Administrivia

- Assignment 4 available (finally!).
- Midterm: March 25 (or state your objection soon!).

Syllabus Notes

- We are finishing filtering. We will touch on the Laplacian of Gaussian filter (§8.3.1), and discuss briefly the Fourier transform (7.3.1). And then onto texture.
- Recommended optional reading (§7.4)
- If you are new to Fourier methods, and could use more background, then you will have to look beyond the book. I may be able to lend / photocopy materials.

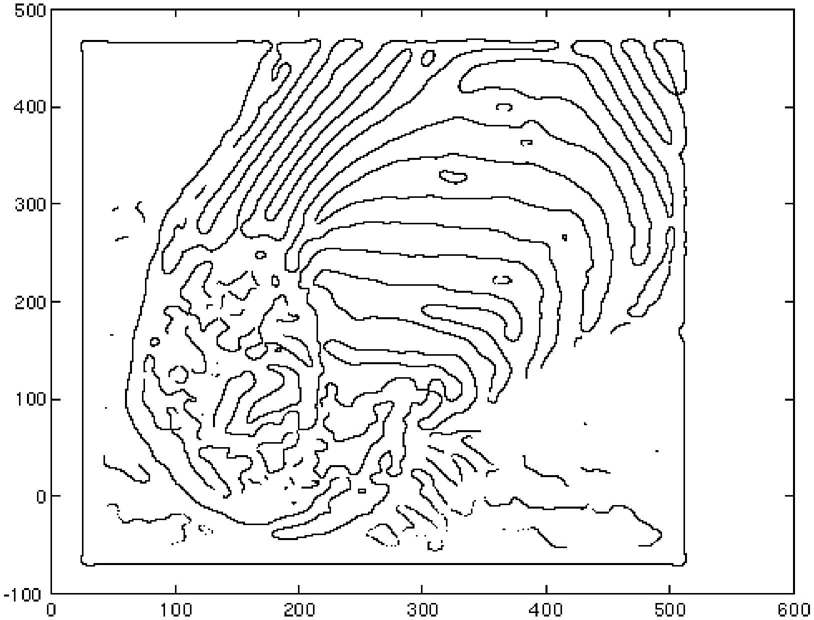
The Laplacian of Gaussian

- Another way to detect an extremal first derivative is to look for a zero second derivative
- Generally not used too much anymore, but it appears in the literature
- Sometimes convenient when you prefer a rotationally invariant filter
- Appropriate 2D analogy of second derivative is the Laplacian operator (rotationally invariant)

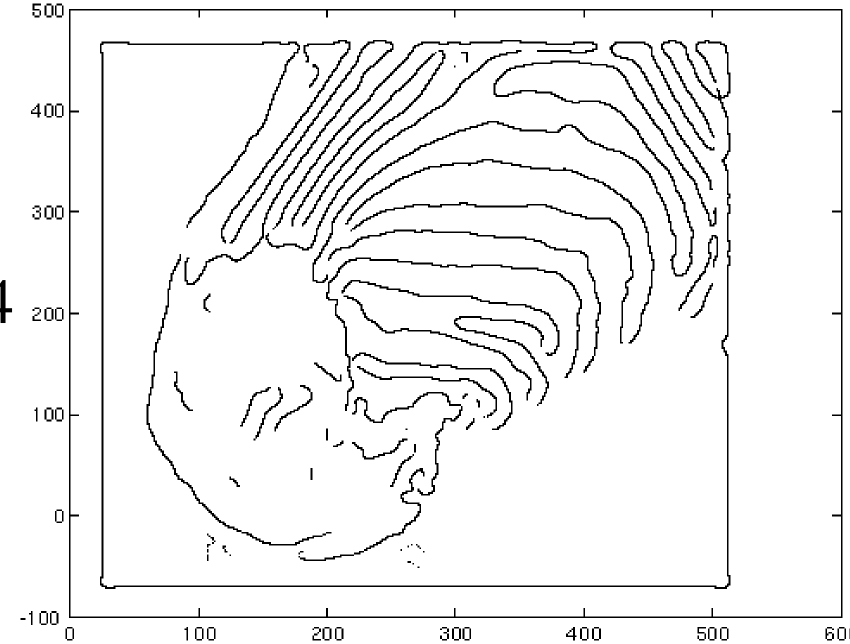
$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

The Laplacian of Gaussian

- Again, it is a bad idea to do this without smoothing
  – smooth with Gaussian, apply Laplacian
  – this is the same as filtering with a Laplacian of Gaussian filter
  – filter is resembles a hat and is sometimes referred to as the "Mexican hat" operator
  – can be approximated by a difference of two Gaussians of different scales
  – this amounts to subtracting the average surround from a center point
  – has been linked to "center-surround" cells in the human vision system (HVS)
- Now mark the zero points *where there is a sufficiently large derivative*



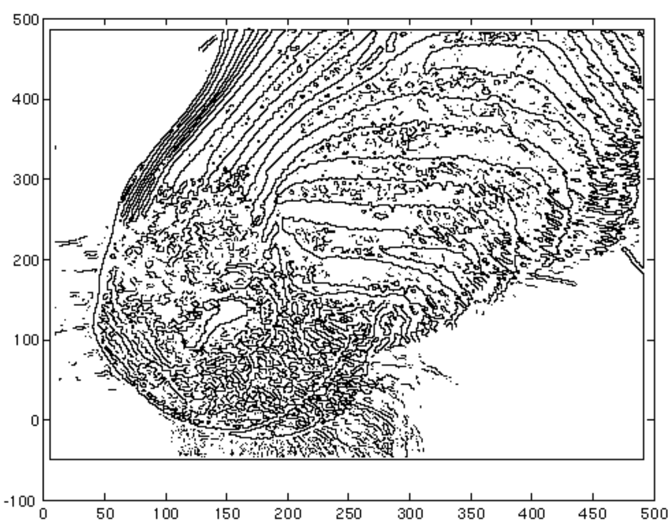
sigma=4



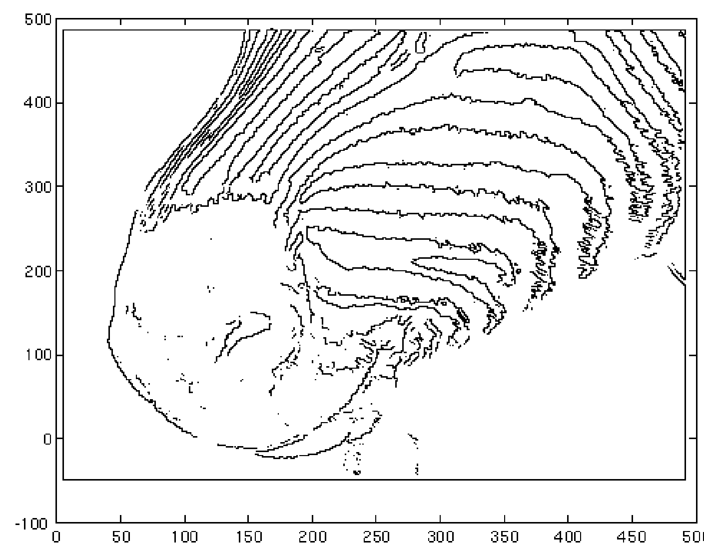
contrast=1

LOG zero crossings

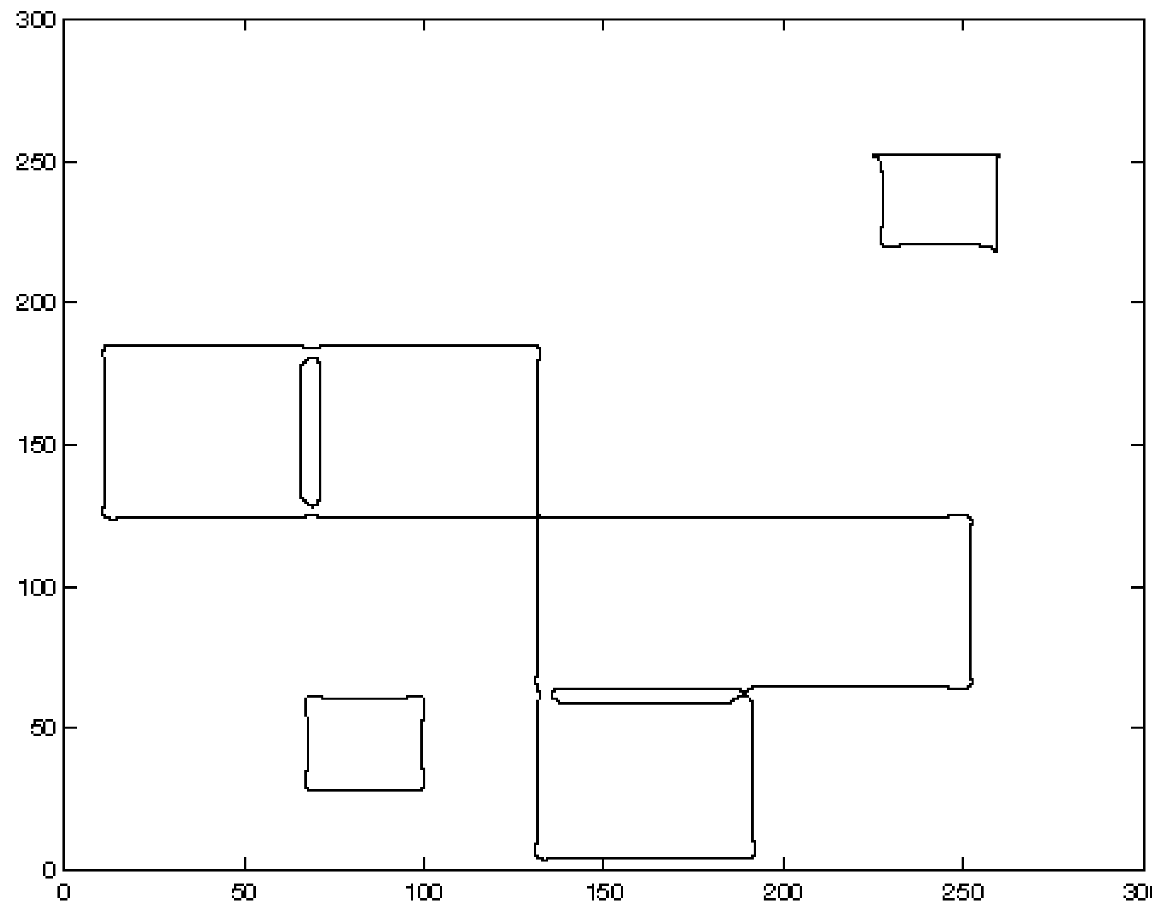
contrast=4



sigma=2



Especially unfortunate behavior at corners (simple edge detectors do not do well at corners in general)



Introduction to Fourier methods

- A periodic function (vector) can be decomposed into a sum of sines and cosines
- Sines and cosines are **orthogonal**
- This forms a new basis for the function (vector)
- A (suitably well behaved) non-periodic function can also be decomposed but now an integral is needed (not just an infinite sum).
- The distinction is blurred in the discrete case--we have a finite number of samples, and are at liberty to assume the image repeats in some way.

Introduction to Fourier methods

- Because the basis functions (sines/cosines) are orthogonal, we find their coefficients by integrating against them (or, in the discrete case, taking dot products).
- Sampling theorem: (roughly)--we can reconstruct a "band limited" signal (composed of a limited number of frequencies) from a limited number of samples
  - This is why adding even more bits to the digital representation of music does not help--you can only hear up to certain frequency, sampling more than that rate does not do any good.

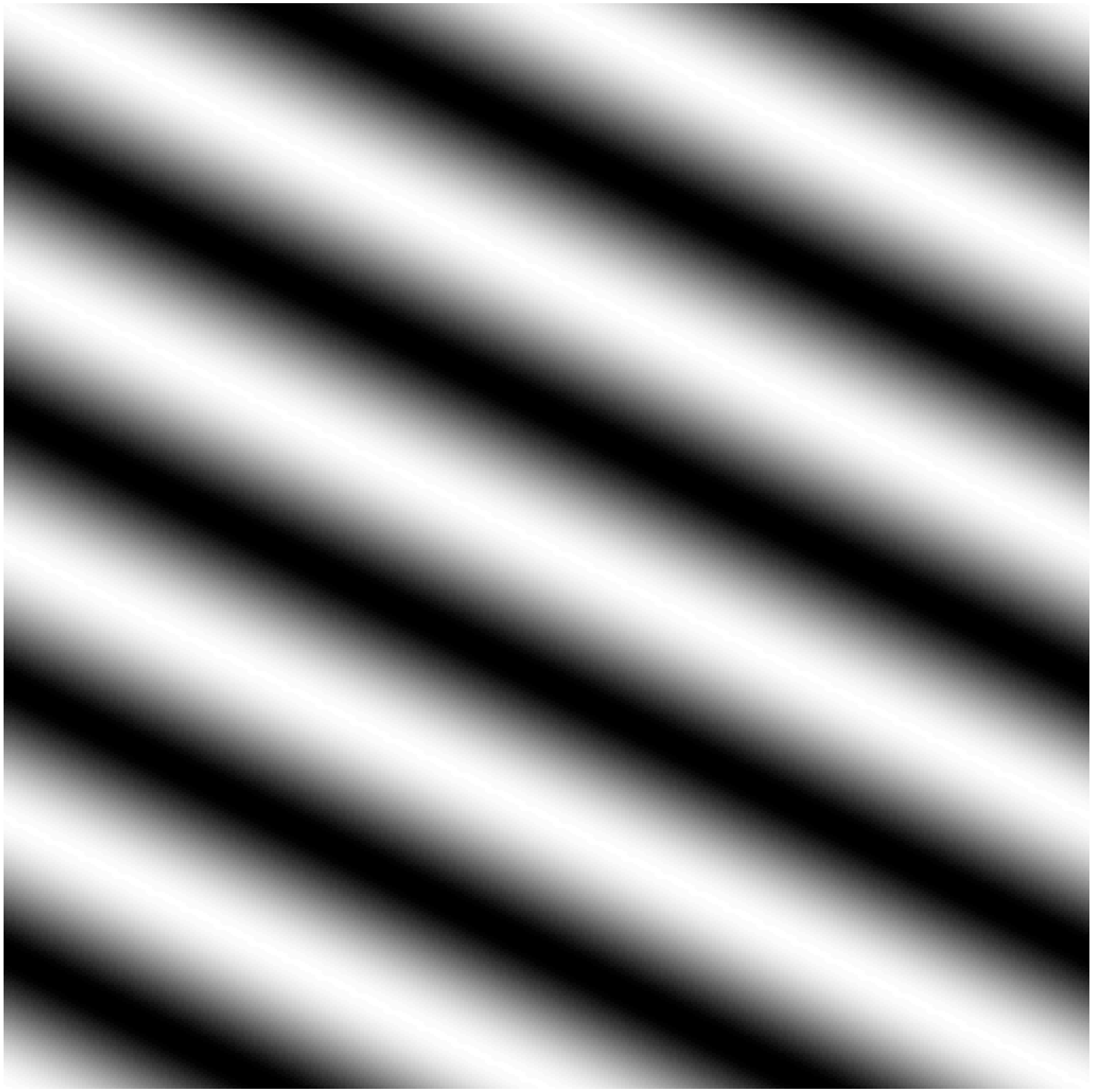
The Fourier Transform

- Represent function on a new basis
  - Think of functions as vectors, with many components
  - We now apply a linear transformation to transform the basis
    - dot product with each basis element

$$F\big(g(x,y)\big)(u,v) = \iint_{\mathbb{R}^2} g(x,y)\, e^{-i2\pi(ux+vy)}\, dxdy$$

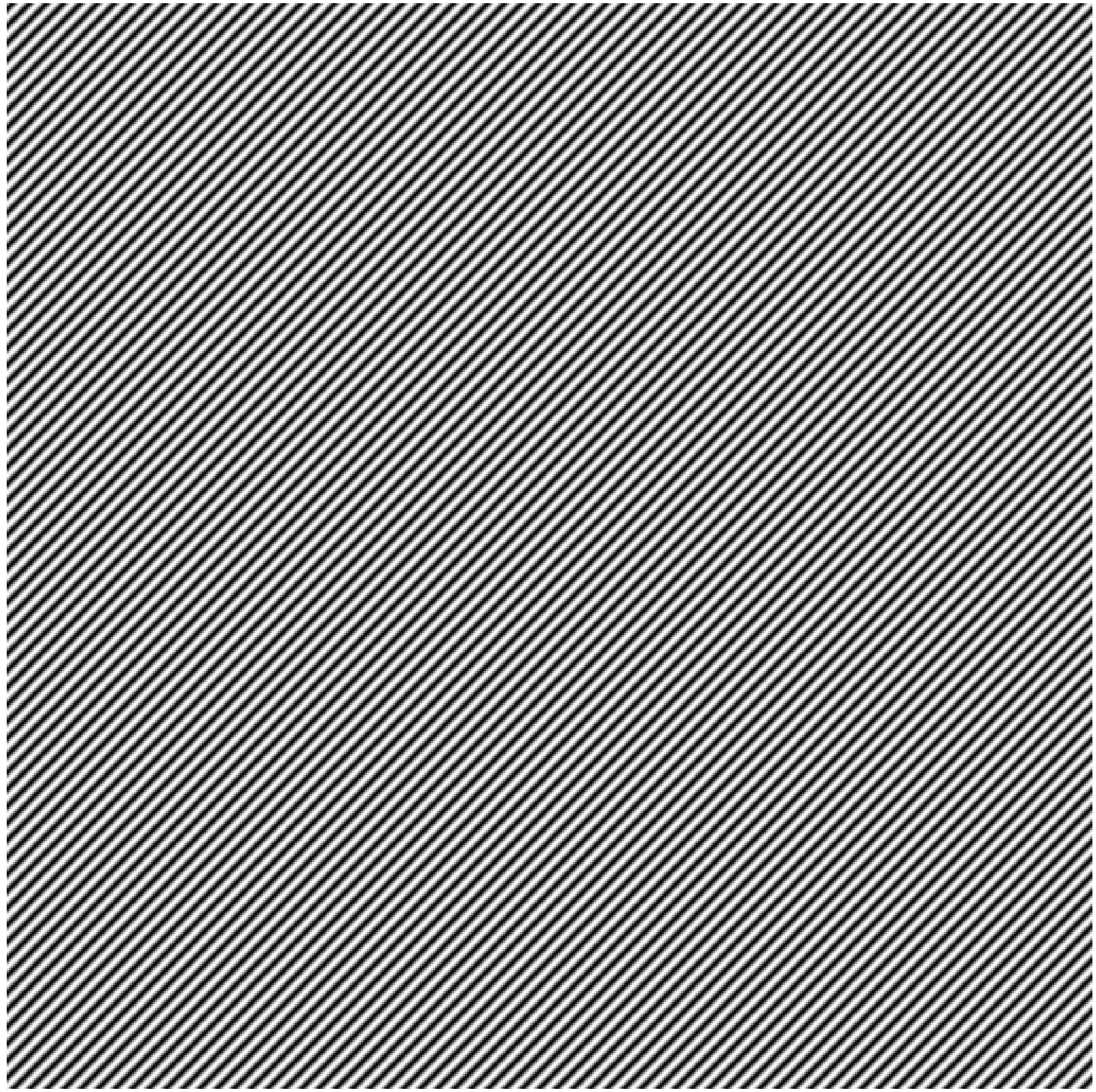
- In the expression, u and v select the basis element, so a function of x and y becomes a function of u and v
- basis elements have the form $e^{-i2\pi(ux+vy)}$

To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.



Here u and v are larger than in the previous slide.

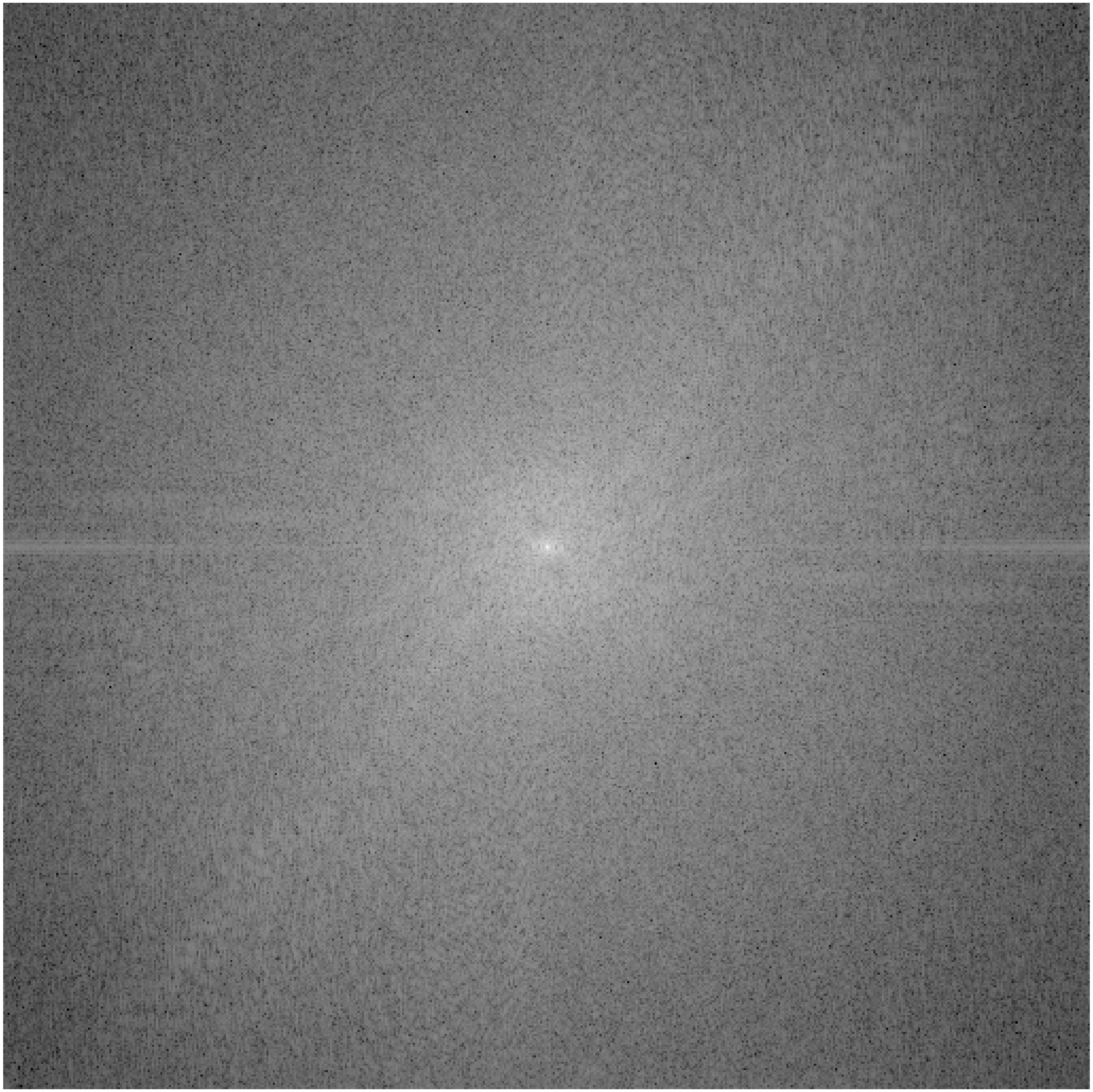
And larger still...



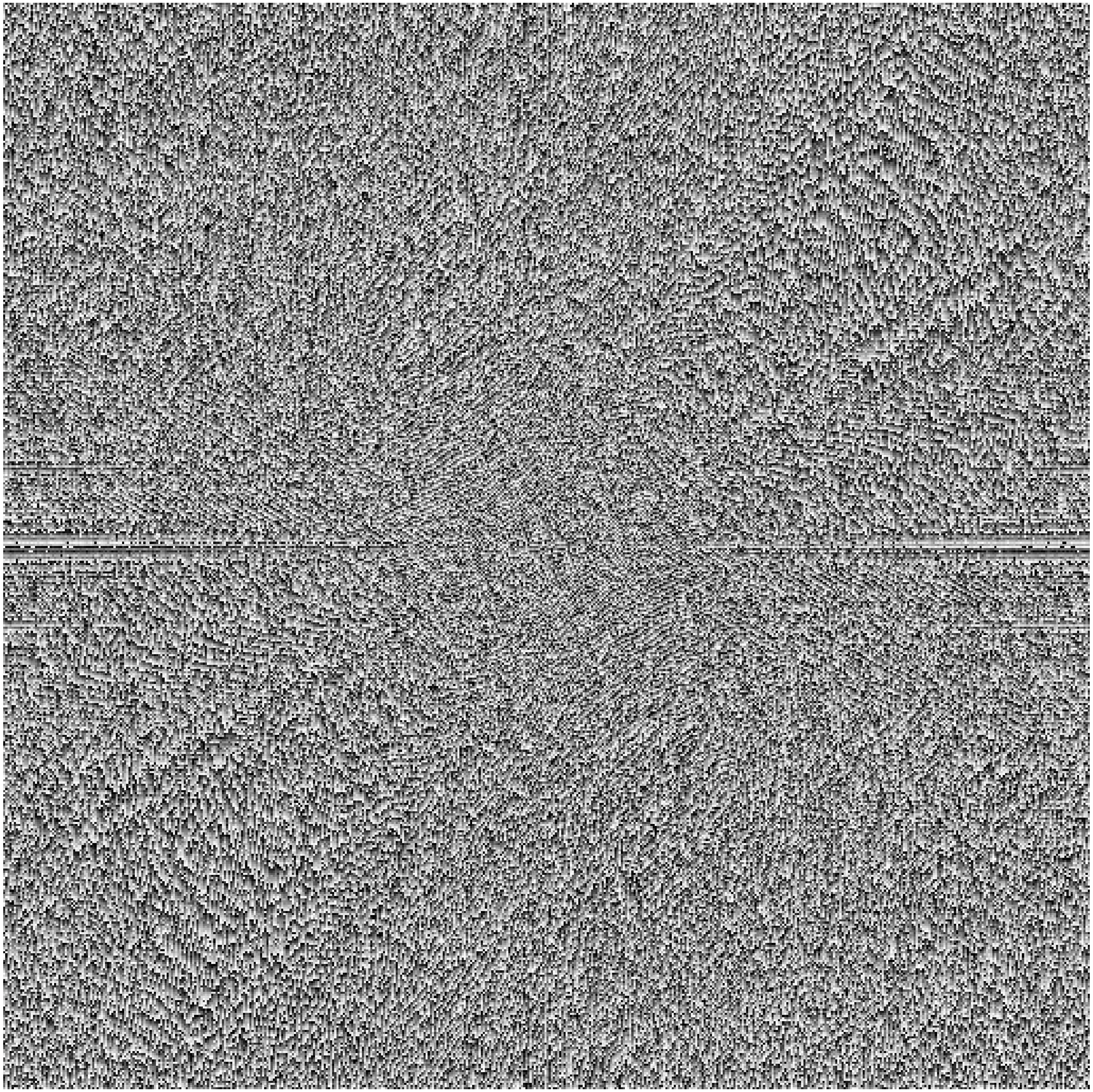
Phase and Magnitude

- Fourier transform of a real function is complex
  - difficult to plot, visualize
  - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
- Magnitude is the magnitude of the complex transform
- Curious fact
  - all natural images have about the same magnitude transform
  - hence, phase seems to matter, but magnitude largely doesn’t
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

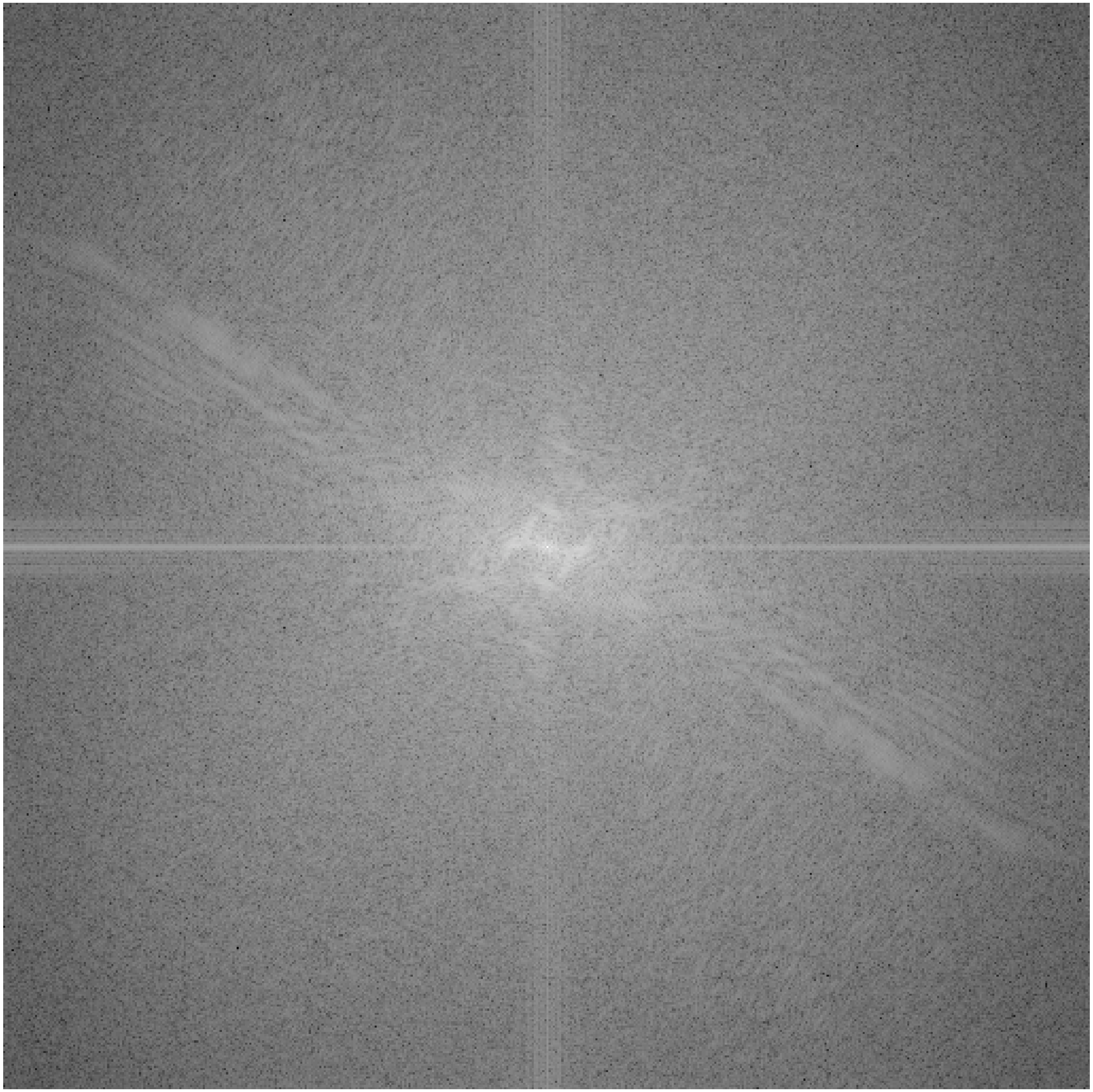
This is the magnitude transform of the cheetah pic



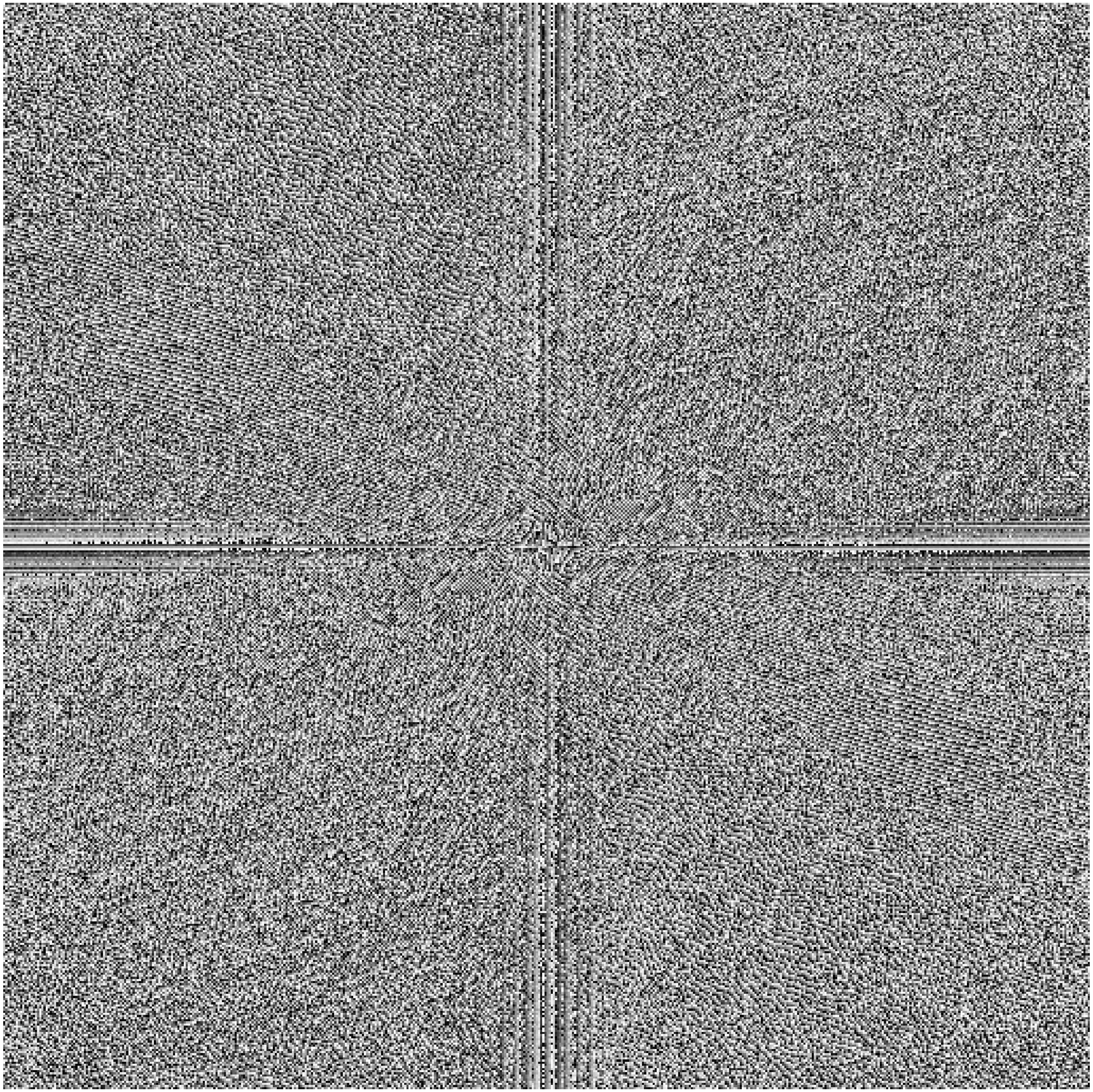
This is the phase transform of the cheetah pic



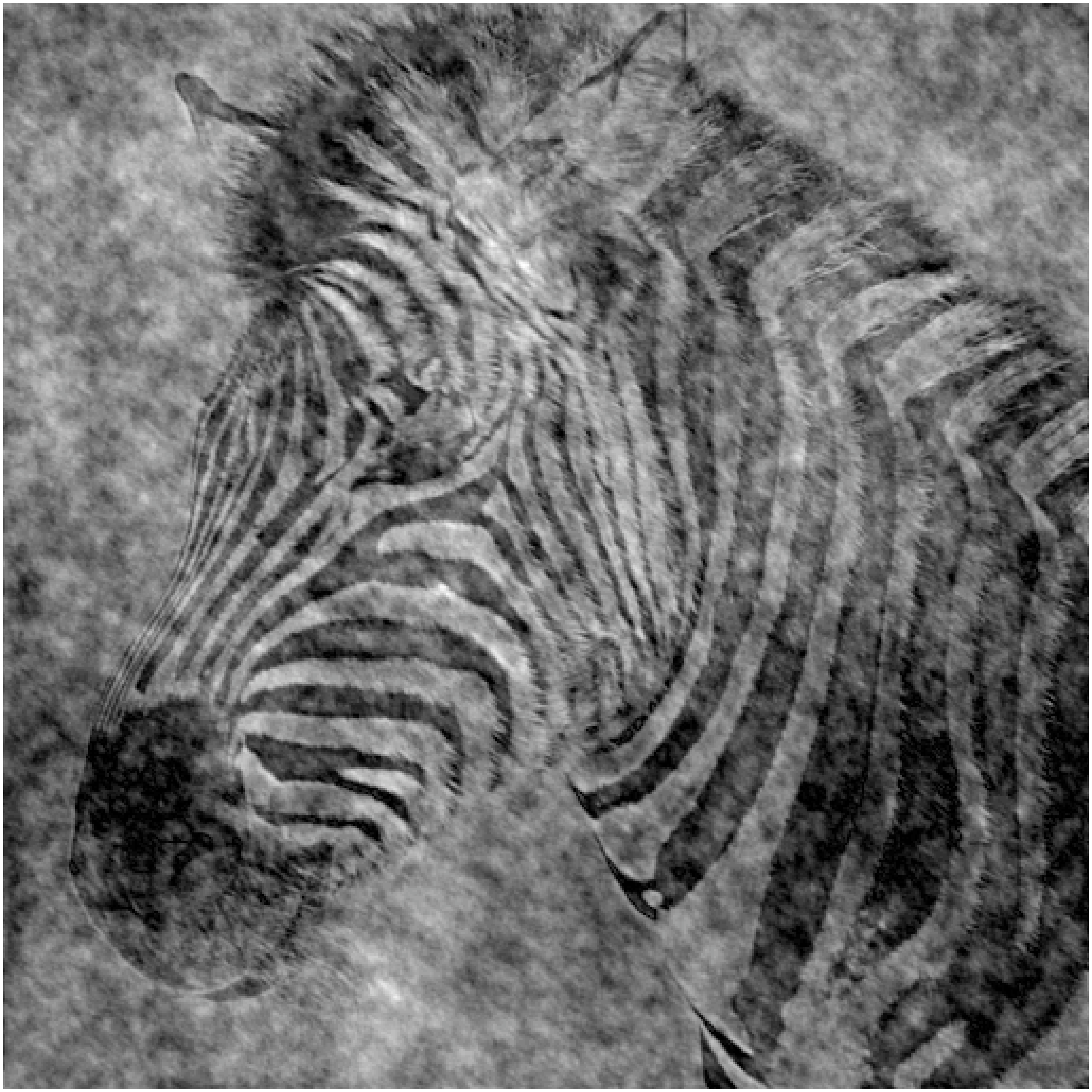
This is the magnitude transform of the zebra pic



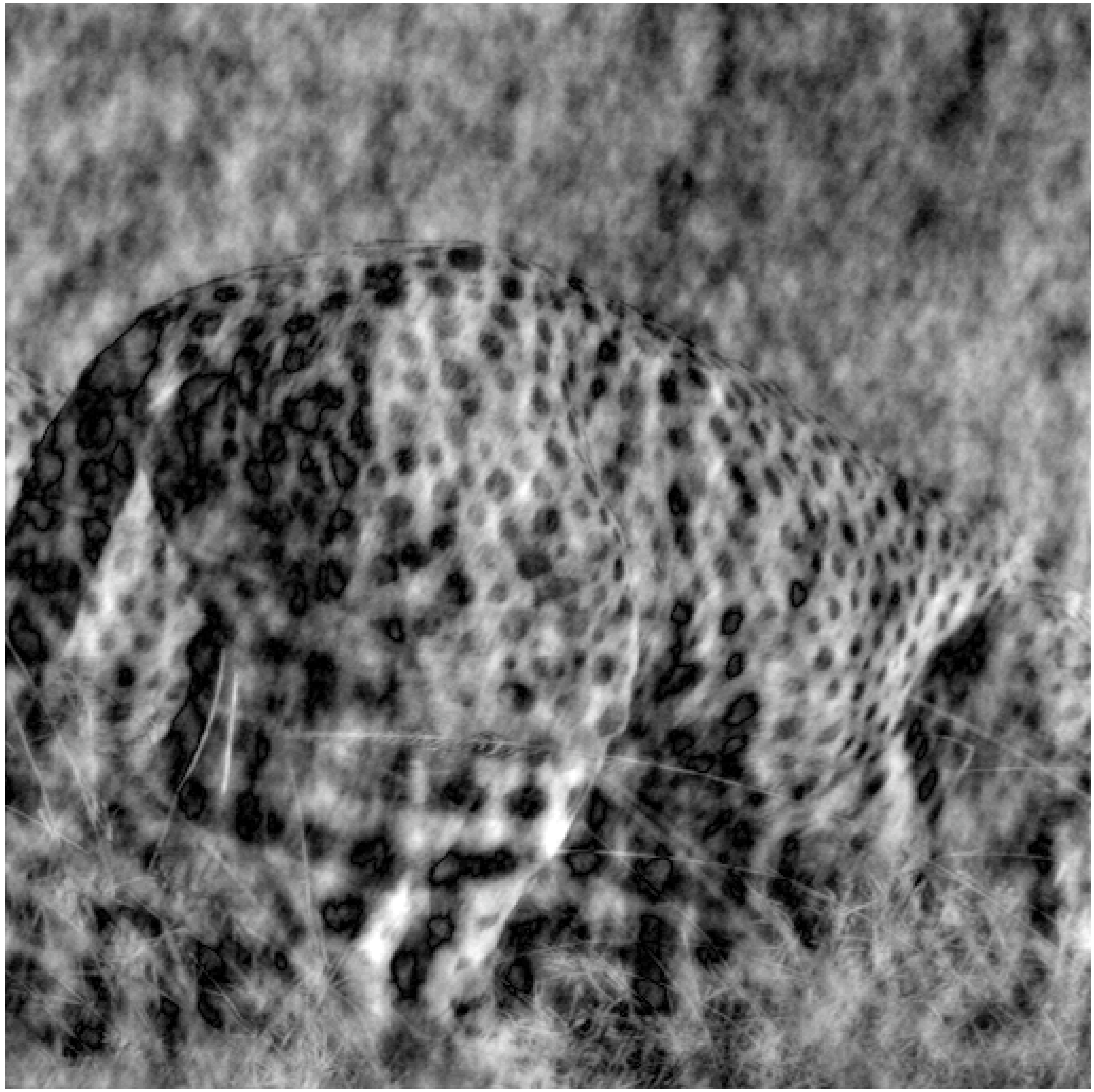
This is the phase transform of the zebra pic



Reconstruction with zebra phase, cheetah magnitude



Reconstruction with cheetah phase, zebra magnitude



Fourier Transform

- Important facts
  - The Fourier transform is linear
  - There is an inverse FT
  - if you scale the function's argument, then the transform's argument scales the other way. This makes sense --- if you multiply a function's argument by a number that is larger than one, you are stretching the function, so that high frequencies go to low frequencies
  - The FT of a Gaussian is a Gaussian.
- The convolution theorem
  - The Fourier transform of the convolution of two functions is the product of their Fourier transforms
  - The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms
- There's a table in the book.