Direct Linear Transform Method

From before \( \lambda' \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \)

so \( \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \) and \( H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \) are parallel, so their cross product should be zero.

i.e., \( \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \parallel \begin{bmatrix} h_1^T \cdot X \\ h_2^T \cdot X \\ h_3^T \cdot X \end{bmatrix} \)

so, for example, the first component of the cross product gives

\[ y'h_3^T \cdot X - h_2^T \cdot X = y'X^T \cdot h_3 - X^T \cdot h_2 = 0 \]
Direct Linear Transform Method

This leads to the following more stable set of equations.

\[
\begin{bmatrix}
0 & -X^T & y'X^T \\
X^T & 0 & -x'X^T \\
-y'X^T & x'X^T & 0
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
h_3
\end{bmatrix} = 0
\]
Direct Linear Transform Method

For homogenous least squares.

\[
\begin{bmatrix}
0 & -X_1^T & y'X_1^T \\
X_1^T & 0 & -x'X_1^T \\
\vdots & \vdots & \vdots \\
-X_4^T & 0 & -x'X_4^T \\
X_4^T & 0 & -x'X_4^T \\
\vdots & \vdots & \vdots \\
-y'X_4^T & x'X_4^T & 0
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
h_3
\end{bmatrix} = 0
\]
Direct Linear Transform Method

The previous system has 3 equations per match, but only two of them are independent (one could be omitted, but no need for least squares methods, and hard to characterize the effect of breaking the symmetry).

By adding rows for additional points, we get the DLT method.
More on and problems
Segmentation/Grouping by EM

• We assume that the observed data is from multiple hidden processes (e.g., clusters)

• A generative statistical model for the data
  - Choose a “cause”, e.g., a cluster, according to $p(c)$.
  - Given the cluster, sample its probability model $p(X|c)$.

• For concreteness, assume Gaussians
  - This is a Gaussian Mixture Model (GMM)
GMM illustrated

(a) Truth

(b) Data

(c) Clustering according to the model
Gaussian Mixture Model (GMM)

• Generative process
  – Chose a mixture component (cluster), \( m \), with probability \( p(m) \)
  – For the component \( m \), consult the particular Gaussian distribution
  – Generate a sample from that distribution

• This models the distribution
  \[
  p(x) = \sum_m p(m)p(x \mid m) \quad \text{where} \quad p(x \mid m) = \mathcal{N}(\mu_m, \Sigma)
  \]

• And for multiple points
  \[
  p(\{x_i\}) = \prod_i \left( \sum_m p(m)p(x \mid m) \right)
  \]