Specifying a camera

World coordinates

y
x
z

View reference point (VRP) ("eye")

View-plane normal (VPN)

Project VUP onto film plane to get v

Computing (u,v,n) in world coordinates

v is the projection of VUP into the view plane which is perpendicular to n

u is perpendicular to the plane formed by n and VUP

So, we can easily compute a vector parallel to u.

Computing (u,v,n) in world coordinates

\[ u \parallel \text{VUP} \times n \]

\[ u = \frac{\text{VUP} \times n}{|\text{VUP} \times n|} = \frac{\text{VUP} \times n}{|\text{VUP}|} \]

What about v?

Specifying a camera

World coordinates

y
x
z

Projection reference point (PRP)—assume it is on n

Window given by \( s_{max}, s_{min} \), \( s_{max}, s_{max} \), denote center by CW
• VRP, VPN, VUP must be in world coords;
• PRP (focal point) could be in world coords, but more commonly, camera coords (which are the same scale as world coords)
• We will use camera coords, and further assume that PRP = (0,0,f).

In world coords

Object in world coordinates (after modeling transforms)

Transform object from world coords to standard camera coordinates

Clip against canonical view frustum

Project using standard camera model

Transform object from world coords to camera coords

Further transform so that frustum is canonical frustum.

All in H&B chapter 7 “3D viewing”

We will first map world coordinates to the camera coordinates (top figure)

However, if clipping against the frustum is difficult because planes bounding the frustum have a complex form

Solution: further transform frustum to a canonical form where clip planes are an easy form to deal with:

\[ z_{=x}, \ z_{=x} \]
\[ z_{=y}, \ z_{=y} \]
\[ z_{=-1}, \ z_{=d} \]

\( u \) and \( v \) can be used to specify a window in the image plane; only this section of image plane ends up on the screen.

This window defines four planes; points outside these planes are not rendered.

Hither and yon clipping planes, which are always given in terms of camera coordinates, and always parallel to the film plane, give a volume known as the view frustum.

Orthographic case: view frustum is cuboid (i.e. all angles right angles, but edges not necessarily of equal length).
If image plane transforms to \( z = f' \) then in new frame, projection is easy:

\[(x, y, z) \rightarrow (-x/z, -y/z)\]

Step 1. Translate the camera at VRP to the world origin. Call this \( T_1 \).
Translation vector is simply negative VRP.

(We are changing the coordinate system of the world, which is the same thing mathematically as moving the camera. We want object world coordinates to change so that the camera location (VRP) becomes the origin).

Step 2. Rotate camera coordinate frame (in w.c.) so that so that \( u \) is \( x \), \( v \) is \( y \), and \( n \) is \( z \). The matrix is:

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(why?)
Object in world coordinates (after modeling transforms)

Transform object from world coordinates to standard camera coordinates

Clip against canonical view frustum

Project using standard camera model

Transform object from world coords to camera coords

Further transform so that frustum is canonical frustum.

Further transform so that frustum is canonical frustum.

Since we are now in camera coordinates, we will often refer to them as (x,y,z) not (u,v,n).

1. Translate focal point to origin
2. Shear so that central axis of frustum lies along the z axis
3. Scale x, y so that faces of frustum lie on conical planes
4. Isotropic scale so that back clipping plane lies at z=-1

Step 1: Translate focal point (PRP) to origin; call this translation Tz. Since we have PRP in camera coordinates (where we now are), the translation vector is simply negative PRP. In particular, in the very common case where PRP is (0,0,f), the translation vector is (0,0,-f)

Window center is:

$$\left(\frac{1}{w_{max}+w_{min}},\frac{1}{v_{max}+v_{min}},0\right)$$

Window center is now:

$$\left(\frac{1}{w_{max}+w_{min}},\frac{1}{v_{max}+v_{min}},-f\right)$$
Step 1 is relatively straightforward, but notice that the location of the clipping planes also gets shifted.

So, before we had the back clipping plane at B (which is negative). Now it is at:  B-f.

Shear $S_1$ takes previous window midpoint \[
\left(\frac{1}{2}(v_{\text{min}} + v_{\text{max}}), \frac{1}{2}(w_{\text{min}} + w_{\text{max}}), -f\right)
\] to (0, 0, -f) - this means that matrix is

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & f \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Step 2: Shear this volume so that the central axis lies on the z-axis. This is a shear, because rectangles on planes $z=\text{constant}$ must stay rectangles. Call this shear $S_1$.

3. Scale x, y so that planes are on $zwk$, $x=x$ and $xzy$ and $x=-y$.
   Call this scale $S_{c_1}$.

4. Isotropic scale so that far clipping plane is $z=1$; call this scale $S_{c_2}$.
3. Scale x, y so that planes are on z=x, z=-x and z=y and z=-y. Call this scale $S_{c_1}$

$$y, z = \left(\frac{1}{2}(v_{\text{max}} - v_{\text{min}}), -f\right)$$

Diagram for $S_y$
(ignoring x coordinate)

- $z$
- $y$

$z = B - f$

Camera plane

4. Scale x, y so that planes are on z=x, z=x and z=y and z=-y. Call this scale $S_{c_1}$

$$\left(\frac{1}{2}(v_{\text{max}} - v_{\text{min}}), -f\right) \rightarrow (f, -f)$$ (because $y=-z$)

$$k, \frac{1}{2}(v_{\text{max}} - v_{\text{min}}) = f$$

$$k, \frac{2f}{(v_{\text{max}} - v_{\text{min}})}$$ (is y scale factor)

5. Now isotropic scale so that far clipping plane is $z=1$; call this scale $S_{c_2}$

$$\begin{align*}
S_{c_1} &= \begin{bmatrix}
2f/(u_{\text{max}} - u_{\text{min}}) & 0 & 0 & 0 \\
0 & 2f/(v_{\text{max}} - v_{\text{min}}) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}$$

Now $y=-z$

- $z$
- $y$

$z = f + B$

Camera plane
(may be on either side of near clipping plane)
5. Now isotropic scale so that far clipping plane is z=-1; call this scale $S_{c_2}$

Currently, at far clipping plane, $z=-f+B$

Want a factor $k$ so that $k(-f+B)=-1$

So, $k = -1 / (-f + B) = 1 / (f - B)$

(Note that $B$ is negative, and $k$ is positive)

$$S_{c_2} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
\frac{f-B}{f-B} & 1 & 0 & 0 \\
0 & 0 & \frac{1}{f-B} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

Note that the focal length, $f$, also gets transformed (needed for the perspective transformation coming up).

It is: $f' = \frac{f}{f-B}$

3D Viewing Pipeline

Then clip (more later), and then project
Further comments on the canonical frustum

$u_{\text{min}}, u_{\text{max}}, v_{\text{min}}, v_{\text{max}}$, are thought of as being in the camera coordinate system $\Rightarrow$ units are that of world coordinate system

For assignment three, you need to choose $u_{\text{min}}, u_{\text{max}}, v_{\text{min}}, v_{\text{max}}$, and $f$.

I suggest simply setting $u_{\text{min}}, u_{\text{max}}, v_{\text{min}}, v_{\text{max}}$, to reflect your understanding of your screen window in world coordinates, and set $f$ accordingly. (Best to keep the aspect ratio the same).

Note the approximate reciprocal relation of $u_{\text{min}}, u_{\text{max}}$ and $v_{\text{min}}, v_{\text{max}}$, with $f$.

$f_1$ and $w_1$ give the same image as $f_2$ and $w_2$, but to see this in the math note that the camera center has shifted.

Because of this shift, the clipping plane values change, and $B_1-f_1 \Rightarrow B_2-f_2$

Determining the screen coordinates

Once you have (x,y) you need to map them back to the screen coordinates.

Use primes (’) for mapped quantities (including projection) and carets (^) for screen quantities.

The canonical frustum gives the screen as a square that is $2f'$ by $2f'$. Note that $f'$ is between 0 and 1 (why?)

Our window on the screen has corners: $(\hat{u}_{\text{min}}, \hat{u}_{\text{max}})$ and $(\hat{v}_{\text{min}}, \hat{v}_{\text{max}})$

(Unless we want to distort things we assume the same aspect ratio as the camera window.)

Determining the screen coordinates

Our screen coordinates are then:

$$\hat{x} = \hat{u}_{\text{max}} + \hat{u}_{\text{min}} \frac{\hat{x}'}{2f'} \cdot (\hat{u}_{\text{max}} - \hat{u}_{\text{min}})$$

$$\hat{y} = \hat{v}_{\text{max}} + \hat{v}_{\text{min}} \frac{\hat{y}'}{2f'} \cdot (\hat{v}_{\text{max}} - \hat{v}_{\text{min}})$$

Or, equivalently:

$$\hat{x} = \hat{u}_{\text{min}} + \hat{u}_{\text{max}} \frac{x + f}{2f} \cdot (\hat{u}_{\text{max}} - \hat{u}_{\text{min}})$$

$$\hat{y} = \hat{v}_{\text{min}} + \hat{v}_{\text{max}} \frac{y + f}{2f} \cdot (\hat{v}_{\text{max}} - \hat{v}_{\text{min}})$$
Plan A: Clipping against the canonical frustum

2D algorithms are easily extended. For line clipping with Cohen Sutherland we use the following 6 out codes:

\[ y \geq z \quad y \leq z \quad x \geq z \quad x < z \quad z < 1 \quad z > z_{\text{min}} \]

Plan A: Clip against canonical frustum (relatively easy—we chose the canonical frustum so that it would be easy!)

Plan B: Clipping in homogenous coords

- For any camera, can turn the view frustrum into a regular parallelepiped (box). We will use the box bounded by \( x = \pm 1, y = \pm 1, z = -1, \) and \( z = 0 \).
- Advantages
  - Simplified clipping in homogenous coordinates
  - Extends to cases where we use homogenous coordinates to represent additional information (and \( w \) could be negative).
  - Can simplify visibility algorithms.
- Approach: clever use of homogenous coordinates

Clipping against the canonical frustum

Clipping polygons in 3D against canonical frustum planes is simpler and more efficient than the general case.

Recall the S.H. gives four cases:

- Polygon edge crosses clip plane going from out to in
  - emit crossing
- Polygon edge crosses clip plane going from in to out
  - emit crossing
- Polygon edge goes from out to out
  - emit nothing
- Polygon edge goes from in to in
  - emit next vertex

(The above is from before, just change "edge" to "plane")
Plan B

Orthographic case

Polygons in 3D

Rotate and translate to place camera at origin

Translate and scale to box

Translate, shear and scale to canonical frustum

Transform frustum to standard box

Divide and clip against box or clip in h.c.'s and then divide

Projection is now trivial

Transforming canonical frustum to box

Transforming canonical frustum to box

Camera plane

The picture should suggest an appropriate scaling for y.

It is ?

Projection in 3D

z=1

z=1

z=1

y=1

y=1

y=z

y=z

y=z

x=z

x=z

x=z

z=\text{z}_{\text{min}}

z=\text{z}_{\text{min}}

z=\text{z}_{\text{min}}

Camera plane

Do this in two steps. One stretch in y (and x), and on stretch in z.
Transforming canonical frustum to box

On top, $y \rightarrow 1$, so scaling is $(1/y)$
Recall that $y = z$ there.

On bottom, $y \rightarrow -1$ so scaling is $(-1/y)$. Recall that $y = z$ there.

So scaling is $y' = y(-z)$
Similarly, $x' = x(-z)$

Transformation is non-linear, but in h.c., we can make $w = (-z)$.

For $z$, we translate near plane to origin. But now box is too small.
Specifically it has $z$ dimension $(1 + z_{min})$ (recall $z_{min}$ is negative)

So we have an extra scale factor $1 / (1 + z_{min})$ and thus $z' = (z - z_{min}) / (1 + z_{min})$

But we want $x$ and $y$ to work nicely in h.c., with $w = -z$, so we use

$z' = ((z - z_{min}) / (1 + z_{min})) / (-z)$
(Thus in our box, depth transforms non-linearly)

In h.c.,
$x \rightarrow x$
$y \rightarrow y$
$z \rightarrow (z - z_{min}) / (1 + z_{min})$
$l \rightarrow -z$

So, the matrix is ?

**Mapping to standard view volume**
*(additional comments)*

* The mapping from $[z_{min}, -1]$ to $[0, -1]$ is non-linear. (Of course, there exists a linear mapping, but not if we want everything else to work out nicely in h.c.).
* So a change in depth of $\triangle D$ at the near plane maps to a larger depth difference in screen coordinates than the same $\triangle D$ at the far plane.
* But order is preserved (important!); the function is monotonic (proof?).
* And lines are still lines (proof?) and planes are still planes (important!).
Clipping in homogeneous coordinates

- We have a cube in \((x,y,z)\), but it is **not** a cube in homogeneous coordinates, so we must divide if we want to take advantage of this particularly nice clipping situation.
- However, dividing before clipping might be inefficient if many points are excluded, so we often clip in homogeneous coordinates.

Clipping in homogeneous coord.'s

- Write h.c.’s in caps, ordinary coords in lowercase.
- Consider case of clipping stuff where \(x>1\), \(x<-1\)
- Rearrange clipping inequalities:

\[
\begin{align*}
\left(\frac{x}{w}\right) &> 1 \\
\left(\frac{x}{w}\right) &< -1
\end{align*}
\]

becomes

\[
\begin{align*}
X > W, \\
X < -W, \\
W > 0
\end{align*}
\]

AND

\[
\begin{align*}
X < W, \\
X > -W, \\
W < 0
\end{align*}
\]

(So far \(W\) has been positive, but negatives occur if we further overload the use of h.c.’s)
Clipping in homogeneous coord.’s

The clipping volume in cross section

Region A
Region B
W=1

Clipping in homogeneous coord.’s

- If we know that W is positive (the case so far!), simply clip against region A
- If we are using the h.c. for additional deferred division, then W can be negative.
- If W is negative, then we use region B. The clipping can be done by negating the point, and clipping against A, due to the nature of A and B.
- Case where object has both positive and negative W is a little more complex.
- Notice that the actual clipping computations are not that different from the case in Plan A—no free lunch!

Reminder of the last steps

In both plans we need to project into 2D.

If we are working in the canonical view space, then we project using the standard camera model (easy) and divide

Recall that the matrix for the standard camera model using homogeneous coordinates is:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & \frac{1}{f}
\end{bmatrix}
\]

Reminder of the last steps

If we are working in homogenous coordinates, then we first divide and then projection is even easier (ignore z coordinate).

The mapping to the box—which was complete once the division was done—implicitly did the perspective projection—essentially we transformed the world so that orthographic projections holds.
Reminder of the last steps

Finally, we may need to do additional 2D transformations.

In the canonical frustum case, our (x,y) coordinates are relative to \((-1,1)\). They need to be mapped to the viewport (possibly implicitly by the graphics package).

In the canonical box case, our (x,y) coordinates are relative to \((-1,1)\). They need to be mapped to the viewport (possibly implicitly by the graphics package).