Homogenous linear least squares

Example 3.1 in book

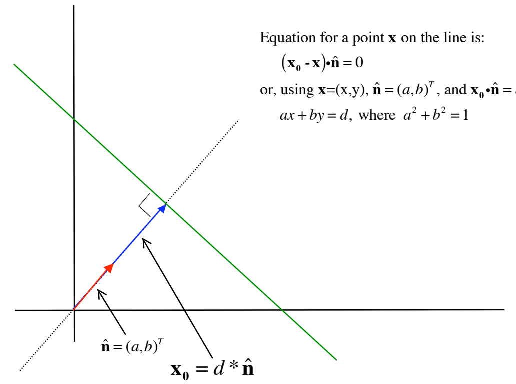
(fitting a line to points, a better way for many applications)

Key initial point: The perpendicular distance from a point x_i, to a line ax+by=d, where a^2+b^2=1 is given by:

$d_i = | ax_i + by_i - d |$

(See next two slides for geometry)

Line Fitting



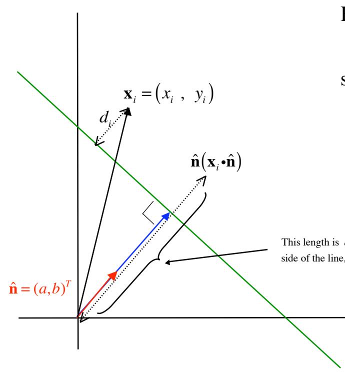
Equation for a point **x** on the line is:

$$(\mathbf{x_0} - \mathbf{x}) \bullet \hat{\mathbf{n}} = 0$$

or, using **x**=(x,y), $\hat{\mathbf{n}} = (a,b)^T$, and $\mathbf{x_0} \bullet \hat{\mathbf{n}} = d$,

$$ax + by = d, \text{ where } a^2 + b^2 = 1$$

Line Fitting



This length is $d_i + d$. If the point was on the other side of the line, we would have $-d_i + d$.

From the picture:

$$\pm d_i + d = \mathbf{x}_i \bullet \hat{\mathbf{n}}$$

so,

$$d_i = |ax_i + by_i - d|$$

Line Fitting (continued)

$$E = \sum d_i^2 = \sum (d - ax_i - by_i)^2$$

$$\frac{\partial E}{\partial d} = 2 \sum (d - ax_i - by_i) = 0$$

So, $d = a\bar{x} + b\bar{y}$

Line Fitting (continued)

$$d = a\bar{x} + b\bar{y} \qquad \text{(from previous slide)}$$

$$E = \sum\left(a\bar{x} - ax_i + b\bar{y} - by_i\right)^2$$

$$= \sum\left(\left(\bar{x} - x_i, \bar{y} - y_i\right)\bullet(a,b)\right)^2$$

$$= |U\mathbf{n}|^2, \text{ where U}=\begin{pmatrix} \bar{x} - x_1 & \bar{y} - y_1 \\ \dots & \dots \\ \bar{x} - x_n & \bar{y} - y_n \end{pmatrix}$$

So, we solve $U\mathbf{n}=0$ in the least squares sense, with $\mathrm{a}^2 + b^2 = 1$

Back to cameras (§3.2.1)

Goal one: Find the matrix M linking world coordinates to image coordinates from image of calibration object.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = MP$$

Recall, that since the above is in terms of homogeneous coordinates we have to work in terms of the observed image coordinates, u=U/W and v=V/W

Recall that we form column vectors from the rows of M and stack the columns on top of one another to get the vector of unknowns, **m**.

Recall that we derived the following equation for **m**, to be solved subject to |**m**|=1 in the least squares sense.

$$\begin{pmatrix} \mathbf{P}_1^T & & -u_1\mathbf{P}_1^T \\ & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \dots & \dots & \dots \\ \mathbf{P}_i^T & & -u_i\mathbf{P}_i^T \\ & \mathbf{P}_i^T & -v_i\mathbf{P}_i^T \\ \dots & \dots & \dots \\ \mathbf{P}_n^T & & -u_n\mathbf{P}_n^T \\ & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix} \begin{pmatrix} \mathbf{m_1^T} \\ \mathbf{m_2^T} \\ \mathbf{m_3^T} \end{pmatrix} = \mathbf{0}$$

So, now we can simply apply the eigenvalue method in the previous slides to solve for **m**.

Intrinsic/extrinsic parameters

Recall goal two: Given M, recover the intrinsic parameters.

See §3.2.2 for the development of some formulas. Grad students will use a simplified version of them in assignment three (relatively straight forward, but a bit complex)

Lambertian surfaces

Simple rule for shading--attenuate brightness by

n • s

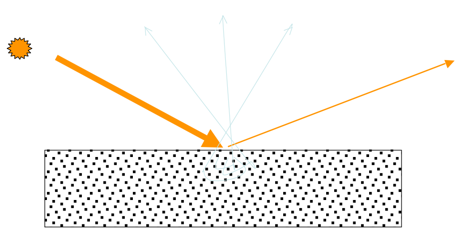
Surface normal

Light source direction

Must know this

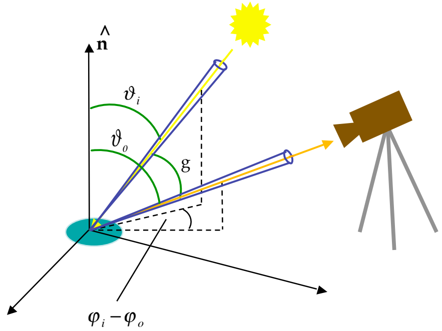
Light interacting with the world

- The light captured by camera caries information about what is in the world **because** what is in the world interacts with it differently depending on what it is.
- Many effects when light strikes a surface. It could be:
  - absorbed
  - transmitted
  - reflected
  - scattered (in a variety of directions!)



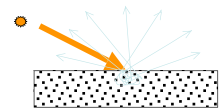
Bidirectional Reflectance Distribution Function (BRDF)

- The BRDF is a technical way of specifying how light from sources interacts with the matter in the world
- Understanding images requires understanding that this varies as a function of materials. The following “look” different
  - mirrors
  - white styrofoam
  - colored construction paper
  - colored plastic
  - gold
- The BRDF is the **ratio** of what comes out to what came in
- What comes out <--> “radiance”
- What goes in <--> “irradiance”
- Details on the BRDF available as supplementary material



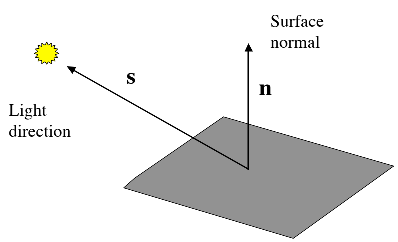
Lambertian surfaces

- Simple special case of reflectance: ideal diffuse or matte surface--e.g. cotton cloth, matte paper.
- Surface appearance is independent of viewing angle.
- Typically such a surface is the result of lots of scattering---the light "forgets" where it came from, and it could end up going in any random direction.



- What counts is how much light power reaches the surface

Lambertian Reflection

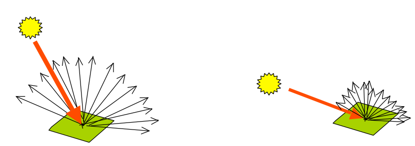


Brightness is proportional to **n•s**

Lambertian Reflection

Why is brightness proportional to **n•s** ?

Intuitive argument: The surface scatters light in all directions equally, but as the angle of the light becomes oblique, the amount of light per unit area received is reduced (foreshortened) by a factor of the cosine of the angle.



The same light is spread over a, giving intensity, i_a, as is spread over b, giving intensity, i_b. This means that:

$$a \bullet i_a = b \bullet i_b$$

or, because a is the length of the perpendicular,

$$i_b = i_a\left(\frac{a}{b}\right) = i_a \cos(\theta)$$