BRDF (Bidirectional reflectance distribution function)

- The irradiance at a point due to a particular angle is
  \[ L_i(x, \theta_i, \phi_i) \cos \phi_i d\phi \]

- The energy leaving (reflected) in a particular outgoing direction is given by:
  \[ L_o(x, \theta_o, \phi_o) \]

- The BRDF is simply the ratio of the output to input.
  \[ \Omega_{bd}(x, \theta_o, \phi_o, \theta_i, \phi_i) = \frac{L_o(x, \theta_o, \phi_o)}{L_i(x, \theta_i, \phi_i) \cos \phi_i d\phi} \]
• Important constraint: The BRDF is symmetric in incoming and outgoing directions (Helmholtz reciprocity)
  – Addendum to what was said in class: A number of books suggest that there is a simple argument for this based on thermodynamics but I cannot find one that is both believable and simple. Furthermore, recent papers suggest that the argument is flawed, and develop not so simple arguments based on electromagnetics. No violations of reciprocity are known in the domain defined by our assumptions.
• Additional constraints (see page 61-62)--basically the function can be large for some directions, but not many because energy coming out must always be less than or equal to that going in.

\[ \mathbf{b}^{bd}(x, \alpha_{o}, \alpha_{o}, \alpha_{i}, \alpha_{i},) = \frac{L_{o}(x, \alpha_{o}, \alpha_{o})}{L_{i}(x, \alpha_{i}, \alpha_{i}) \cos \alpha_{i} d\alpha} \]

Units are inverse steradians (sr\(^{-1}\))
BRDF

- The “distribution” part of the name is a hint that we need to integrate the function to get some light.
- To compute the brightness of a surface viewed from a given direction, we add up the contributions from all the input directions:

\[
\int \int \int_{\Omega} b_d(x, \Omega_o, \Omega_o, \Omega_i, \Omega_i) L_i(x, \Omega_i, \Omega_i) \cos \Omega_i d\Omega_i
\]

(by definition, this is the output in the direction (\(\Omega_o, \Omega_o\)))
Note that what we have developed so far is mostly notation, definitions, and descriptions.

Two approaches to obtaining BRDF’s--measure and model.

Measuring BRDF is painful (but there is some data available on-line).

Developing physics based approximations for the BRDF for simple classes of surfaces is complicated but possible--this is still an active research area.

Adding color to the BRDF is easy (one more variable). The full form has additional variables for fluorescence and polarization.
Isotropic surfaces

The BRDF for many surfaces can be well approximated as a function of 3 variables (angles), not 4. We replace the input and output non-azimuthal angles by their difference. In this case, turning the surface around the normal has no effect. The surface is said to be *isotropic*. 
Lambertian surfaces

- First special case: Lambertian surface (ideal diffuse or matte surface--e.g. cotton cloth, matte paper).

- Surface appearance is independent of viewing angle.

- Typically such a surface is the result of lots of scattering---the light “forgets” where it came from, and it could end up going in any random direction.

- Thus the BRDF is a constant ($\frac{d}{d}$, where $d$ is the albedo).
Lambertian surfaces

- Surface brightness is only a function of the foreshortening of the incident light (the more oblique it is, the less bright the surface).

- Question: Is the moon a Lambertian reflector?
Lambertian surfaces

\[ L_o(x, \mathbf{n}_o, \mathbf{n}_o) = \frac{\mathbf{bd}(x, \mathbf{n}_o, \mathbf{n}_o, \mathbf{n}_i, \mathbf{n}_i) L_i(x, \mathbf{n}_i, \mathbf{n}_i) \cos \mathbf{d}_i d}{\mathbf{d}} \]

\[ L_o(x) = \frac{\mathbf{d} L_i(x) \cos \mathbf{d}_i d}{\mathbf{d}} \]

Simple rule to shade an object--attenuate brightness by

\[ \mathbf{n} \cdot \mathbf{s} \]

Surface normal \quad Light source direction

Must know this
Ideal Mirrors

The opposite extreme case from Lambertian is a mirror

Recall that we integrate the BRDF over incident light from **incoming** directions to get reflected energy as a function of direction.

In the case of a mirror, this integral is positive only when the directions are correct for mirror reflection.
Reflection Direction

\[ \hat{s} + \hat{r} = k\hat{n} \]
\[ \hat{n} \cdot \hat{s} + \hat{n} \cdot \hat{r} = k \quad \text{where} \quad k = 2\hat{n} \cdot \hat{s} \]

So \[ \hat{r} = 2(\hat{n} \cdot \hat{s})\hat{n} \times \hat{s} \]

The three vectors are coplanar!
Should know this

Optional--Just in case you need it for something!
Ideal Mirrors

So the BRDF should be proportional to

\((J_e - J_i)(J_e - J_i - \theta)\)
Another important class of surfaces is specular (somewhat mirror-like).

- Specular surfaces reflect a significant amount of energy in the specular (mirror) direction.
- A significant amount may also be reflected in a direction roughly in the mirror direction (specular lobe).
- Typically there is a diffuse component as well.
- Writing a BRDF approximation is possible, but beyond the scope of this course, but it should be clear that it is valid break it into the 2 or 3 components mentioned above.
from
http://www.geocities.com/SiliconValley/Horizon/6933/shading.html
Radiosity

• In many situations, we do not need angle coordinates at all
  – e.g. cotton cloth, where the reflected light is not dependent on angle
• Radiometric unit is radiosity
  – total power leaving a point on the surface, per unit area on the surface (Wm\(^{-2}\))

• Radiosity from radiance?
  – sum radiance leaving surface over all exit directions

\[ B(x) = \int L_o(x, \theta, \phi) \cos \phi \, d\phi \]

Optional
Sources and Exitance

• Exitance of a source is
  – the internally generated power radiated per unit area on the radiating surface

• A source will have both
  – radiosity, because it reflects
  – exitance, because it emits

Radiosity leaving = Exitance + Radiosity due to incoming light