Important

Non-homogeneous linear least squares (example two---naïve line fitting)

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So form
- a matrix U with rows (x_i 1)
- a vector **y** with elements y_i
- a vector of unknowns **x**=(a,b)

and use the formula to solve U**x**=**y**

Camera parameters (§2.2)

- Camera might not be at the origin, looking down the z-axis
  - extrinsic parameters (position and orientation of the camera)
- Units in camera coordinates are not the same as units in world coordinates
  - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Notes:
We set f=1, and push the actual value of f into the intrinsic parameter matrix (see §2.2.1)

Actual pixel coords are (u,v) = (U/W, V/W)

Camera parameters (§2.2)

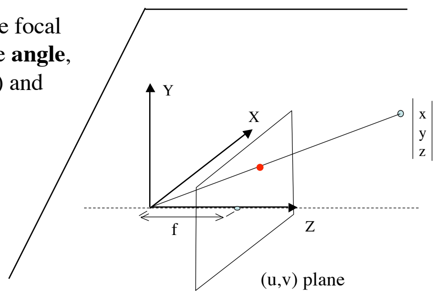
- Extrinsic parameters
  - position of the camera (3)
  - and orientation of the camera (3)
- Intrinsic parameters
  - focal length (1)
  - aspect ratio (ratio of pixel horizontal size to vertical size) (1)
  - principal point (intersection of viewing direction with camera plane) (2)
  - angle between axes of image plane---usually very close to 90 degrees (1)

Intrinsic parameters (focal length)

Recall that $u = f * (x/z)$ and $v = f * (y/z)$

The natural, easy to measure, units for (u,v) are pixels.

This means that the focal length transfers the **angle**, as encoded in (x/z) and (y/z) into **pixels**.



Intrinsic parameters (focal length)

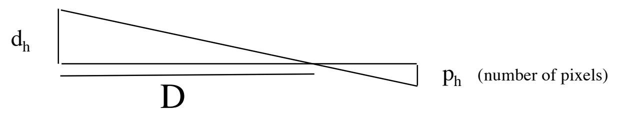
To transform a focal length in meters to pixels you would need to know the size of a pixel in meters. But you can easily measure the focal length in pixels (which is usually what you want)*.

However pixels are not always square. The ratio of width to height is called the aspect ratio.

Hence it is common to instead use two other parameters giving the horizontal and vertical focal lengths, □ and □, in pixel units

* If you have the focal length both in pixels and in meters, then you can compute the size of a pixel (if you wanted it for some reason)

Measuring focal lengths



$$□ = D \frac{p_h}{d_h}$$

Intrinsic parameter matrix

Conversion of projected coords into pixel units is achieved by simple **scalings** based on □ and □.

Translate coords to so that line through pinhole (or center of lens) perpendicular to projection plane is at origin.

Compensate for non-perpendicular pixel axis (if needed, usually this is OK) using a **shear** transformation.

Since these operations are all achievable with matrices, we see that a camera can be modeled with M composed of the three parts (intrinsic, projection, extrinsic).

Camera calibration (§3)

- Want to find out:
  - what is the camera matrix? (intrinsic+extrinsic)
  - what are intrinsic parameters of the camera?
- General strategy:
  - view calibration object
  - identify image points
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix
- Error minimization:
  - Linear least squares
    - easy problem numerically
    - solution can be rather bad
  - More robust methods exist, including ones that don't require a calibration object

Typical setup for calibration

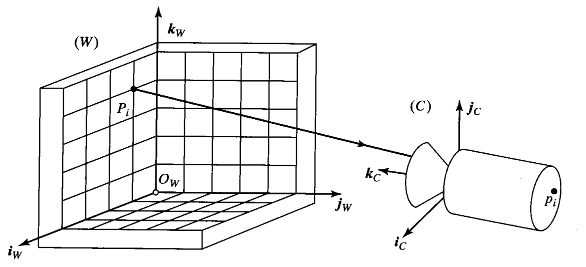


Figure 3.1 Camera calibration setup: In this example, the calibration rig is formed by three grids drawn in orthogonal planes. Other patterns could be used as well, and they may involve lines or other geometric figures.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \underbrace{\begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix}}_{\text{Camera matrix, } M} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Goal one: find M from image of calibration object

Goal two: given M, find the two matrices

Is goal one feasible?

More specifically, given (X,Y,Z) and corresponding u=(U/W) and v=(V/W), can we compute M?

First observation---if M is a solution then, because of homogeneity, k*M is an equivalent solution.

Thus, it only makes sense to recover M up to a scaling constant, and we can set the scale of M in advance for our convenience.

Is goal one feasible?

Since we are allowed to collect as much data as we need, goal one seems feasible.

(Details to follow soon).

Is goal two feasible?

Reason by counting parameters.

We have 11 numbers, as M is 3 by 4, and we can fix the scale.

The number of parameters (degrees of freedom) are the number of intrinsic parameters *plus* the number of extrinsic parameters.

Extrinsic parameters: ?

Intrinsic parameters: ?

The number of parameters are the number of intrinsic parameters *plus* the number of extrinsic parameters.

Extrinsic parameters:		
location	(3)	
orientation	(3)	
Intrinsic parameters:		
focal length	(1)	Or □ and □.
pixel aspect ratio	(1)	
principal point	(2)	
skew	(1)	
	11	

$\boxed{\text{Often assume skew is zero}}$

Is goal two feasible?

Yes (provided the points are not “degenerate”).

The 11 numbers (knowns) from M match the unknowns (camera parameters).

If some camera parameters are known, then a more robust computation is possible.

Finding M (goal one) (§3)

Find M from an image of calibration object. The equation relating world coordinates to image coordinates is:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = MP$$

If we identify enough non-degenerate points whose *world coordinates are known* then we can estimate M from their *location in the image*.

Specifically we have points in space, P, and corresponding observed image coordinates, u=U/W and v=V/W