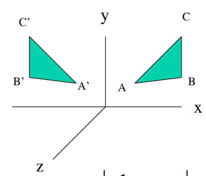
Transforming the normal vector for reflection

- Reflection of a triangle in the xy plane through y-axis
  - Not to be confused with a rotation around y-axis
- Geometric argument suggests normal should remain the same because what is in front of the triangle remains in front
- (B-A) × (C-A) is out of page by right hand rule
- But (B'-A') × (C'-A') is into page by right hand rule
- Resolve this by noting that a reflection maps a right-hand coordinate system into a left-hand one. So, we need to use the "left-hand" rule.



$$\mathbf{M} = \begin{vmatrix} -1 & & \\ & 1 & \\ & & 1 \end{vmatrix}$$

$$\hat{\mathbf{n}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Details optional

Transforming the Normal Vector

M is a transformation in ordinary coordinates (not homogenous, for simplicity)

Let **t** be a vector tangent to the polygon, and **n** the normal

M**t** is on the transformed polygon

Want a transformation N so that N**n** is perpendicular to all M**t**

$\mathbf{n} \bullet \mathbf{t} = \mathbf{n}^{\mathrm{T}}\mathbf{t} = 0$ and so (trick!) $\left(\mathbf{n}^{\mathrm{T}} M^{-1}\right)\left(\mathrm{M}\mathbf{t}\right) = 0$

So, $\mathbf{n}^{\mathrm{T}} M^{-1}$ is the row vector version of the transformed normal

So, $\mathrm{N}\mathbf{n} = (\mathbf{n}^{\mathrm{T}} M^{-1})^{\mathrm{T}} = (M^{-1})^{\mathrm{T}}\mathbf{n}$

And $\mathrm{N} = (M^{-1})^{\mathrm{T}}$

Transforming the Normal Vector

Derived transformation for the normal: $\mathrm{N} = (M^{-1})^{\mathrm{T}}$

Note that Nn is not necessarily a unit vector

The formula proves that orthogonal transformations also transform the normal because orthogonal transformations satisfy:

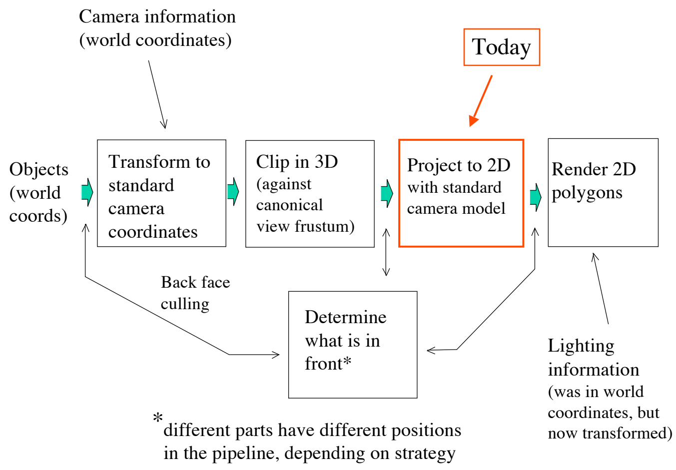
$$\mathrm{M} = (M^{-1})^{\mathrm{T}}$$

(The inverse of an orthogonal matrix is its transpose)

3D Graphics Concepts

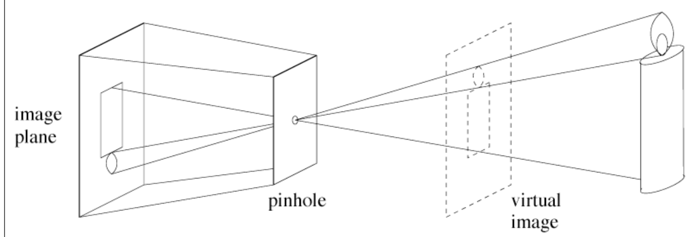
(H&B ch, 7, Watt ch. 5, Foley et al ch. 6)

- Modeling: For now, objects will be collections of polygons in 3D. Complex shapes will be many small polygons.
- Issues:
  - Which polygons can be seen? (some polygons hide others, and some are outside the relevant volume of space and need to be clipped).
  - Where do they go in the 2D image? (key abstraction is a virtual camera)
  - How bright should they be? (for example, to make it look as if we are looking at a real surface)

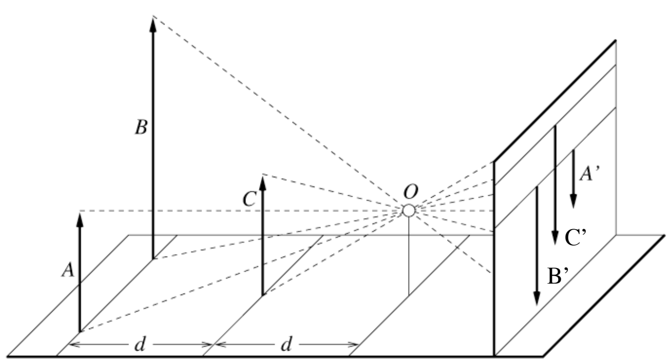


Pinhole cameras

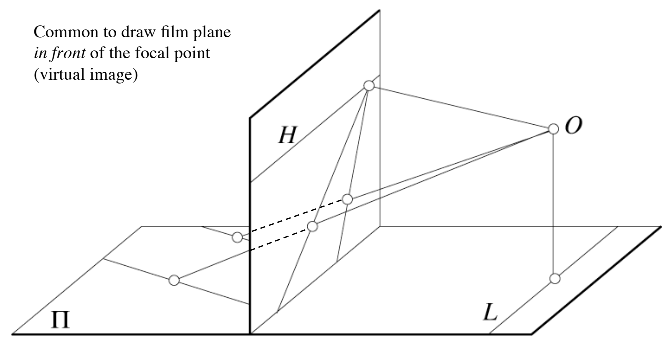
- Abstract camera model--box with a small hole in it
- Pinhole cameras work for deriving algorithms--a real camera needs a lens



Distant objects are smaller



Parallel lines meet*



*Exceptions?

Vanishing points

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
  - Standard horizon is the horizon of the ground plane.
- One way to spot fake images