Basic Vector Operations

Let $\mathbf{X} = (x_1, x_2, x_3)$ *and* $\mathbf{Y} = (y_1, y_2, y_3)$

Sum $\mathbf{X} + \mathbf{Y} = (x_1 + y_1,\ x_2 + y_2,\ x_3 + y_3)$

Difference $\mathbf{X} - \mathbf{Y} = (x_1 - y_1,\ x_2 - y_2,\ x_3 - y_3)$

Scale $a\mathbf{X} = (x_1, x_2, x_3) = (ax_1, ax_2, ax_3)$

Magnitude $|\mathbf{X}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

Representations for lines and segments

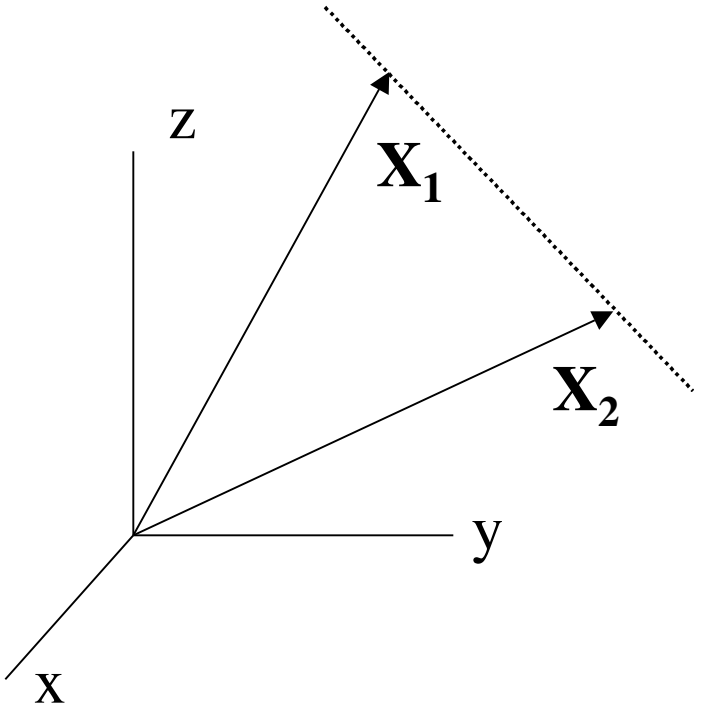
Cartesian

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{y - y_o}{x - x_o} \quad \Rightarrow \quad y = mx + b$$

Specification with this approach for 3D lines is more complex

Representations for lines and segments

Vector representation



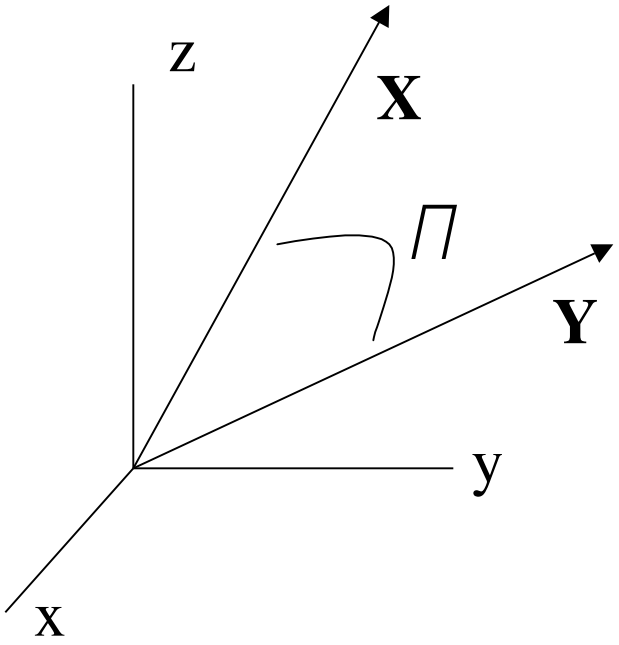
$$t\mathbf{X}_1 + (1 - t)\mathbf{X}_2$$

Works in any dimension

Simplifies representing *segments*

More Vector Operations

Dot Product (any number of dimensions)

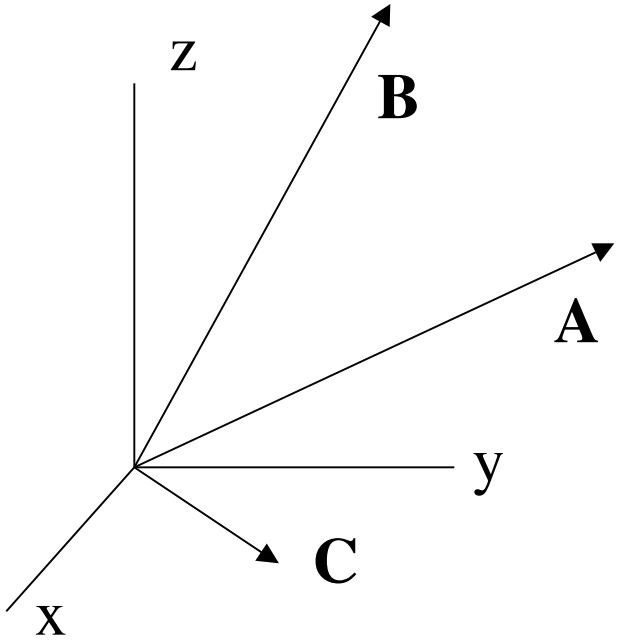


$$\mathbf{X} \bullet \mathbf{Y} = (x_1 y_1 + x_2 y_2 + x_3 y_3)$$
$$= |\mathbf{X}||\mathbf{Y}| \cos \theta$$

Orthogonal $\perp$ $\mathbf{X} \bullet \mathbf{Y} = 0$

More Vector Operations

Vector (cross) product (3D)



$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

$\mathbf{C} \perp \mathbf{A}$ *and* $\mathbf{C} \perp \mathbf{B}$

Use Right Hand Rule

$$|\mathbf{C}| = |\mathbf{A}||\mathbf{B}|\sin\theta$$

$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{bmatrix}$$

Representations for planes (1)

A plane passes through a point and has a given “direction”

Direction of plane is given by its normal

$$(\mathbf{X} - \mathbf{X_0}) \bullet \hat{\mathbf{n}} = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{ax + by + cz = k}$$

A half space is defined by $(\mathbf{X} - \mathbf{X_0}) \bullet \hat{\mathbf{n}} \geq 0$

Representations for planes (2)

Three points determine a plane

(Can make it the same as previous approach---how?)

Direct vector representation

$$v(uA + (1 - u))B + (1 - v)C$$

$$t = uv \quad and \quad s = v$$

$$C + t(A - B) + s(B - C)$$

(linear combination of two vectors, offset by another)