\[
\begin{align*}
X & = X_r \ X_g \ X_b \\
Y & = Y_r \ Y_g \ Y_b \\
Z & = Z_r \ Z_g \ Z_b \\
\end{align*}
\]

\[
\begin{align*}
X & = 75 \\
Y & = 100 \\
Z & = 150 \\
\end{align*}
\]
Colour Reproduction
(Monitors & Projectors)

Find (R,G,B)

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{apple}} = M
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{\text{apple}}
\]

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{\text{apple}} = M^{-1}\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{apple}}
\]

Possible problems?
XYZ color space

XYZ color space is a linear transformation of the matches to standard lights.

The transformation is used to ensure that all color coordinates are positive.

This means that XYZ corresponds to matches of fictitious (physically impossible) lights.
Qualitative features of CIE x, y

- Linearity implies that colors obtainable by mixing lights with colors A, B lie on line segment with endpoints at A and B
- Monochromatic colours (spectral colors) run along the "Spectral Locus"

Why the funny shape?

One measurement of human cone absorption

XYZ response curves
Matching is only for “aperture” color

- When color is viewed in the context of other colors numerous effects occur which complicate the characterization of color (simultaneous contrast, color constancy, etc)
- Other complications include chromatic aberration in the eye and different spatial resolution for different colors (these are linked)

The quest for uniform colour spaces

- Definition of uniform: equal (small!) steps give the same perceived color changes.
- XYZ is not uniform!
- Uniformity only applies to small differences. There is no theory for numerically deciding if yellow is perceptually closer to green or red.

Colour Reproduction

Key point--color reproduction is based on “metamerism”

Metameric match--colors which match, despite different spectra.

Duplicating spectra would work, but for practical reasons, we duplicate the match.

For reflective surfaces, e.g prints, this means that the match can change if the illumination changes.
Mixing pigments in CIE

A typical image encoding is NOT linear. Often a gamma correction is included. This leads to no end of confusion.

A “gamma” corrected image is ready to drive a CRT monitor, and has advantages that quantization (8 bits) errors are roughly uniformly distributed--that fact that this works is a convenient accident.

Shading values for colored surfaces

• Simplest:
  – Use appropriate shading model in 3 channels, instead of one
  – Implies red albedo, green albedo, blue albedo, etc.
  – Works because the shading model is independent of wavelength.
  – Can lead to somewhat inaccurate colour reproduction in some cases - particularly coloured light on coloured surfaces

• Better
  – Use appropriate shading model at many different wavelength samples - 7 is usually enough
  – Estimate receptor response in eye using sum over wavelength
  – Set up pixel value to generate that receptor response

Monitor Gamma

Due to the physics involved, CRT monitor brightness is proportional to voltage^2.5

This is further hacked to give the “standard” gamma of 2.2

So, if an image looks good on a CRT, it is likely to be non-linear by pow(1/2.2)

LCD--more linear, but then hardware/software can be hacked to be like CRT

Confusing? Yes!