Syllabus Notes

• We are finishing filtering. We will finish the Fourier transform (7.3.1). And then onto texture (last topic for midterm).

• Recommended optional reading--handouts
The Fourier Transform (review)

• The Fourier Transform is an example of a very important idea--represent function on a new basis of orthogonal elements
  – Discrete, 1D, example
  – We find the coefficients by a linear transform
• Recall that in the continuous domain, dot product (inner product) becomes integration
• In the Fourier Transform, our basis are sines and cosines
The Fourier Transform (review)

• We use complex number for convenient representation

• In the Fourier transform the basis is $e^{i2\pi(ux+vy)}$

• Recall that $e^{i2\pi(ux+vy)} = \cos(2\pi x) + i \sin(2\pi y)$

• The transform is given by

$$F(g(x,y))(u,v) = \int_{\mathbb{R}^2} g(x,y) e^{i2\pi(ux+vy)} \, dx \, dy$$
The Fourier Transform

• Have both cosines (gives real part) and sines (imaginary part)
• Recall that for an even (symmetric) function \( f(-x)=f(x) \), and for an odd (anti-symmetric) one \( f(-x)=-f(x) \)
• Sine gives odd part of function, cosine even part
• If the function is even there are only cosine terms, and the result is real (cosine transform)

Example bases with different \((u,v)\)
Fourier Transform (continued)

- Fourier transform of a real function is complex
  - difficult to visualize
  - we can think of the phase and magnitude of the transform
- \( z = a + bi \)
- Phase is the phase (angle) of the complex transform
  \( \theta = \arctan(b/a) \)
- Magnitude is the magnitude of the complex transform
  \( |z| = \sqrt{a^2 + b^2} \)

- Important observation
  - The Fourier transform is global--the value for each \((u,v)\) is a function of the entire image.
  - (This is why it is difficult to visualize/understand)
  - The magnitude gives an idea of the distribution of energies at each frequency, and is similar across many natural images (recall the cheetah/zebra swap)
This is the magnitude transform of the cheetah pic.
This is the phase transform of the cheetah pic
This is the magnitude transform of the zebra pic
This is the phase transform of the zebra pic
Reconstruction with zebra phase, cheetah magnitude
Reconstruction with cheetah phase, zebra magnitude
The Convolution Theorem

• Important result which can have practical impact (convolution theorem)

\[ F(a \circledast b) = F(a)F(b) \]

• (Depending on your workflow, using the DFT for convolution can save time).
Fourier Transform (practice)

- Because of the convolution theorem, the FT gives a convenient way to invert the effect of convolution. For example, often blurring can be modeled as a convolution, and the FT gives a convenient way to think about de-blurring.

- Fast (O(n*\log(n))) methods exist to compute discrete version of Fourier transform (DFT2 in Matlab, IDFT2 for the inverse).

- If we assume that the image is periodic and symmetric then only the cosine terms count and we can avoid imaginary components which can speed up and simplify some tasks (cosine transform; DCT2 in Matlab, IDCT2 for the inverse).
Texture

- Texture always has a scale (leaf -> bush -> forest)
- Key issue: representing texture
- Texture based matching
  - obvious thing to do, little is known
- Texture segmentation
  - key issue: representing texture
- Texture synthesis
  - useful; also gives some insight into quality of representation
- Shape from texture
  - cover superficially
Representing textures

• Textures are made up of quite stylized sub-elements, repeated in meaningful ways
• Representation:
  – find the sub-elements, and represent their statistics
• But what are the sub-elements, and how do we find them?
  – recall normalized correlation
  – find sub-elements by applying filters, looking at the magnitude of the response
• What filters?
  – experience suggests spots and oriented bars at a variety of different scales
  – details probably don’t matter
• What statistics?
  – within reason, the more the merrier.
  – At least, mean and standard deviation
  – better, various conditional histograms.
Smaller Scale

Larger Scale
(Image from previous slide made larger to compare)