Administrivia

Access to BSE 324 has been requested, and your CAT card should enable you to enter the lab now (or very soon). After hours, entry into the building also requires CAT card access together with a PIN which will be set to your month and date of birth in the form MMDD

Honors: See www.honors.arizona.edu/contracting.html

First assignment as well as first three lectures are available on web page. Access to course material will require either connecting from a UA machine, OR a login id ("me") and password ("vision4fun")

TA: Scott Morris is awaiting your questions (smorris @ cs.arizona.edu)
Office 710D Office hours: TBA

Syllabus Notes

We are currently doing §1.1.1 with extra detail, and a few bits which appear in §2. I recommend reading §1 and much of §2. We will later revisit some parts of §2 not covered in the next few.

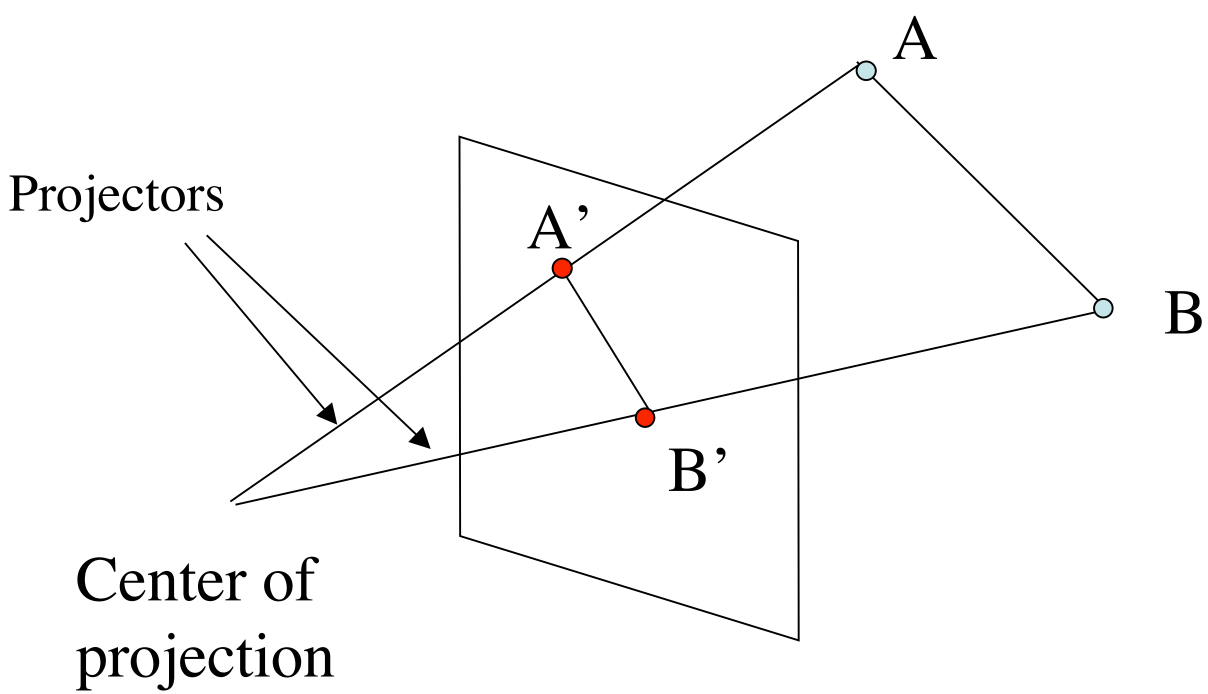
Students who took graphics will notice some similarities between what is discussed in §2.2 and mapping to the canonical frustum. (Graphics students may want to consider how the rotation matrix we used to rotate world coordinates to camera coordinates is a special case of the rotation matrix in the table on page 36).

The section in §3 regarding least squares is recommended. We will use this.

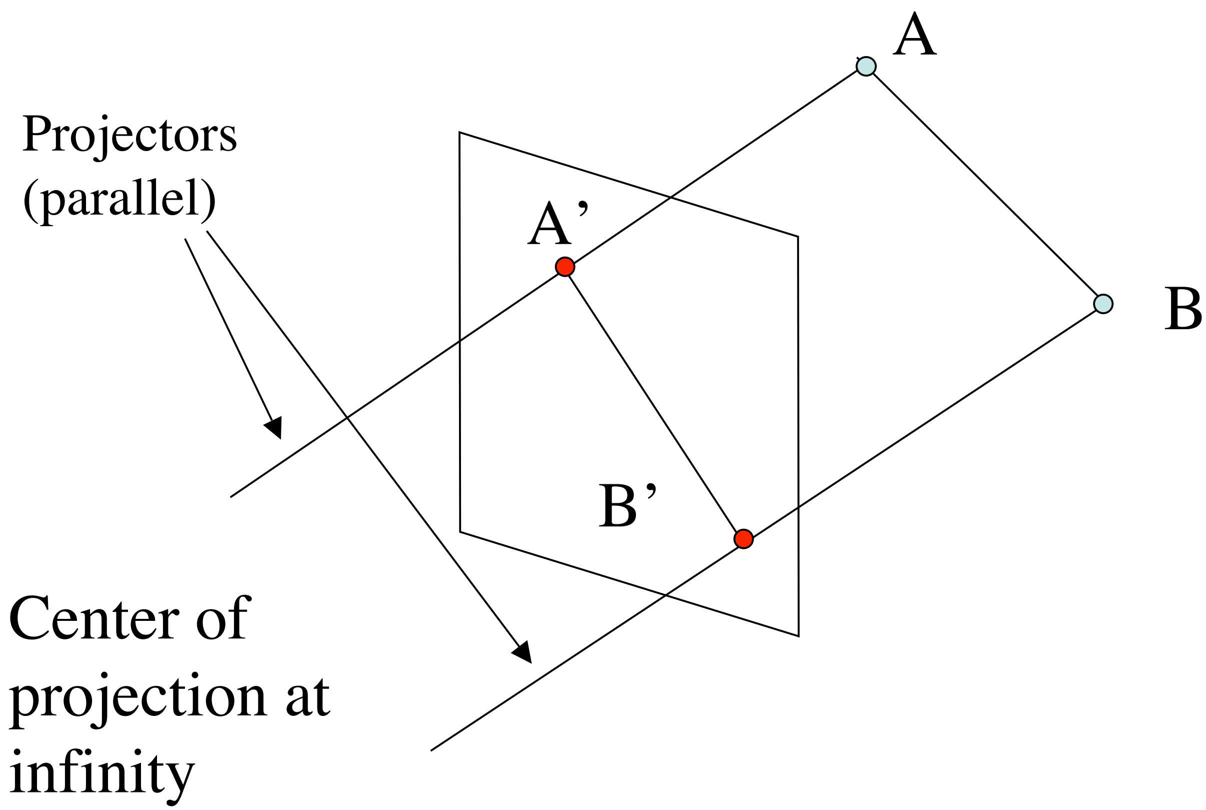
Projections

- Want to think about geometric image formation as a mathematical transformation taking points in the 3D world and mapping them into an image plane.
- Mathematical definition of a projection: PP=P
- (Doing it a second time has no effect).
- Generally rank deficient (non-invertable)--exception is P=I
- Transformation looses information (e.g., depth)
- Given a 2D image, there are many 3D worlds that could have lead to it.

Projections



Parallel Projection

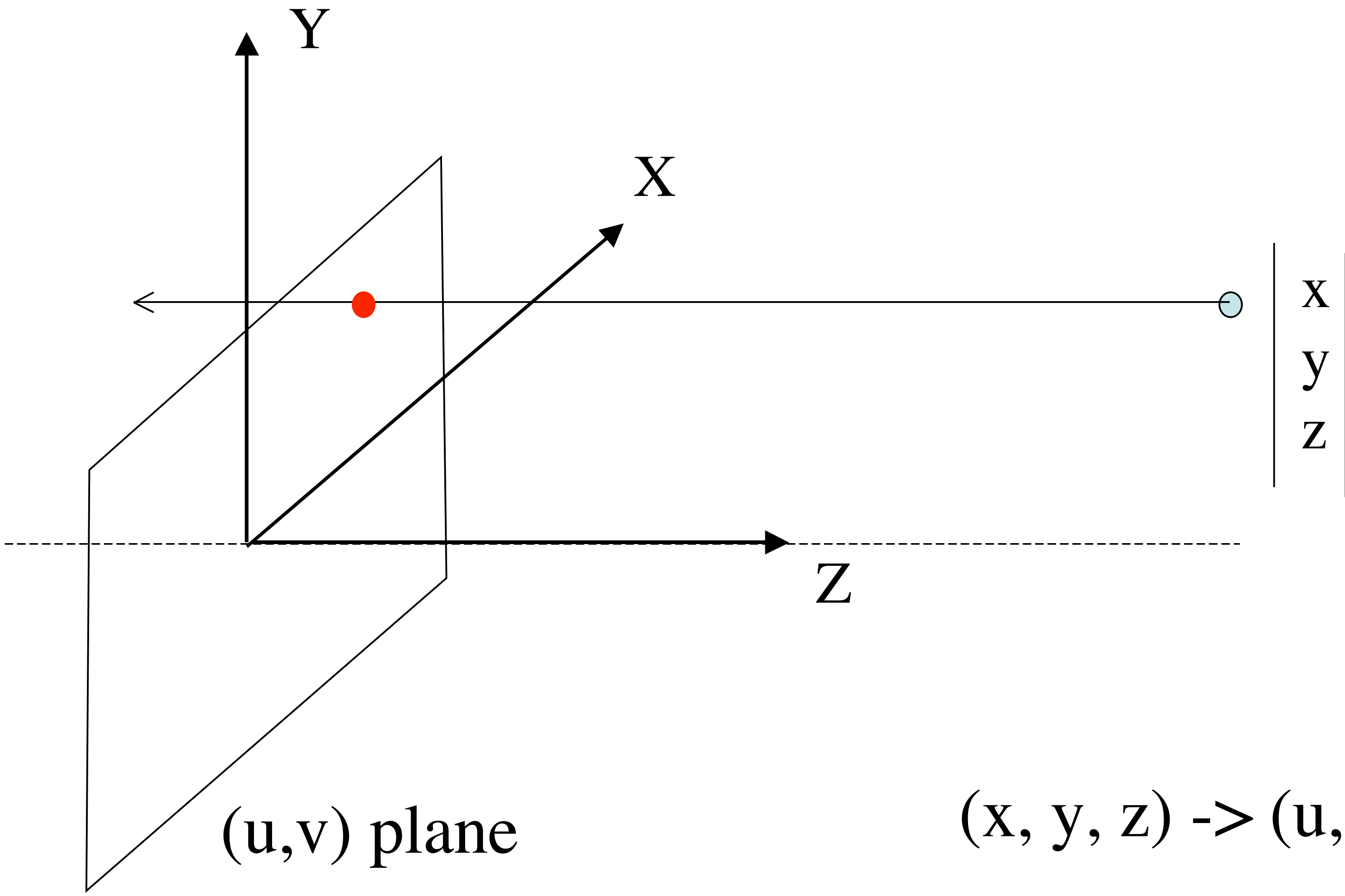


Parallel Projection

Parallel lines remain parallel, some 3D measurements can be made using 2D picture

If projection plane is perpendicular to projectors the projection is orthographic

Orthographic example (onto z=0)



(u,v) plane

(x, y, z) -> (u,v)

The equation of projection (orthographic, onto z=0)

- In homogeneous coordinates

$$(x,\ y,\ z,\ 1) \Rightarrow (x,\ y,\ 1)$$

- Graphics course survivors: You will notice slight changes in style to be consistent with the book. Perhaps most notably we will explicitly, rather than implicitly, ignore the third projected coordinate, so projection matrices will be 3 by 4 not 4 by 4.

The projection matrix

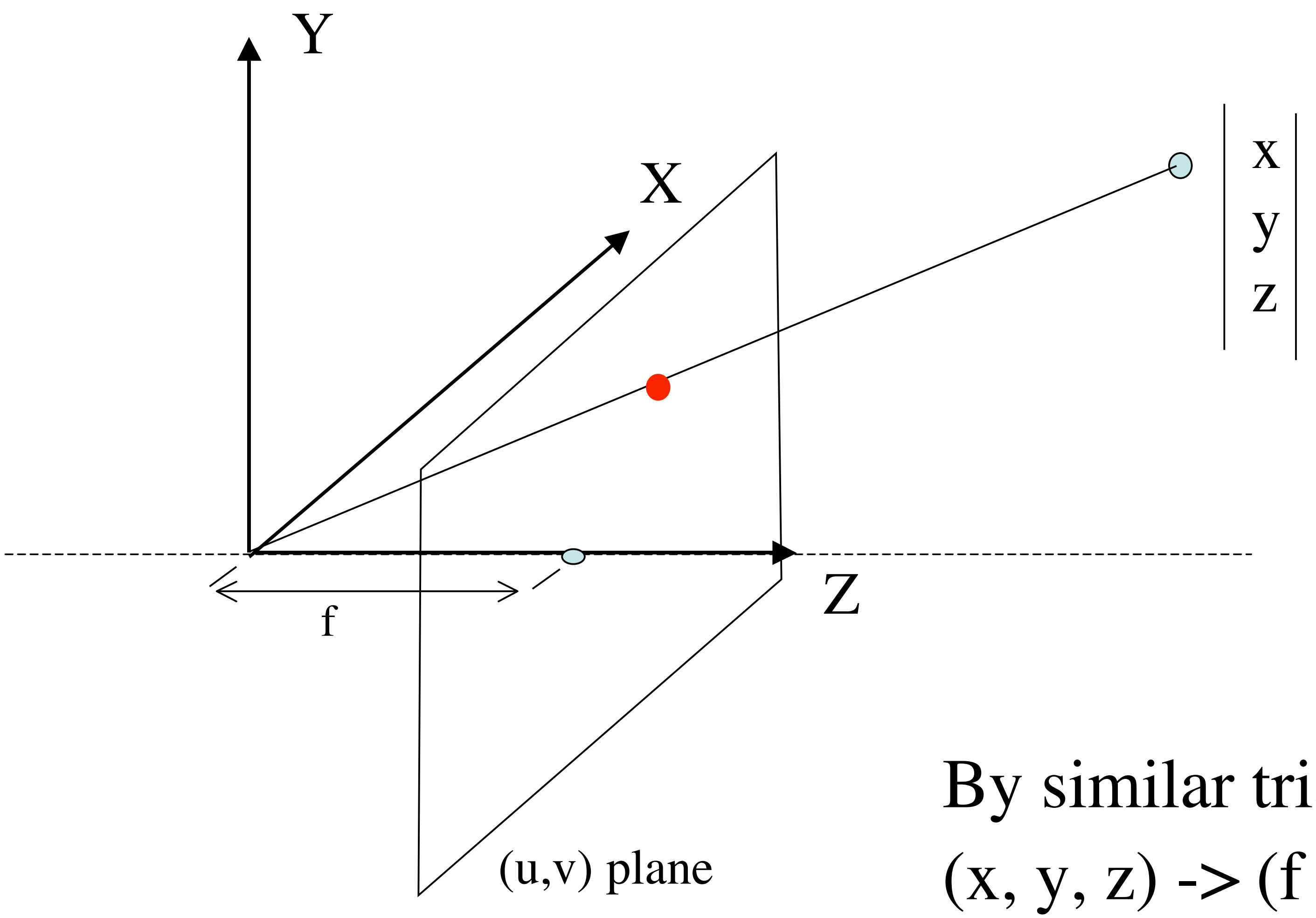
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} & & & \\ & ? & & \\ & & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$(x,\ y,\ z,\ 1) \rightarrow (x,\ y,\ 1)$

The projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective example (onto z=f)



By similar triangles,
(x, y, z) -> (f x/z, f y/z, f)

The equation of projection

- In homogeneous coordinates

$$(x,\ y,\ z,\ 1) \rightarrow (f\frac{x}{z},\ f\frac{y}{z},\ 1)$$

- Equivalently

$$(x,\ y,\ z,\ 1) \rightarrow (x,\ y,\ \frac{z}{f})$$

- Homogeneous coordinates are being used to store foreshortening

- Note that using regular coordinates does **not** yield a linear transformation (inconvenient!).

The projection matrix

$$\begin{bmatrix} x \\ y \\ \frac{z}{f} \end{bmatrix} = \begin{bmatrix} & & \\ & ? & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

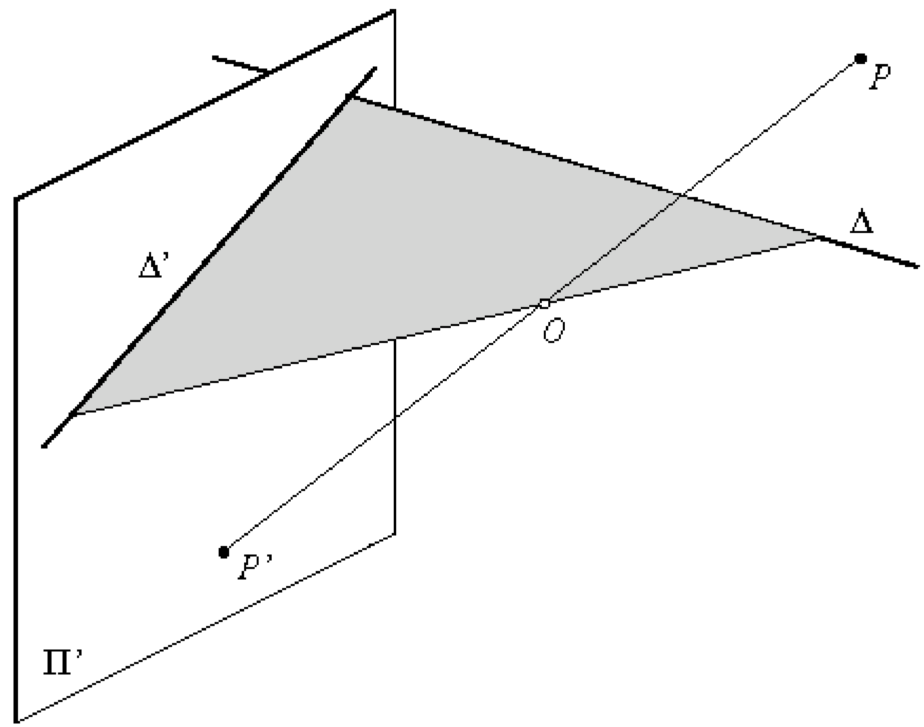
$$(x,\ y,\ z,\ 1) \rightarrow (x,\ y,\ \frac{z}{f})$$

The projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix}$$

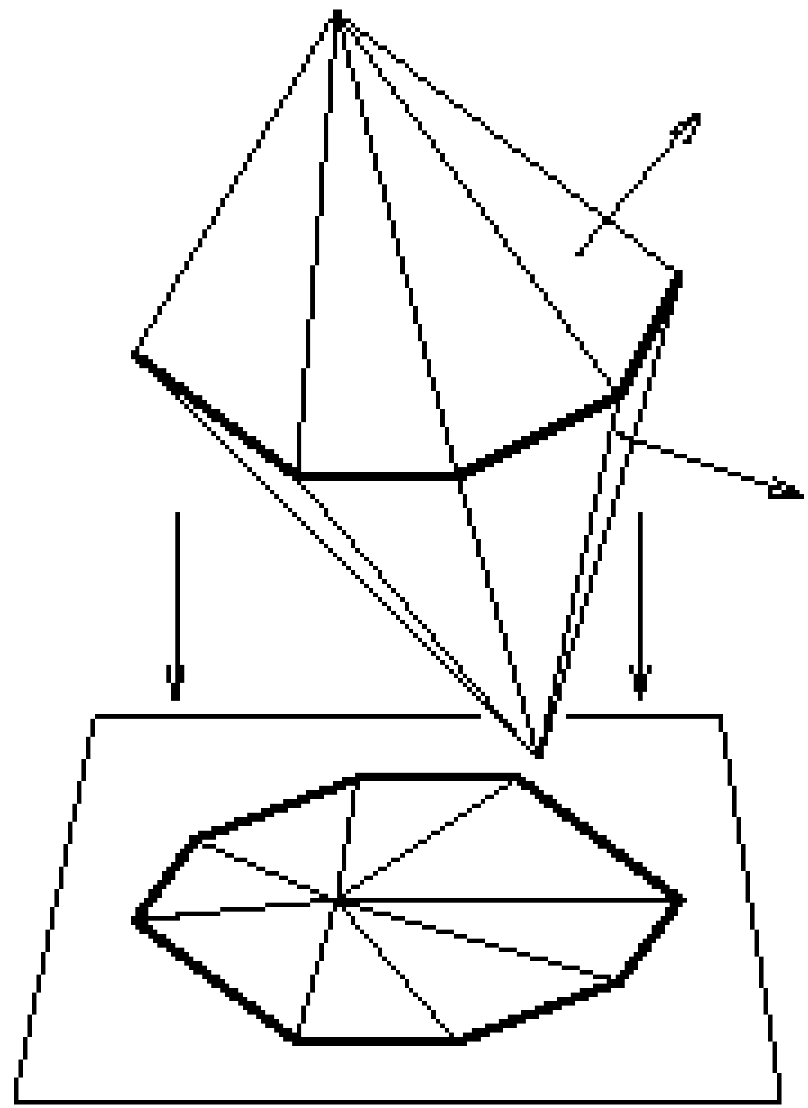
Geometric properties of projection

- Points go to points
- Lines go to lines
- Polygons go to polygons
- Degenerate cases
  - line through focal point projects to a point
  - plane through focal point projects to a line



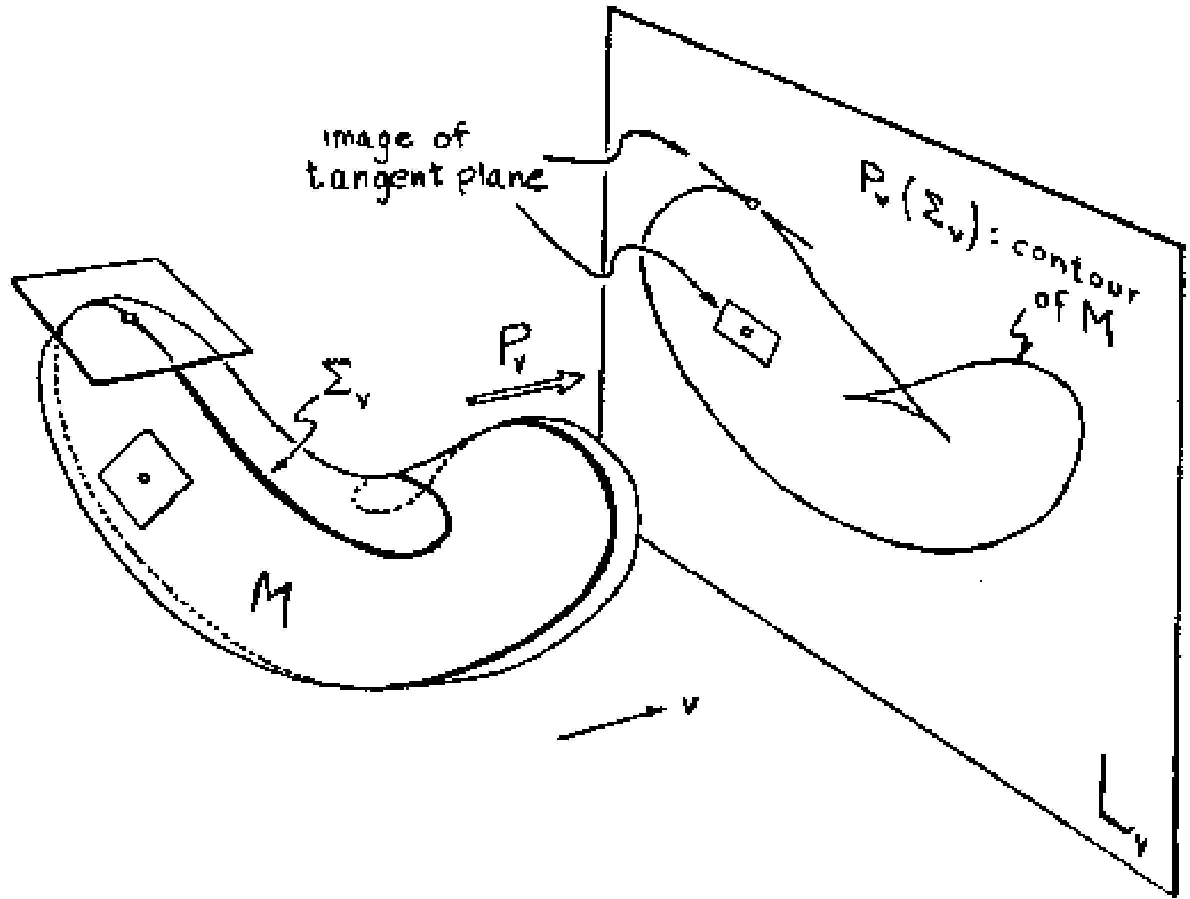
Polyhedra project to polygons

- Because lines project to lines



Curved surfaces are much more interesting

- Crucial issue: outline is the set of points where the viewing direction is tangent to the surface
- This is a projection of a space curve, which varies from view to view of the surface



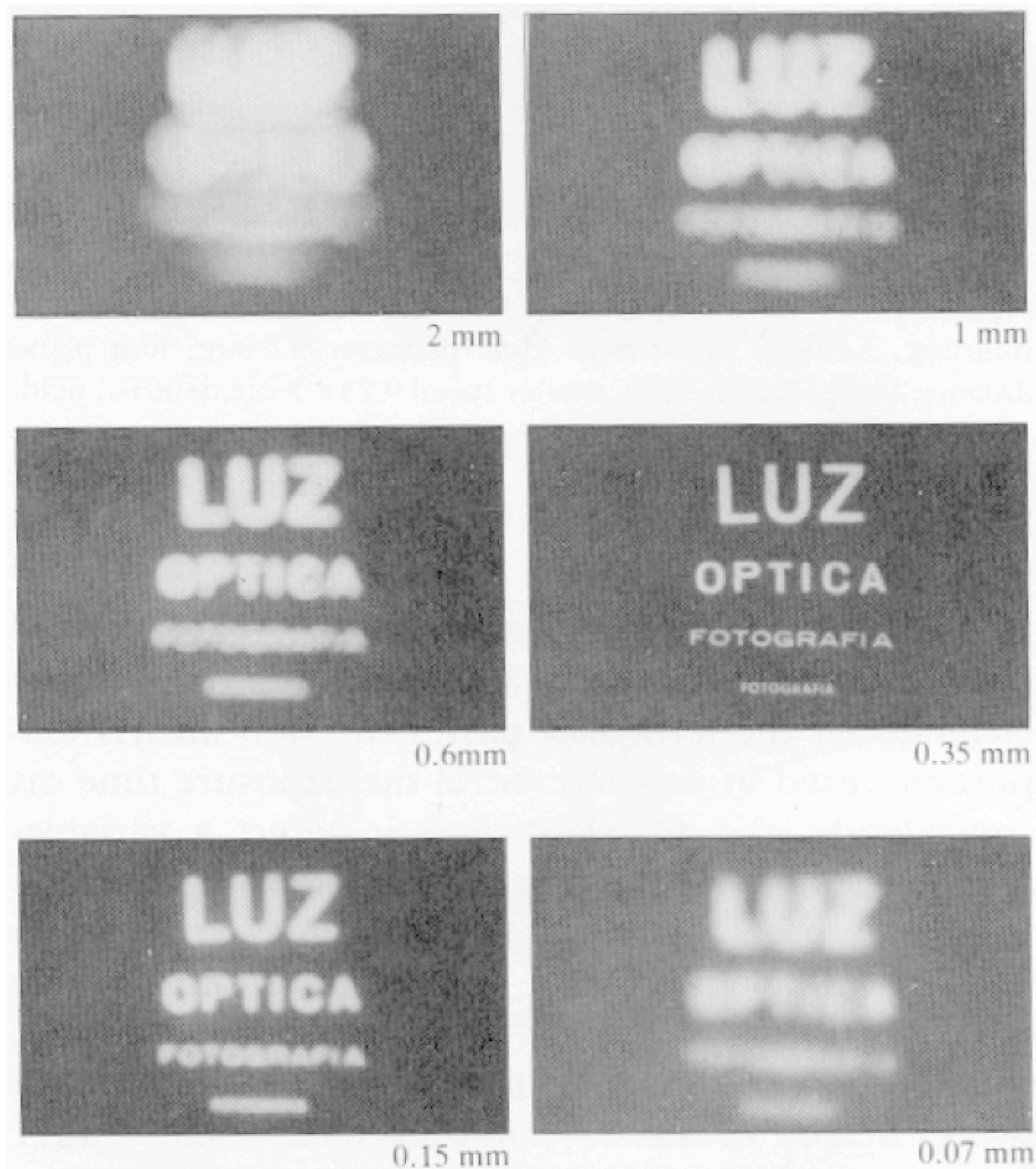
Real Cameras

Real Pinhole Cameras

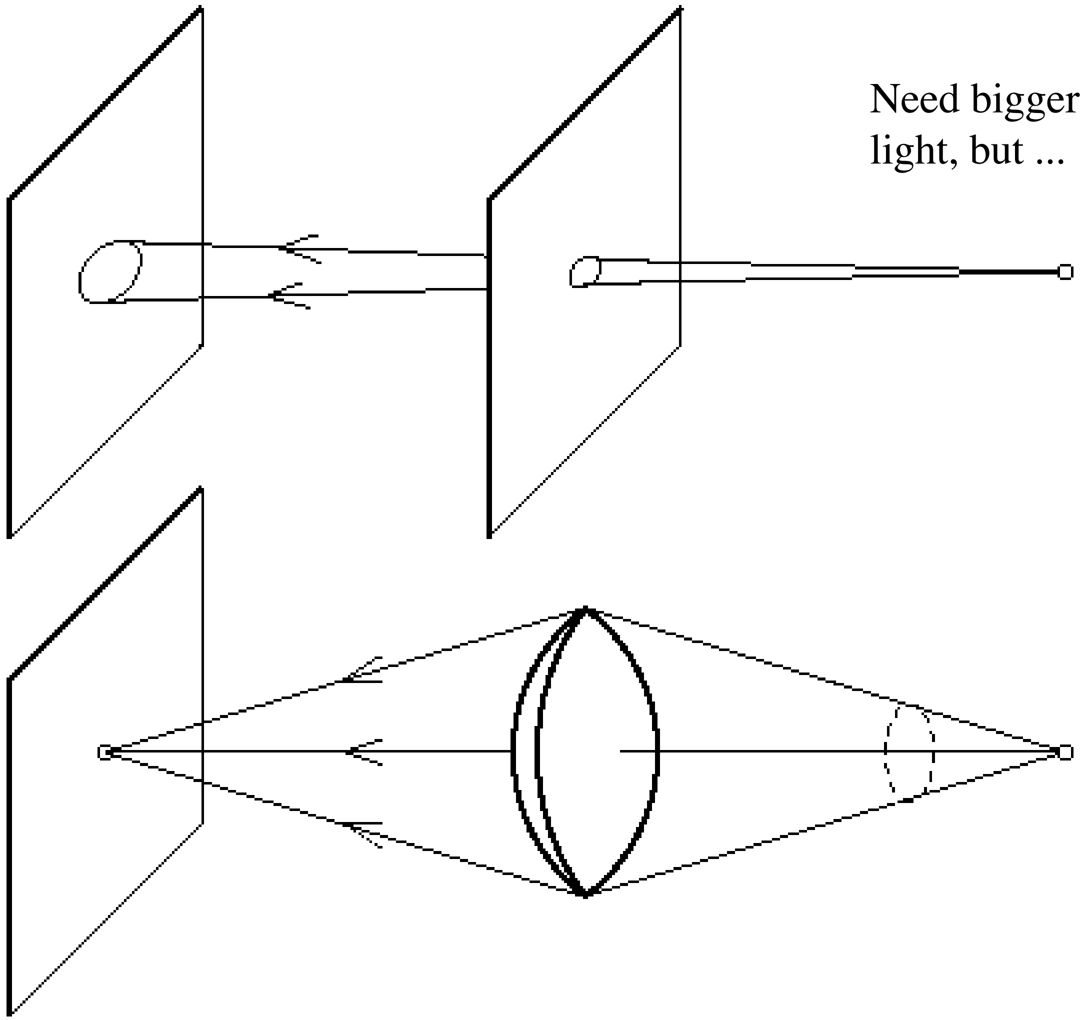
Pinhole too big -many directions are averaged, blurring the image

Pinhole too small-diffraction effects blur the image

Generally, pinhole cameras are ***dark***, because a very small set of rays from a particular point hits the screen.



The reason for lenses

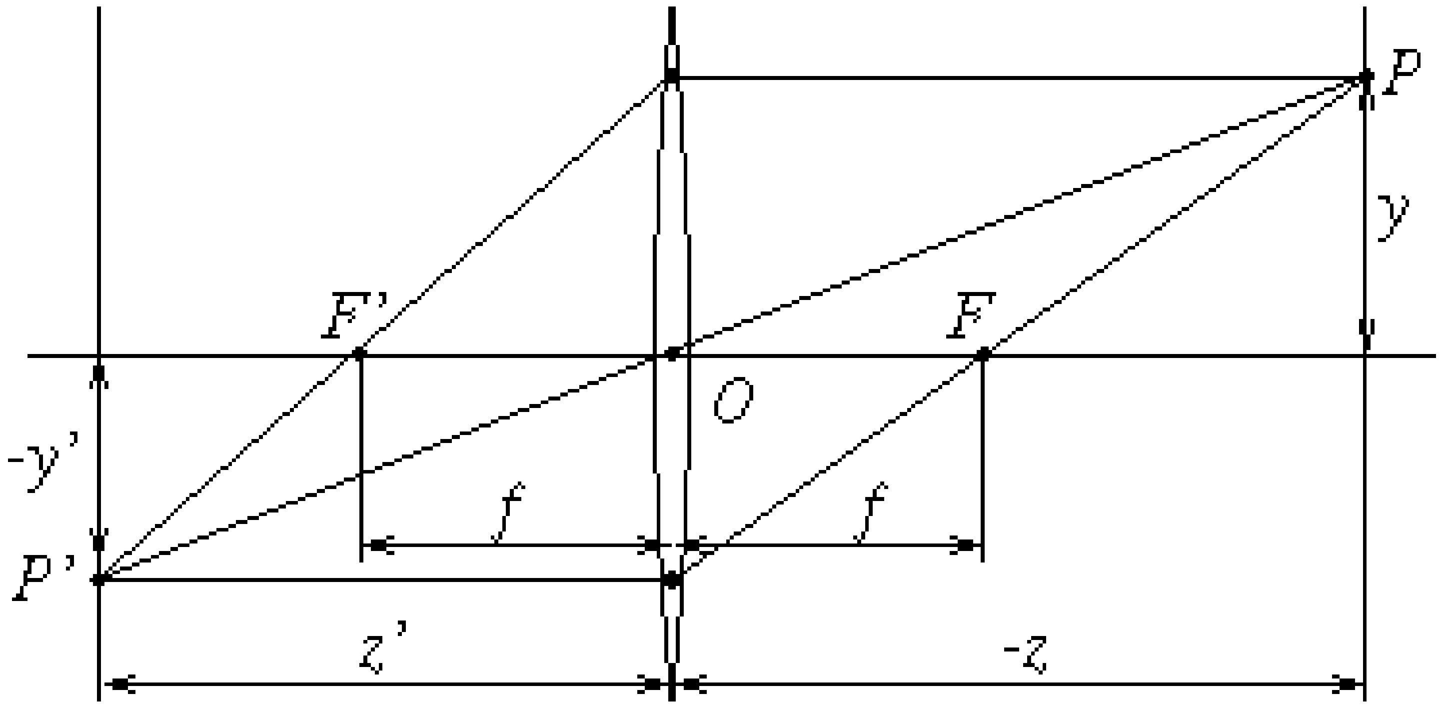


Need bigger hole to get more light, but ...

Depth of Field

Unlike a pinhole camera, a camera with a lenses has limited depth of field (only a limited range of depths are in focus at once)

The thin lens*

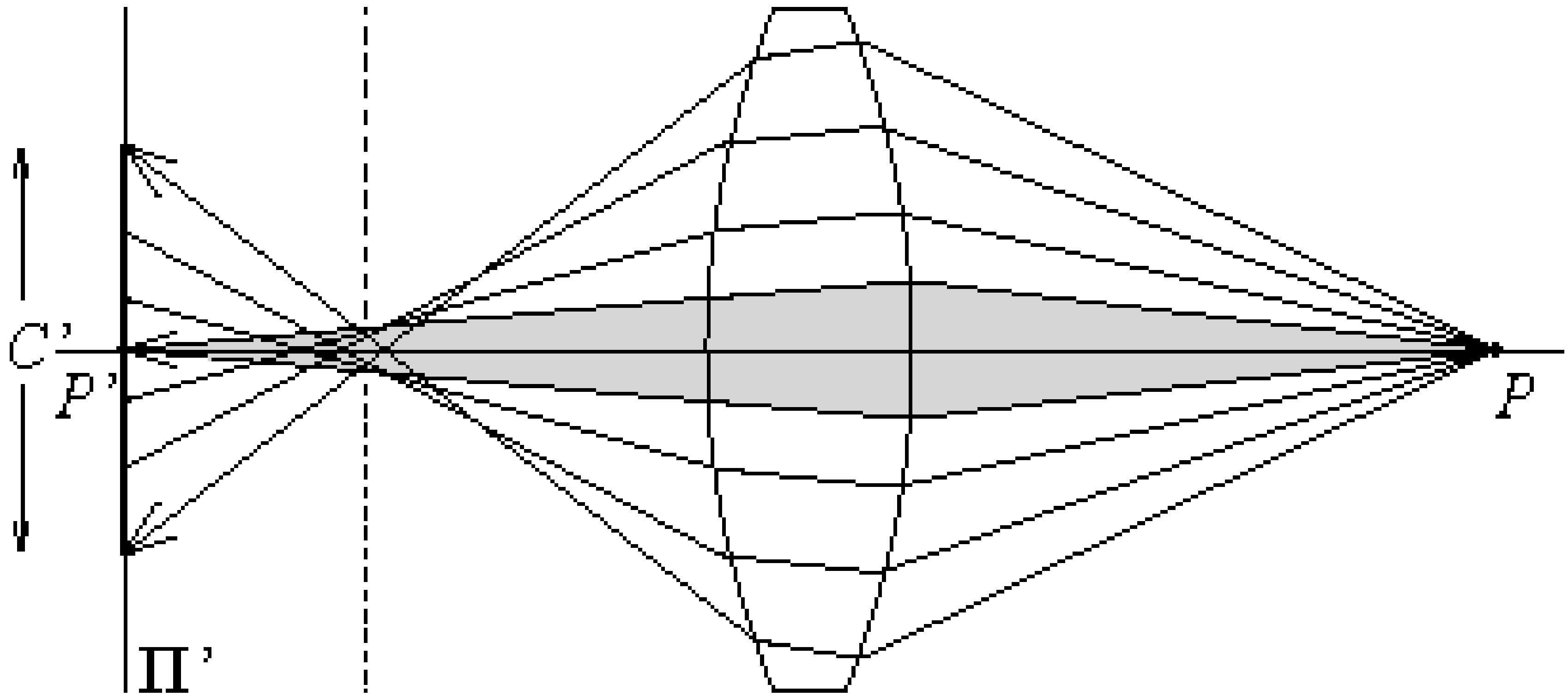


Focus depends on depth When z=infinity, this is same equation as a pinhole.

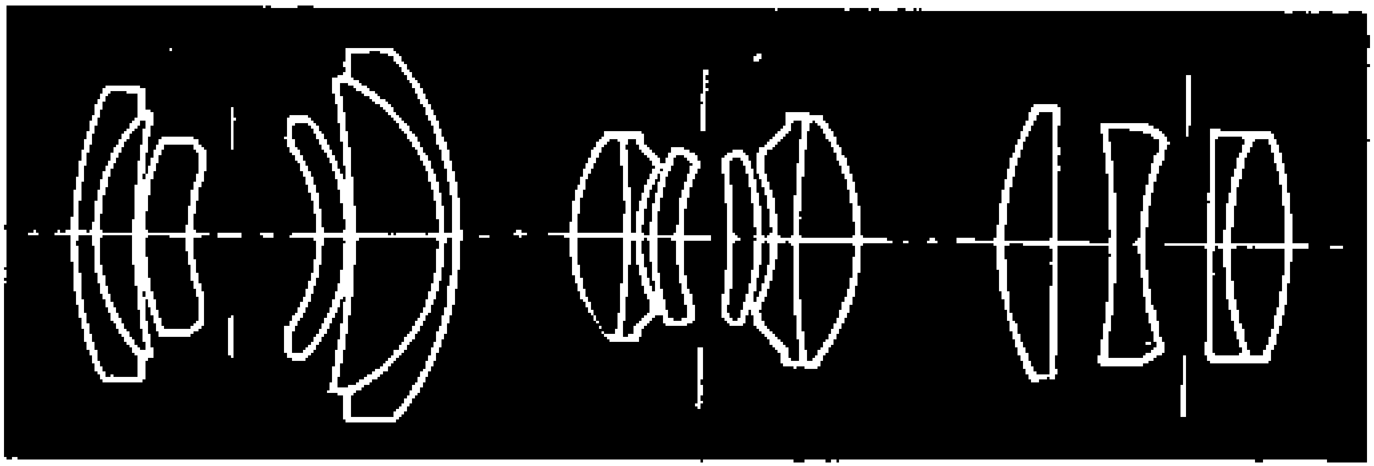
$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Spherical aberration*

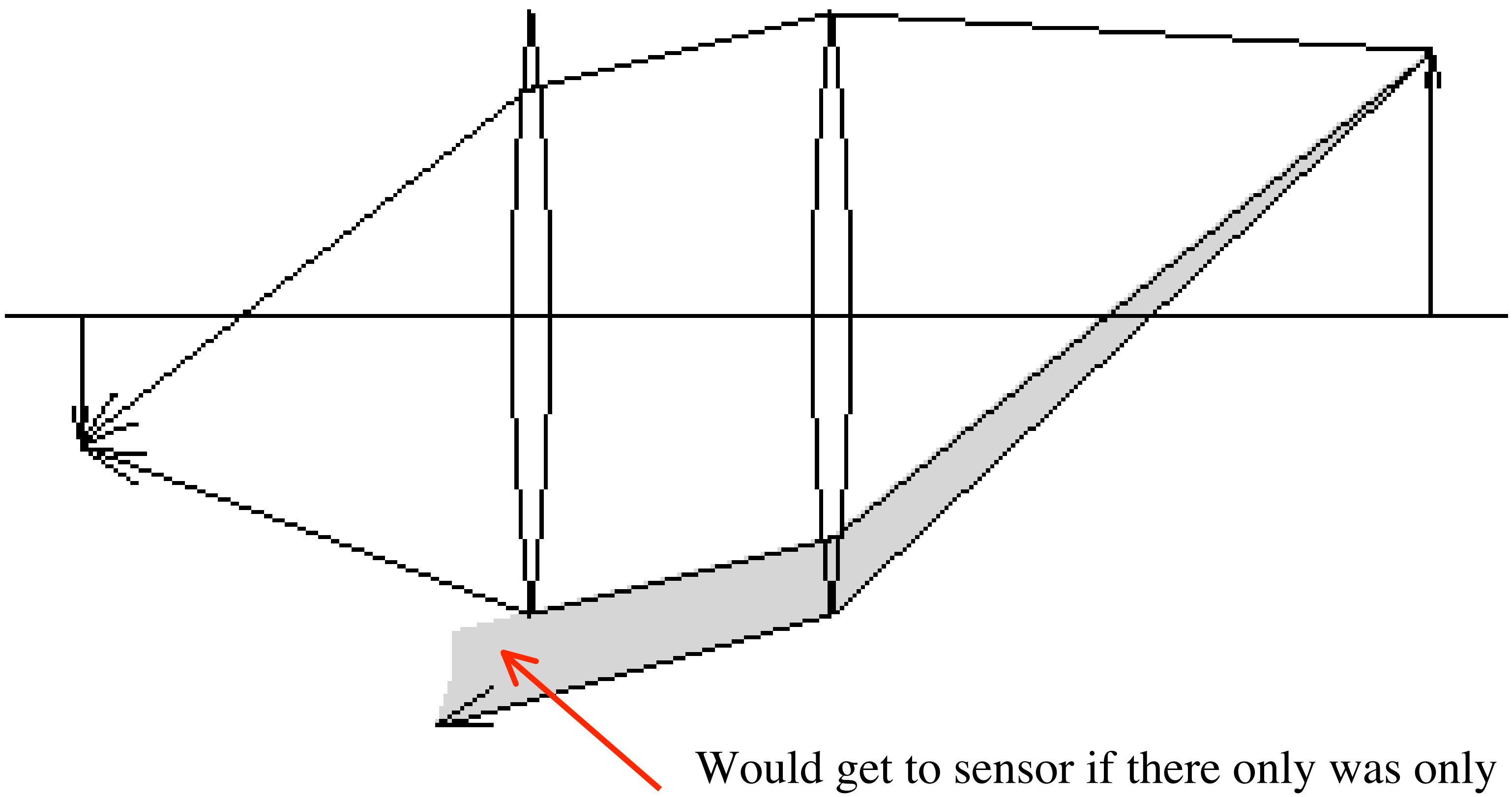
Imaging equation is not exact



Lens systems*



Vignetting: Fall-off in brightness is due to blockage of the optical path occurring more for off-axis light.



Would get to sensor if there only was only one lens element, or if it was less off axis.

Image plane brightness

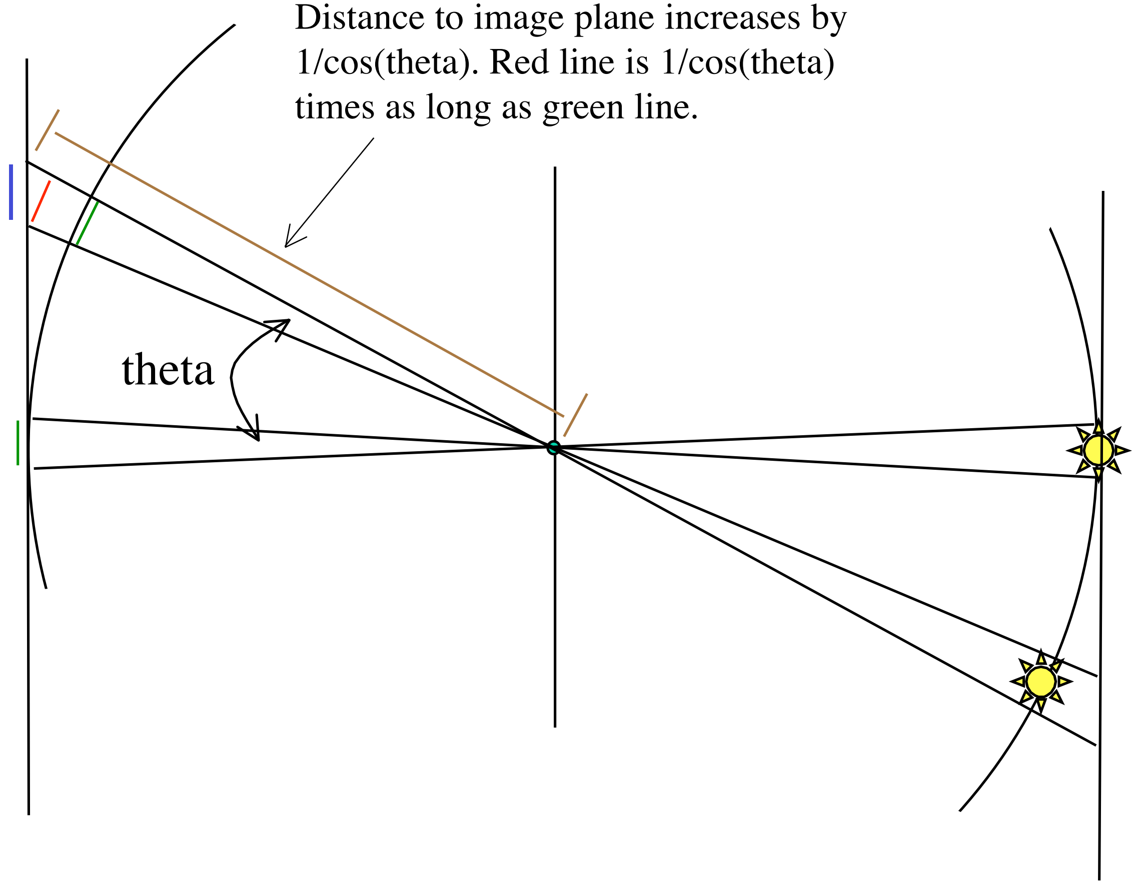
There is a $\cos^4(\theta)$ brightness fall-off towards the edges of the image due to foreshortening (true for pinhole cameras also)---this is a consequence of using a flat projection plane (less of a problem with the somewhat spherical projection of the eye).

This is in addition to any brightness reduction towards the edges due to vignetting

Understanding the picture on the next page will be helpful when we consider radiometry. (Radiometry is tricky---consider this a warm-up).

Image is spread out by a factor of cos(theta) due to oblique angle--blue line is longer than red line by approximately a factor of 1/cos(theta)

Distance to image plane increases by 1/cos(theta). Red line is 1/cos(theta) times as long as green line.



What counts is the amount of light per direction. If the vertical line was a wall which was equally bright in all directions, then the amount of light going through the “sun” is the same in both cases. The area is bigger at the bottom, but the angle is oblique (foreshortening). The two effects cancel each other.

Taking both effects into account, the blue line is longer than the green one by a factor of $1/\cos^2$(theta). Since images are 2D, expansion in one direction needs to be squared to give the change in area. So, the area receiving light from the “sun” increases by a factor of $1/\cos^4$(theta), and the brightness falls off by a factor of $\cos^4$(theta).