Sensor/light interaction example

R = (0, 0, 1, 3, 7, 4, 2, 0, 0, 0)

L = (0, 8, 6, 3, 1, 2, 7, 4, 3, 0)

Multiply lined up pairs of numbers and then sum up

R • L = (0*0, 0*8, 1*6, 3*3, 7*1, 4*2, 2*7, 0*4, 0*3, 0*0)

= (0, 0, 6, 9, 7, 8, 14, 0, 0, 0)

L • P = 0 + 0 + 6 + 9 + 7 + 8 + 14

= 44

Image Formation (Spectral)

- Note that by this model, light capture is linear.
- Formally this means:
  \[ L_i(\lambda) \rightarrow r_i^{(k)} \text{ and } L_2(\lambda) \rightarrow r_2^{(k)} \]
- Then:
  \[ aL_1(\lambda) + bL_2(\lambda) \rightarrow ar_1^{(k)} + br_2^{(k)} \]

- Note that image formation loses spectral information
- This means that two quite different spectra can map into the same color
One tricky bit

Electronic capture (e.g. “CCD”) is linear, but typically the circuitry will put the sensor responses through a non-linear mapping (e.g. approximate square root).

This is because display is usually either non-linear due to physics (CRT) or by design (to be like a CRT). This is better because there is less relative noise where humans will notice it.

(A bit more on this later).

Causes of color

• The sensation of color is caused by the brain.
• One way to get it is through a response of the eye to the presence/absence of light at various wavelengths.
• Dreaming, hallucination, etc.
• Pressure on the eyelids

Trichromaticity

Empirical fact--colors can be approximately described/matched by three quantities (assuming normal color vision).

Need to reconcile this observation with the spectral characterization of light

Color receptors

“Long” cone  “Medium” cone  “Short” cone

Some understanding results from an analogy with camera sensors

Directly determining the camera like sensitivity response is hard!
Colour Reproduction

Motivates specifying color numerically (there are other reasons to do this also)

General (man in the street) observation--color reproduction *sort of works.*
Trichromacy

Experimental fact about people (with “normal” colour vision)---matching works (for reasonable lights), provided that we are sometimes allowed negative values.

Our “knob” positions correspond to (X,Y,Z) in the standard colorimetry system.

Technical detail: (X,Y,Z) are actually arranged to be positive by a linear transformation, but these “knob” positions cannot correspond to any physical light.

Specifying Colour

We don’t want to do a matching experiment every time we want to use a new color!

Grassman’s Contribution

Colour matching is linear
Matching is Linear (Part 1)

C1 is matched with $(X_1, Y_1, Z_1)$

$C = a \cdot C_1$

$C$ is matched with $a \cdot (X_1, Y_1, Z_1)$
Matching is Linear (formal)

\[ C = a*C1 + b*C2 \]

- \( C1 \) is matched with \((X1,Y1,Z1)\)
- \( C2 \) is matched with \((X2,Y2,Z2)\)

\[ C \text{ is matched by} \quad a*(X1,Y1,Z1) + b*(X2,Y2,Z2) \]

Specifying Color

On my monitor it’s \((R,G,B) = (75,150,100)\)
Specifying Colour

But what is (R,G,B)?

Specifying Colour

R matches \((X_r, Y_r, Z_r)\)
G matches \((X_g, Y_g, Z_g)\)
B matches \((X_b, Y_b, Z_b)\)

Specifying Colour

\[
X = 75 \times X_r + 150 \times X_g + 100 \times X_b
\]
\[
Y = 75 \times Y_r + 150 \times Y_g + 100 \times Y_b
\]
\[
Z = 75 \times Z_r + 150 \times Z_g + 100 \times Z_b
\]

(No need to match--just compute!)
Specifying Colour

…, now that we have specified the colour, I can print it!

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
75 \\
100 \\
150
\end{bmatrix}
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]