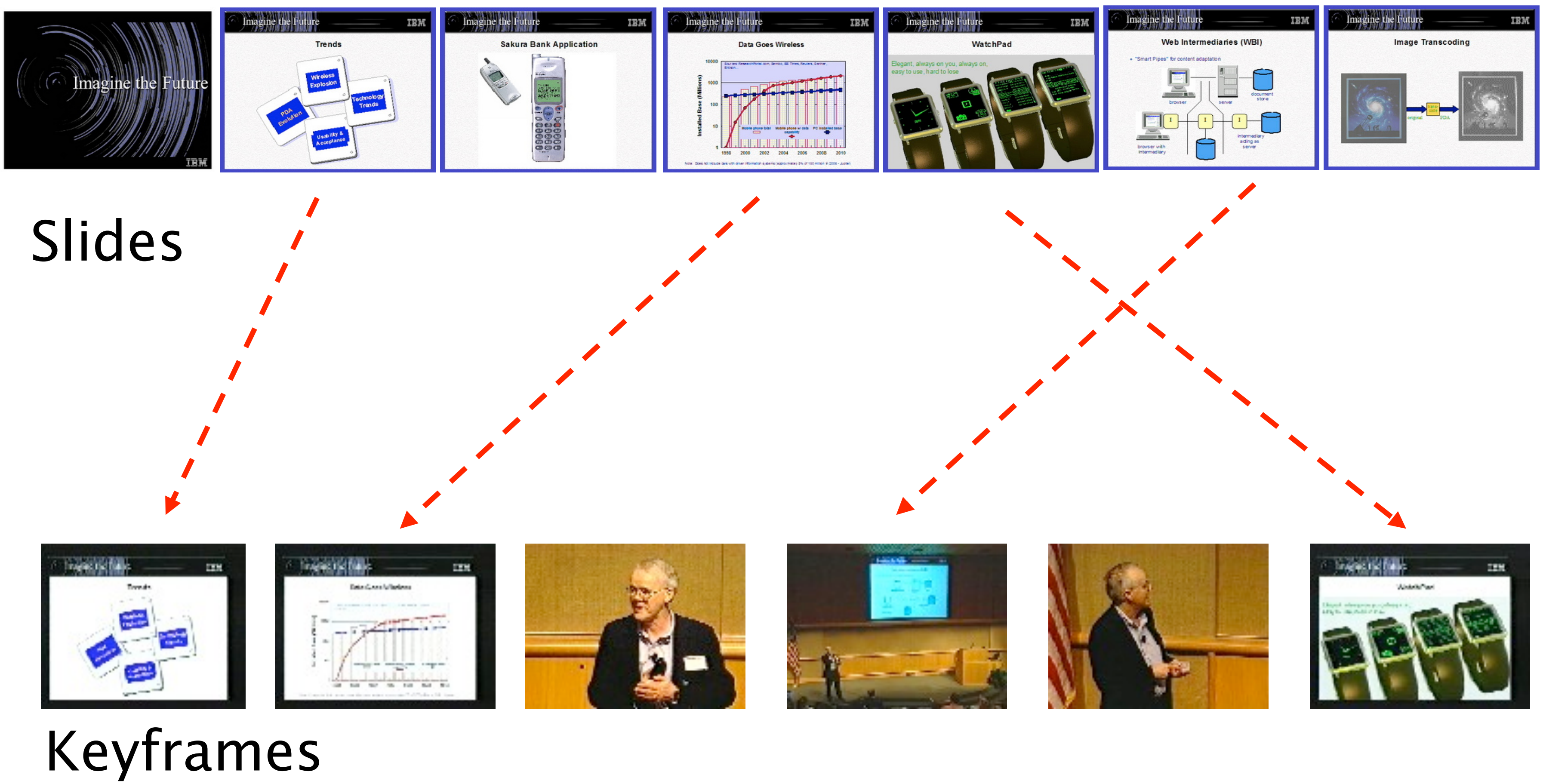
RANSAC and SIFT

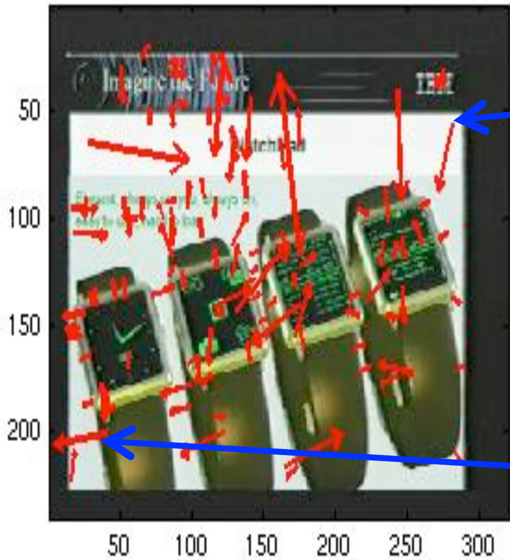
- Powerful combination to find objects in images
- Exemplar image and image being studied typically have different camera angle or position.
- Recall that:
  - SIFT descriptors are relatively invariant to camera changes
  - SIFT matching leads to lots of “false” matches
- The main idea is that true matches should “agree”
- For planar objects, the definition of “agree” is quite simple
  - They link via a homography (covered soon)

Matching Slides to Presentation Videos



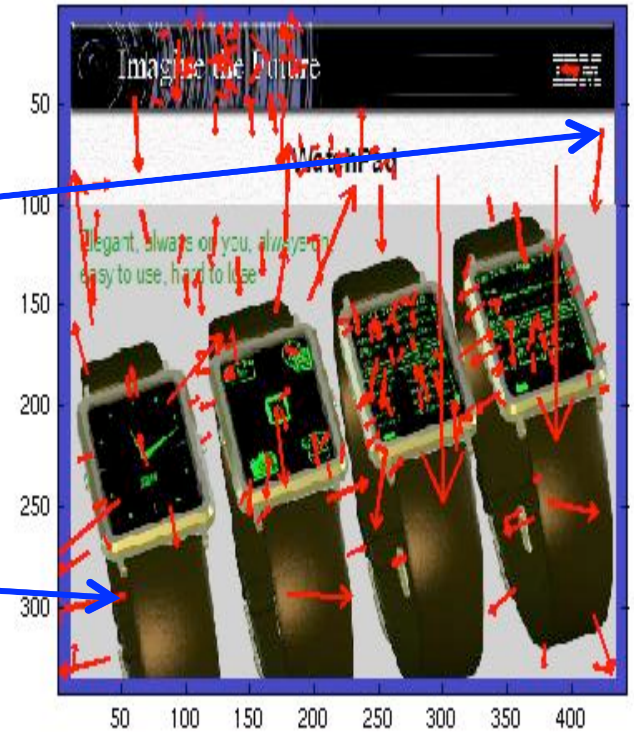
SIFT (**S**cale **I**nvariant **F**eature **T**ransformation) keypoints review

Keyframe



Only a quarter of keypoints shown!

Slide

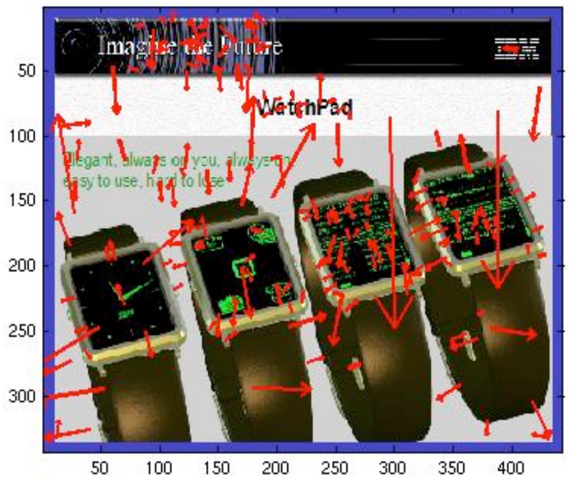


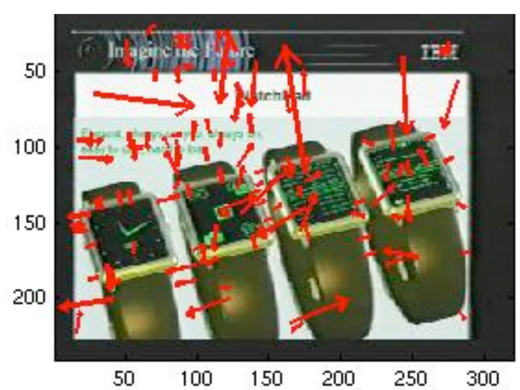
local feature descriptors

location, scale, orientation and a feature vector with 128 elements

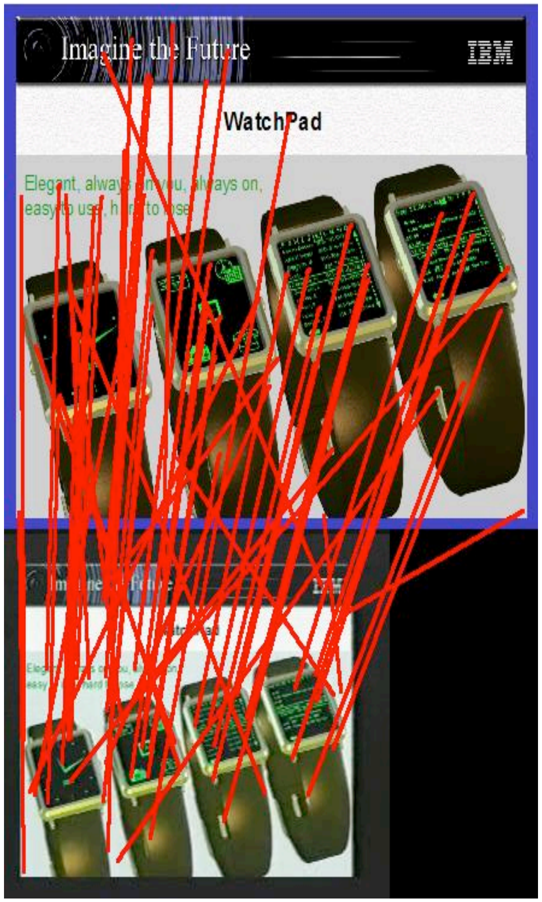
Shown as a vector in the image

Nearest neighbor ratio has many outliers





Nearest Neighbor search



Only a quarter of matches shown!

Planar Homography

Mappings of points on a plane in 3D satisfy a simple relation

$$\lambda' \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

frame keypoints

slide keypoints

X' = H X

Optional

Derivation of Planar Homography

Consider a point on a plane given by

$$X = X_o + sX_1 + tX_2$$

under the two projective transforms

$$P = \begin{bmatrix} A & \mathbf{b} \end{bmatrix} \quad \text{and} \quad P' = \begin{bmatrix} A' & \mathbf{b}' \end{bmatrix}$$

This leads to two image points, $\lambda\mathbf{p}$ and $\lambda'\mathbf{p}'$.

Optional

Derivation of Planar Homography

$$\lambda \mathbf{p} = \begin{bmatrix} A & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{X}_o + s\mathbf{X}_1 + t\mathbf{X}_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} A & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_o \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$$

3x4 4x3

$$= \begin{bmatrix} A\mathbf{X}_1 & A\mathbf{X}_2 & A\mathbf{X}_0 + \mathbf{b} \end{bmatrix} \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$$

3x3

$$= V \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$$

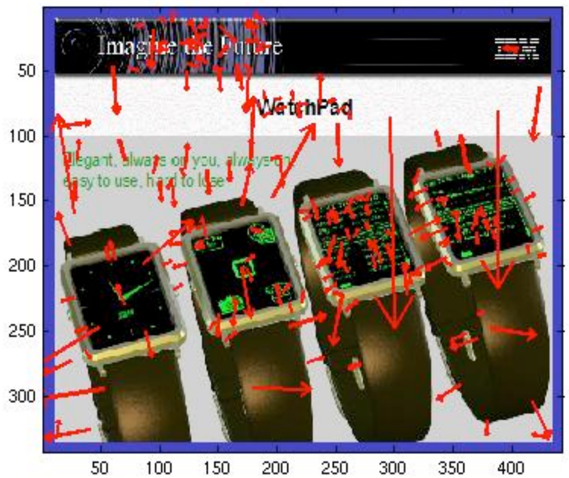
Optional

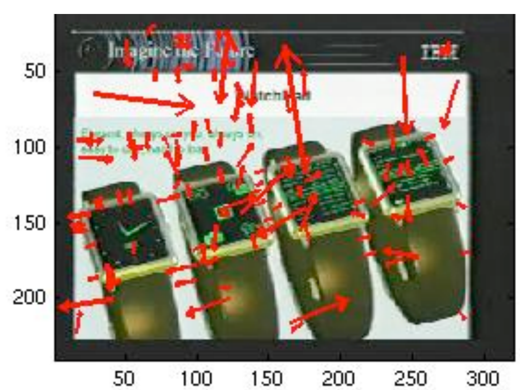
Derivation of Planar Homography

Similarly, $\lambda' \mathbf{p}' = V' \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$

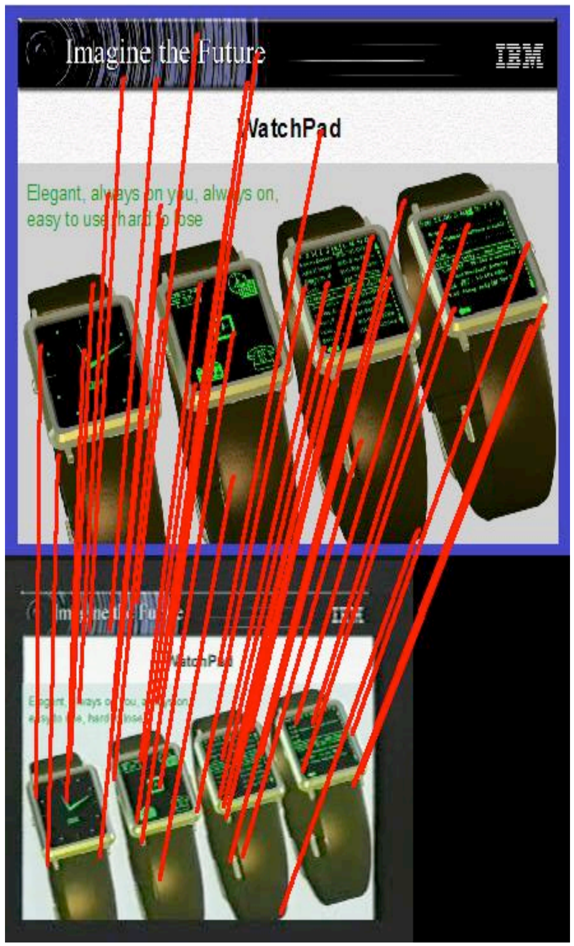
and so $\lambda' \mathbf{p}' = V' \; V^{-1} \; \lambda \mathbf{p} = H \lambda \mathbf{p}$

Constraining matches by homography





Fit homography with RANSAC



Only a quarter of matches shown!

RANSAC approach

Repeat many times

Randomly select enough matches to fit homography

Compute homography

Using that homography, measure error on best (say) 50%

Output best one found

Computing Homography

Seek H where $\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

H is only determined up to a scale factor (eight unknowns).

Let the rows of H be h_1^T, h_2^T, h_3^T.

$x' = \dfrac{u'}{w'}$ so $x'w' = u'$. Similarly, $y'w' = v'$

Also, $u' = h_1^T X$ and $v' = h_2^T X$ and $w' = h_3^T X$

Computing Homography

Each match then gives two linear equations

$$x' h_3^T X = h_1^T X \quad \text{and} \quad y' h_3^T X = h_2^T X$$

Hence four matches are OK.

This can be solved with homogenous least squares*, but this is a bit unstable. A better way is the DLT (direct linear transform) method.

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* Doing this is part of the homework. You may want to review how we set up the equations for camera calibration. Notice that in the expressions above, X can be a row vector and the h_x column vectors. Referring to camera calibration, X is playing the role that P did, and h_x are playing the role of m_x (rows of M).