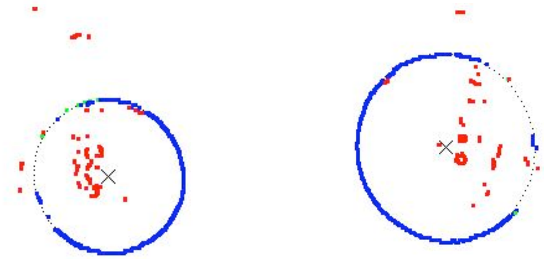
Lecture given by Prasad due to NSF panel travel.

The general thrust is relevant for the exam.

Computer vision based eye-gaze tracking

Prasad Gabbur

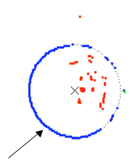
Ellipse fitting and tracking in the presence of outliers



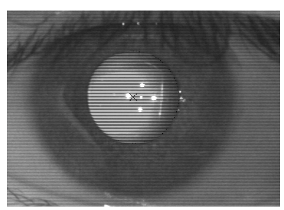
- Seems easy?
- Hard problem!
  - Presence of noisy data and outliers

Pupil (ellipse) tracking

- Pupil center is a feature for eye-gaze estimation
- Track pupil boundary ellipse

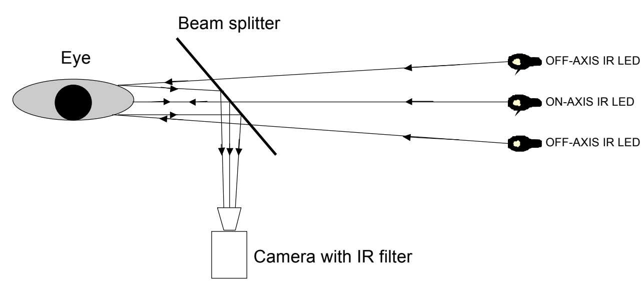


Pupil boundary edge points



Ellipse overlaid on the eye image

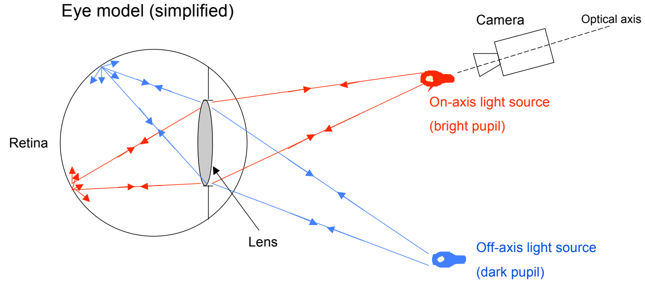
Setup (simplified)



- Even field => ON-AXIS LED is ON
- Odd field => ON-AXIS LED is OFF

Bright and dark pupil

Eye model (simplified)



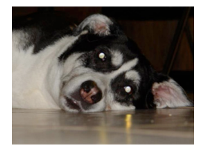
Bright pupil formation is similar to red-eye effect in photography.

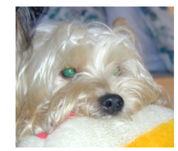
Red-eye effect

Red-eye is due to the image of the retinal surface

Red in humans due to retinal blood vessels

Other animals show other colors due to a reflective layer—tapetum*—helps them in night vision

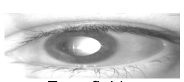


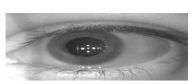


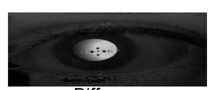
*http://www.colorpilot.com/redeye_effect.html

Even and odd field images

- Even field – bright pupil formation (similar to red-eye effect).
- Odd field – dark pupil.
- Difference image – pupil region enhanced.

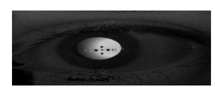

Even field


Odd field

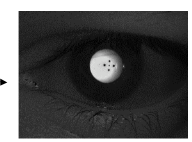

Difference

Pupil localization

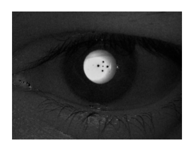
1. Correct aspect ratio

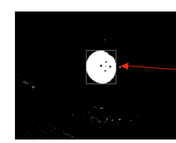

Difference field

Interpolate


Difference frame

2. Threshold and localize pupil region


Difference frame

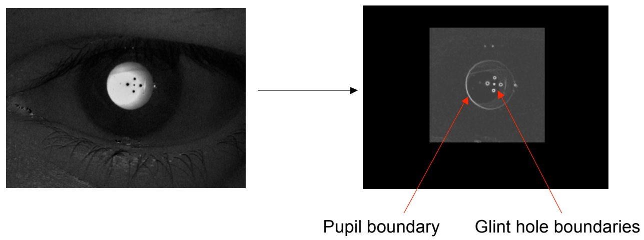


Largest connected component

Localized pupil in the thresholded frame

Pupil edge detection

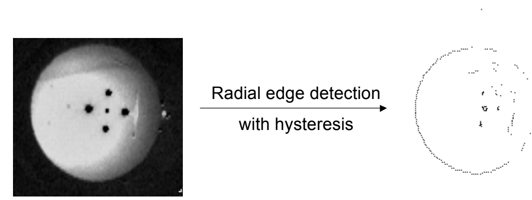
- Gradient magnitude within the processing window



- Large gradient magnitude => potential edge

Pupil edge detection

- Radial edge detection with hysteresis
  - 180-200 edge points
  - spurious edge pixels (e.g., glint hole boundary)



Ellipse fitting

- Bunch of points
  - How to fit an ellipse?
  - Noise in localization
  - Outliers

- Hough transform
  - Time intensive!
- Least squares
  - Minimize sum squared of some distance measure (?)

Algebraic Distance (AD)

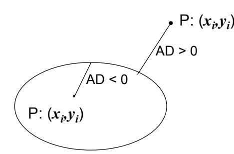
Equation of ellipse

$$F(x,y) = ax^2 + bxy + cy^2 + dx + ey + f = 0 \quad \text{(Conic)}$$

$$4ac - b^2 > 0 \quad \text{(Constraint for ellipse)}$$

Algebraic distance

$$AD(x_i, y_i) = F(x_i, y_i) = ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f$$



To avoid sign of AD, use squared AD: $AD^2(x_i, y_i) = F^2(x_i, y_i)$

Least squares ellipse fitting

- Edge points: $(x_i, y_i), i = 1 \dots N$
- Minimize sum of square of ADs:

$$E = \sum_{i=1}^{N} F^2(x_i, y_i)$$

- Possible to convert to linear least squares (?)
  - Non-homogeneous
  - Homogeneous

Least squares ellipse fitting

Algebraic distance

$$AD(x_i, y_i) = F(x_i, y_i) = ax_i^2 + bx_i y_i + cy_i^2 + dx_i + ey_i + f$$

$$AD(x_i, y_i) = \begin{bmatrix} x_i^2 & x_i y_i & x_i^2 & x_i & y_i & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = u_i^T p$$

Possible to convert to linear least squares (Homogeneous)

Least squares ellipse fitting

- Let $U = \begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_N^T \end{bmatrix}$ $\quad p = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$
- Minimize $E = \sum_{i=1}^{N} F^2(x_i, y_i) = (Up)^T Up = p^T U^T Up$
- Inequality constraint $4ac - b^2 > 0$
- Equality constraint $4ac - b^2 = 1$
  - Avoid trivial solution ***p* = 0**
  - Force a scale on ***p*** because if ***p*** is a solution, then any scalar multiple of ***p*** is also a solution

Least squares ellipse fitting

- Quadratic constraint $4ac - b^2 = 1$
- is equivalent to*

$$p^T \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} p = 1$$

$$p^T C p = 1$$

* Fitzgibbon et al, “Direct least squares fitting of ellipse”, PAMI 1999.

Least squares ellipse fitting

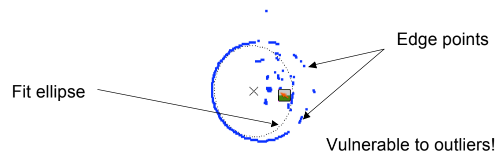
- Solution is the positive generalized eigenvector of

$$U^T U p = \lambda C p$$

- Least squares
  - Is it a perfect (flawless) solution?
  - Problems?

Least squares ellipse fitting

- Direct least squares ellipse fitting*



- Use RANSAC (robust to outliers)
  - Random (RAN) Sample (SA) Consensus (C)

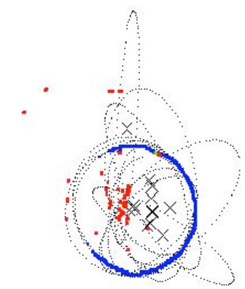
* Fitzgibbon et al, “Direct least squares fitting of ellipse”, PAMI 1999.

RANSAC

- Robust statistical model selection technique in the presence of outliers
- Idea
  - Fit model using minimum # training points chosen at random
  - Evaluate the model against the remaining points according to an error threshold
  - Points agree (consensus) or disagree (outlier)
  - Compute % of consensus points
  - Repeat the above for certain #iterations
  - Keep the model with the highest % consensus
  - Refit using consensus + training

RANSAC for ellipse fitting

- Min # of training points: 6
- Randomly sample
- Fit using least squares
- Evaluate
- Choose the best



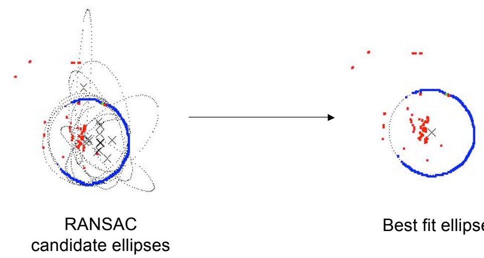
RANSAC

- Number of iterations

$$n = \frac{\log(1-\varepsilon)}{\log(1-\alpha^k)}$$

- $\boldsymbol{n}$ – Number of iterations
- ε - Probability of finding at least one outlier free sample
- α - Probability of inliers in your data
- $\boldsymbol{k}$ – Number of training points sampled every iteration

Sample result

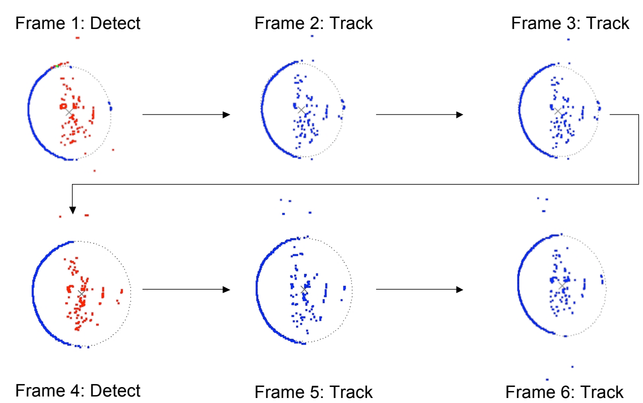


Tracking

- Brute force: Detect ellipse every video frame
  - RANSAC: Computationally intensive
- Better: Detect + Track
  - Ellipse usually does not change too much between adjacent frames

- Principle
  - Detect ellipse in a frame
  - Predict ellipse in next frame
  - Refine prediction using data available from next frame
  - If track lost, re-detect and continue

Detect+Track



Tracking example