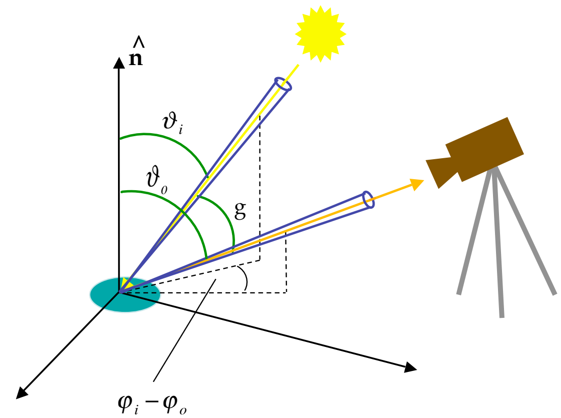
Bidirectional Reflectance Distribution Function (BRDF)

- The BRDF is a technical way of specifying how light from sources interacts with the matter in the world
- Understanding images requires understanding that this varies as a function of materials. The following “look” different
  - mirrors
  - white styrofoam
  - colored construction paper
  - colored plastic
  - gold
- The BRDF is the **ratio** of what comes out to what came in
- What comes out <--> “radiance”
- What goes in <--> “irradiance”



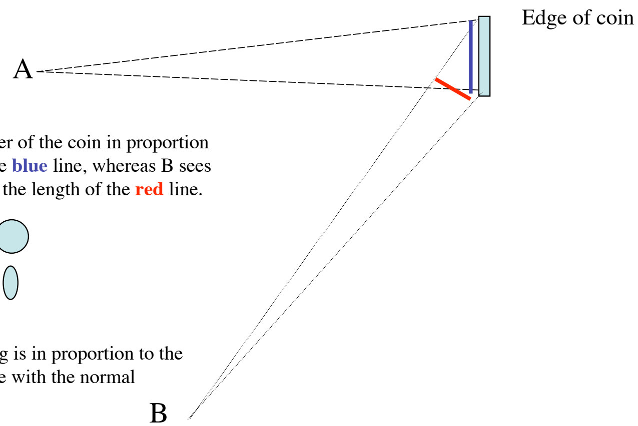
Bidirectional Reflectance Distribution Function (BRDF)

- BRDF parameters
  - direction of source
  - direction of viewer
  - wavelength of source
  - wavelength reflected (in the case of fluorescence--we will ignore this)
  - polarization (we will also ignore this)
- We will assume that the relation of light coming in and light going out is local---it can be reflected in an arbitrary direction, but it exits from (roughly) the same point it entered.
- We will assume that surfaces do not emit light---sources will be handled separately

Specifying light in a direction

- Use “steradians”
- First we need to understand “foreshortening”
  - Consider imaging a circular disk. When it is perpendicular to the camera, it is circular. When it is tilted, it gets smaller in the tilt direction and becomes an ellipse.

Foreshortening illustrated



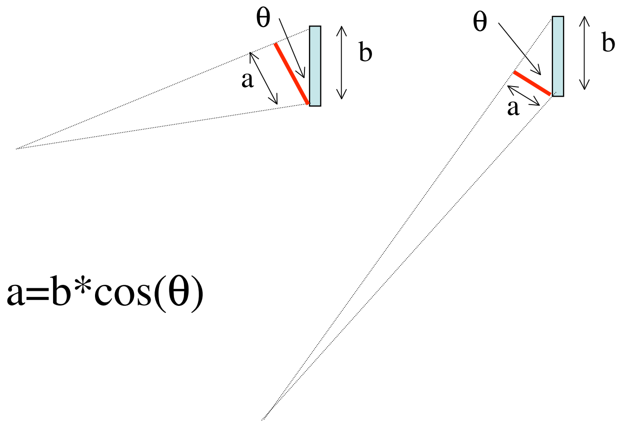
A sees the diameter of the coin in proportion to the length of the **blue** line, whereas B sees it in proportion to the length of the **red** line.

Further, A sees

But B sees

The foreshortening is in proportion to the cosine of the angle with the normal (next slide)

Foreshortening illustrated



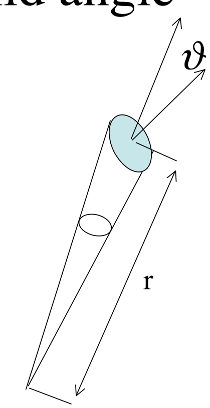
$a=b*\cos(\theta)$

Measuring light in a given direction using solid angle

- Analogous to measuring angles in radians
- The solid angle subtended by a patch area dA is given by

$$d\Omega = \frac{dA \cos\vartheta}{r^2}$$

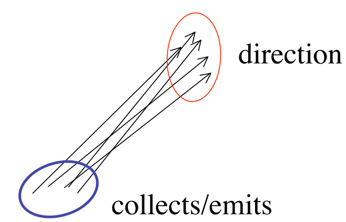
- Units are steradians (sr)



Radiance

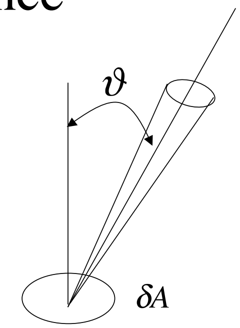
- Amount of light at a point in a particular direction
- Think of a small **area** either emitting or collecting the light
- Property is: *Radiant power per unit foreshortened area per unit solid angle*
  - This foreshortening and area are different than the one used to define the solid angle!
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

$$L(\underline{x},\vartheta,\varphi)$$

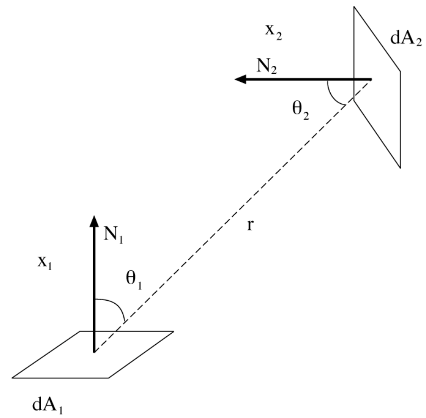


Radiance

$$L(\underline{x},\vartheta,\varphi)=\frac{\delta P(\underline{x})}{\delta\omega\ \delta A\cos\vartheta}$$



Radiance leaving a point, p, in the direction of a point, q, is the same as radiance arriving at q from p.



- Power 1->2, leaving 1:

  $$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

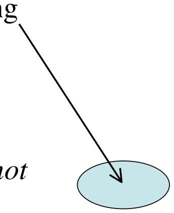
  $$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

- Since energy is conserved, these are equal, and so are the radiances.

Irradiance

- Irradiance is the amount of light (power) falling on a surface per unit area.
- Units are watts/m^2
- Generally a function of direction
- Irradiance is the incident power per unit area *not foreshortened*.

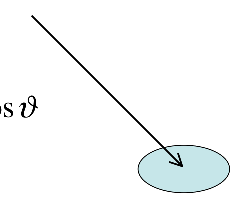
$$E(\underline{x},\vartheta,\varphi)=\frac{\delta P(\underline{x})}{\delta A}$$



Irradiance

Recalling that $L(\underline{x},\vartheta,\varphi)=\frac{\delta P(\underline{x})}{\delta\omega\ \delta A\cos\vartheta}$

$$E(\underline{x},\vartheta,\varphi)=\frac{\delta P(\underline{x})}{\delta A}=L(\underline{x},\vartheta,\varphi)\ \ \delta\omega\ \ \cos\vartheta$$



Thus, a surface experiencing radiance L(x ,θ,ϕ) coming from dω experiences irradiance $L(\underline{x},\vartheta,\varphi)\cos\vartheta\ \ d\omega$

Irradiance

- Total power arriving at the surface is given by adding irradiance over all incoming angles.
- Total power is:

$$\int_{\Omega} L(\underline{x}, \vartheta, \varphi) \cos\vartheta \;\; d\omega$$

- Or

$$\int_0^{\frac{\pi}{2}} \int_0^{2\pi} L(\underline{x}, \vartheta, \varphi) \cos\vartheta \;\; \sin\vartheta \;\; d\vartheta \;\; d\varphi$$

For integration in polar coords

BRDF (Bidirectional reflectance distribution function)

- The irradiance at a point due to a particular angle is

$$L_i\left(\underline{x}, \vartheta_i, \varphi_i\right) \cos\vartheta_i d\omega$$

- The energy leaving (reflected) in a particular outgoing direction is given by:

$$L_o\left(\underline{x}, \vartheta_o, \varphi_o\right)$$

- The BRDF is simply the ratio of the output to input.

$$\rho_{bd}\left(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,\right) = \frac{L_o\left(\underline{x}, \vartheta_o, \varphi_o\right)}{L_i\left(\underline{x}, \vartheta_i, \varphi_i\right) \cos\vartheta_i d\omega}$$

BRDF

$$\rho_{bd}\left(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,\right) = \frac{L_o\left(\underline{x}, \vartheta_o, \varphi_o\right)}{L_i\left(\underline{x}, \vartheta_i, \varphi_i\right) \cos\vartheta_i d\omega}$$

Units are inverse steradians (sr^{-1})

- Important constraint: The BRDF is symmetric in incoming and outgoing directions (Helmholtz reciprocity)
  - A number of books suggest that there is a simple argument for this based on thermodynamics but I cannot find one that is both believable and simple. Furthermore, recent papers suggest that the argument is flawed, and develop **not so simpl**e arguments based on electromagnetics. No violations of reciprocity are known in the domain defined by our assumptions.
- Additional constraints (see page 61-62)--basically the function can be large for some directions, but not many because energy coming out must **always** be less than or equal to that going in.

BRDF

- The "distribution" part of the name is a hint that we need to integrate the function to get some light.
- To compute the brightness of a surface viewed from a given direction, we add up the contributions from all the input directions:

$$\int_{\Omega} \underbrace{\rho_{bd}\left(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,\right) L_i\left(\underline{x}, \vartheta_i, \varphi_i\right) \cos\vartheta_i d\omega_i}$$

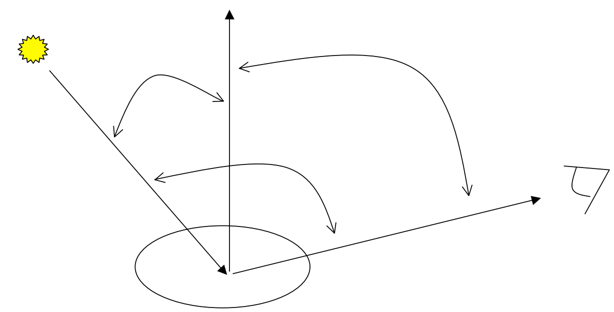
(by definition, this is the output in the direction (ϑ_o, φ_o))

BRDF

- Note that what we have developed so far is mostly notation, definitions, and descriptions.
- Two approaches to obtaining BRDF's--measure and model.
- Measuring BRDF is painful (but there is some data available on-line).
- Developing physics based approximations for the BRDF for simple classes of surfaces is complicated but possible--this is still an active research area.
- Adding color to the BRDF is easy (one more variable). The full form has additional variables for fluorescence and polarization.

Isotropic surfaces

The BRDF for many surfaces can be well approximated as a function of 3 variables (angles), not 4. We replace the input and output non-azimuthal angles by their difference. In this case, turning the surface around the normal has no effect. The surface is said to be *isotropic*.



Lambertian surfaces

- First special case: Lambertian surface (ideal diffuse or matte surface--e.g. cotton cloth, matte paper).
- Surface appearance is independent of viewing angle.
- Typically such a surface is the result of lots of scattering---the light "forgets" where it came from, and it could end up going in any random direction.
- Thus the BRDF is a constant (ρ_d/π. where ρ_d is the albedo).

Lambertian surfaces

$$L_o(\underline{x},\vartheta_o,\varphi_o)=\rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,)L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega$$

$$L_o(\underline{x})=\frac{\rho_d}{\pi}L_i(\underline{x})\cos\vartheta_i d\omega$$

Simple rule to shade an object--attenuate brightness by

$$\mathbf{n} \bullet \mathbf{s}$$

Surface normal

Light source direction

Must know this

Ideal Mirrors

So the BRDF should be proportional to

$$\delta(\vartheta_e - \vartheta_i)\delta(\varphi_e - \varphi_i - \pi)$$