Administrivia

Assignment two is now available (a few adjustments may come some). I will touch up some intro docs to the vision lab software which some may want to use very soon.

More projects---I have some specs from Hong (not added yet), and another biology problem.

Linear Least Squares Problem One Summary

(the part you need to know)

You should be able to set up

$$U\mathbf{x} = \mathbf{y}$$

You should know that it is solved by

$\mathbf{x} = U^{\dagger}\mathbf{y}$ where $U^{\dagger}$ is the pseudoinverse of U

You can assume that you can look up

$$U^{\dagger} = (U^{T}U)^{-1}U^{T}$$

*You should also keep in mind that for numerical stability, one may have to use a different approach to solve (without matrix inversion) the following

$$U^{T}U\mathbf{x} = U^{T}\mathbf{y}$$

Linear Least Squares Problem One
(example one)

Fit best line to a bunch of points (bad way, but note that if you ask a software package to do linear regression, this is what you get).

Assume that a line is specified by:

$$y=mx + b$$

You have a bunch of (x,y)

What is m and b?

Linear Least Squares Problem One

(example one)

Can write y=mx + b as:

(x 1)*(m b) = y

Linear Least Squares Problem One

(example one---naïve line fitting)

Can write y=mx + b as:

(x 1)*(m b) = y

So form

a matrix U with rows (x_i 1)

a vector **y** with elements y_i

a vector of unknowns **x=**(x,y)

and use the formula to solve U**x=y**

Linear Least Squares Problem One

(example two---naïve spectral camera calibration)

Remember the fact that the camera has a spectral sensitivity $R(\lambda)$. So how do we find it out?

Recall that $\rho = \int L(\lambda) R(\lambda) d\lambda$

has the discrete version

$$\rho = \mathbf{L} \bullet \mathbf{R}$$

(previously we accounted for multiple channels with the superscript (k), but here we just consider each channel separately)

Linear Least Squares Problem One

(example two---naïve spectral camera calibration)

Strategy: measure some spectra entering the camera, $\mathbf{L}_i$, and note the response, $\square_i$.

So we have, for a bunch of measurements, i:

$$\square_i = \mathbf{L}_i \bullet R$$

If we don’t have enough measurements, then the problem is under constrained. To account for noise, we want to use multiple measurements.

Linear Least Squares Problem One

(example two---naïve spectral camera calibration)

From:

$$\square_i = \mathbf{L}_i \bullet \mathbf{R}$$

The path is clear. Just form a matrix L with rows $\mathbf{L}_i$, a vector **P** with elements $\square_i$, and solve the least squares equation

$$U\mathbf{R} = \mathbf{P}$$

Linear Least Squares problem two (still §3.1.1)

Recall the second problem inspired by our camera calibration problem:

$$\text{Solve } \; U\mathbf{x} = \mathbf{0} \; \text{ subject to } \; |\mathbf{x}| = 1$$

We will sketch the solution even **more** briefly

Linear Least Squares problem two (still §3.1.1)

Because we solve $U\mathbf{x} = \mathbf{0}$ as best we can, the error vector is $U\mathbf{x}$

The squared error is then

$$(U\mathbf{x})^{\mathbf{T}}(U\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\left(U^{\mathbf{T}}U\right)\mathbf{x}$$

Since $U^{\mathbf{T}}U$ is symmetric it has an eigenvalue decomposition (diagonalization) with real eigen - values

Recal that a matrix A has an eigen - vector, $\mathbf{e}$, with eigen - value λ if
$\mathrm{A}\mathbf{e} = \lambda\mathbf{e}$

IE : $U^{\mathbf{T}}U = V\Lambda V^{T}$ where V is an orthonormal basis made of the eigenvectors, $\mathbf{e}_{\mathrm{i}}$, of $U^{\mathbf{T}}U$, and Λ is a diagonal matrix of the eigenvalues

Linear Least Squares problem two (still §3.1.1)

Critically, since $U^T U$ it is positive semidefinite, the eigenvalues are **positive**

Recall that a matrix A is positve semidefinite if $\mathbf{x}^T A\mathbf{x}$ is never negative.
(Clearly $U^T U$ it is positive semidefinite because $\mathbf{x}U^T U\mathbf{x}$ is $|U\mathbf{x})|$

Note : The book uses $\square_i^2$ in the equation at the top of page 41 which is confusing.
The $\square_i$ are in fact equal to the square of the "singular values of U", and so we will write them as $\square_i = \square_i^2$. This is explicitly reminds us that they are positive.

Linear Least Squares problem two (still §3.1.1)

We can write $\mathbf{x}$ in terms of the eigenvector basis :

$\mathbf{x} = \sum u_\mathrm{i} \mathbf{e}_\mathrm{i}$ where $\sum u_i^2 = 1$ (why?)

(Because $\mathbf{x}^\mathrm{T}\mathbf{x} = \sum u_\mathrm{j} \mathbf{e}_j^\mathrm{T} \sum u_\mathrm{i} \mathbf{e}_i = \sum \sum u_\mathrm{i} u_\mathrm{j} \mathbf{e}_j^\mathrm{T} \mathbf{e}_i = \sum u_i^2$)

The error is $\mathbf{x}^\mathbf{T}\left(V\Lambda V^T\right)\mathbf{x} = \left(\mathbf{x}^\mathbf{T} V\right)\Lambda\left(V^T\mathbf{x}\right) = \sum u_i^2 \lambda_i = \sum u_i^2 \lambda_i^2$

(Note that $\left(V^T\mathbf{x}\right)$ is a vector whose componants are u_i)

Linear Least Squares problem two (still §3.1.1)

From the previous slide the error to be minized is $\square\, u_i^2 \square_i^2$

We are stuck with the values $\square_i^2$ and $\square\, u_i^2 = 1$

So it should be clear that the best we can do is to set $u_i = 1$

for the minimum value of $\square_i = \square_i^2$ and zero for the others.

Thus the minimum is reached when x is set to the eigenvector

corresponding to the minimum eigenvalue of $U^{\mathrm{T}}U$

Linear Least Squares problem two (still §3.1.1)

Example 3.1 in book (fitting a line to points, a slightly better way)

Back to cameras (§3.2.1)

Goal one: Find the matrix M linking world coordinates to image coordinates from image of calibration object.

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix} = MP$$

Recall, that since the above is in terms of homogeneous coordinates we have to work in terms of the observed image coordinates, u=U/W and v=V/W

Recall that we form column vectors from the rows of M and stack the columns on top of one another to get the vector of unknowns, **m**.

Recall that we derived the following equation for m, to be solved subject to |**m**|=1 in the least squares sense.

$$\begin{bmatrix} \mathbf{P}_1^T & & -u_1\mathbf{P}_1^T \\ & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \cdots & \cdots & \cdots \\ \mathbf{P}_i^T & & -u_i\mathbf{P}_i^T \\ & \mathbf{P}_i^T & -v_i\mathbf{P}_i^T \\ \cdots & \cdots & \cdots \\ \mathbf{P}_n^T & & -u_n\mathbf{P}_n^T \\ & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{m}_1^\mathbf{T} \\ \mathbf{m}_2^\mathbf{T} \\ \mathbf{m}_3^\mathbf{T} \end{bmatrix} = \mathbf{0}$$

So, now we can simply apply the eigenvalue method in the previous slides to solve for **m**.

Intrinsic/extrinsic parameters

Recall goal two: Given M, recover the intrinsic parameters.

See §3.2.2 for the development of some formulas. You will use a simplified version of them in assignment two (relatively straight forward)