From Lowe,
IJCV 2004

Can you find the locomotive? Can a computer program?

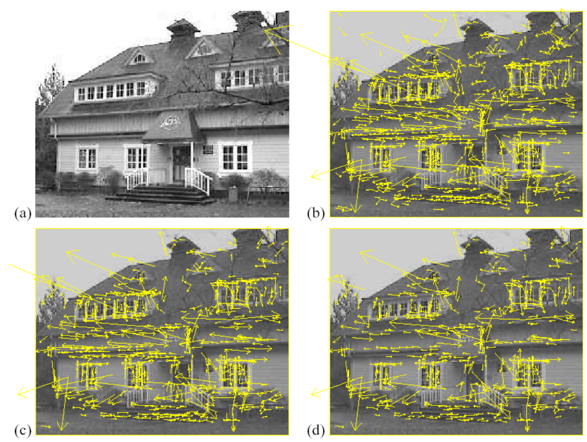
Invariant feature detection*

- Consider representing an image of an object with a collection of descriptive local features
- Most useful if these occur in "edgy" areas.
- Common modern strategy is do detect somewhat robust "interest points" and form a descriptor for the local area.
- Example descriptor is a histogram of edge orientations (local texture).

*Good reference is Lowe, IJCV, 2004

Distinctive Key-Points

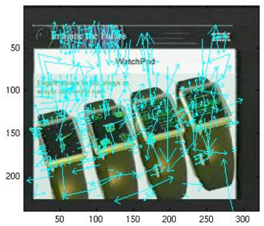
- Edges are interesting, but are they really distinctive?
  - Not for many applications because they do not have good localization
- More distinctive is a corner or a grid point
- Various strategies exist for finding "key-points" that are distinctive and localizable
- One idea is to look for edgy areas where one edge direction does NOT overly dominate the other
  - EG, a corner has both horizontal and vertical responses
- Consider at different scales

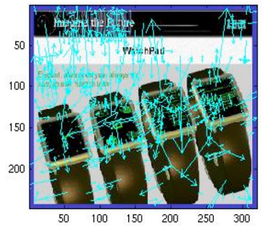


From Lowe, IJCV 2004

Invariant feature detection

- To “find” the object, match the local features

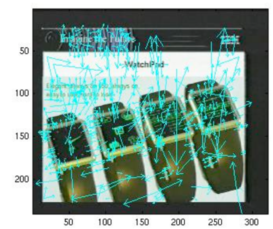


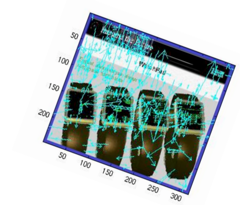


Invariant feature detection

Invariant feature detection

- Problems
  - Consistently determining which features goes with which
    - Covered later
  - Camera view changes
    - Approximately affine
    - Further approximated by scale and rotation

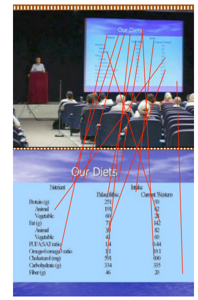




Invariant feature detection

- Dealing with to camera view changes
  - Scaling and rotation can approximate camera view changes for small patches (locally planar)
  - Consider detector scale and direction (gradient)
  - This sets up a 2D coordinate system that is invariant to scale and rotation
  - One strategy is to make edge histogram grid with scaled bins and aligned with direction
  - Now, local feature description is invariant to scale and rotation.

Initial matching



Before pruning outliers

After pruning outliers

(Covered later)

Syllabus Notes

- Next topics segmentation, grouping and fitting.
- We will do perhaps half each of §14, §15, and §16.

Segmentation, Grouping, and Fitting

- Collect together tokens that belong together
- Gives a compact representation from an image/motion sequence/set of tokens that can be significantly easier to deal with
- What is the “right” group is often dependent on the application
- Broad theory is not known at present (and may not exist)
- These are general concepts--apply to many things, not just breaking images into regions of the same color.

Segmentation, Grouping, and Fitting

- Terminology varies and the usage and the meaning of segmentation, grouping, and fitting overlap. However somewhat common usage:
  - Grouping (or clustering) is quite general sometimes suggest a relatively high level (group the black and white halves of a penguin together)
  - Segmentation is suggestive of the grouping is done at a low level and is quite spatially (or temporally coherent) given regions in time or space
  - Fitting when the focus is on a model associated with tokens. Issues:
    - which model?
    - which token goes to which element in the model (correspondence)?
    - how many elements in the model (how complex should it be)?

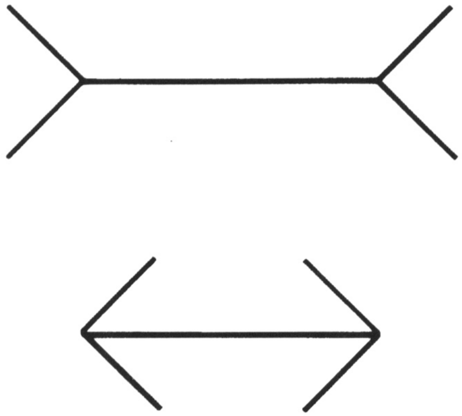
General ideas

- Tokens
  - whatever we need to group (e.g. pixels, points, surface elements)
- Top down segmentation
  - tokens belong together because they lie on the same object
- Bottom up segmentation
  - tokens belong together because they are locally coherent
- These two are not mutually exclusive

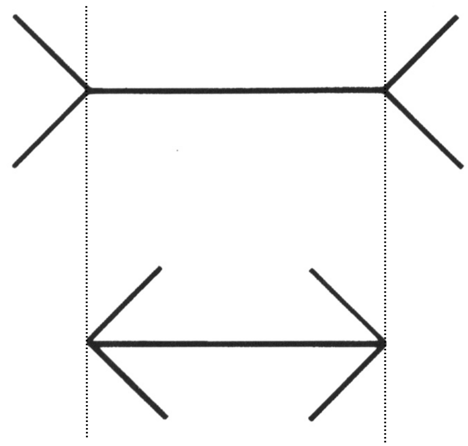
Why do these tokens belong together?

Basic ideas of grouping in humans

- Figure-ground discrimination
  - grouping can be seen in terms of allocating some elements to a figure, some to ground (impoverished theory)
- Gestalt properties
  - Elements in a collection of elements can have properties that result from relationships (e.g. Muller-Lyer effect)
  - A series of factors affect whether elements should be grouped together
    - Gestalt factors



The Muller-Lyer illusion; the horizontal bar has properties that come only from its membership in a group



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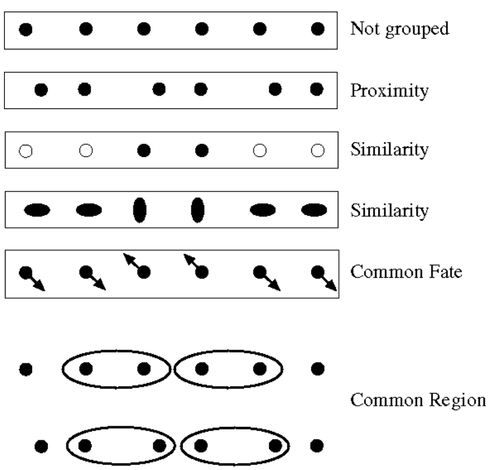
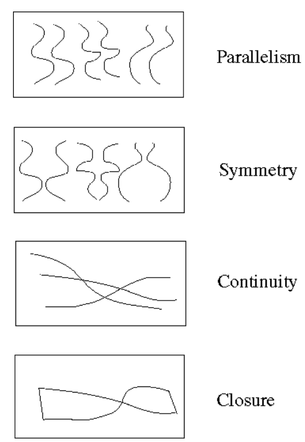
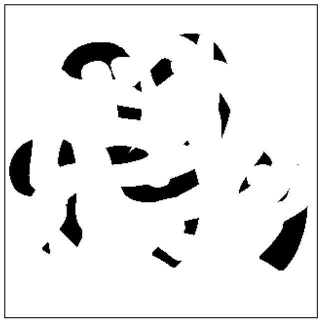


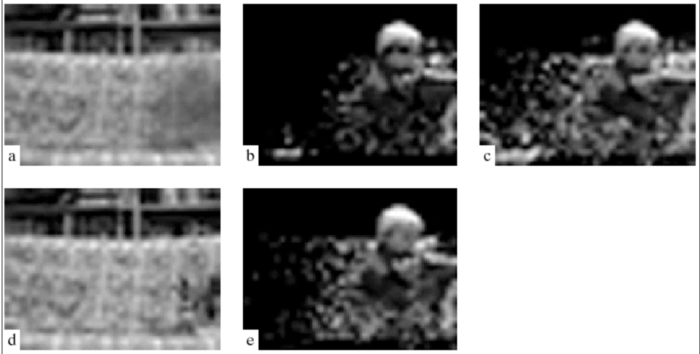




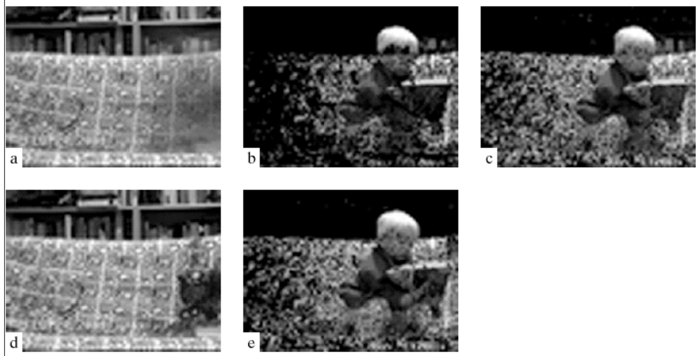
Background Subtraction

Background Subtraction

- If we know what the background looks like, it is easy to identify “interesting bits”
- Applications
  - Person in an office
  - Tracking cars on a road
  - Surveillance
- Approach:
  - Use a moving average to estimate background image
  - Subtract from current frame
  - Large absolute values are interesting pixels
    - trick: use morphological operations to clean up pixels (remove “holes”)



a) average of sequence b) difference from background > T1



Higher resolution version of previous

Segmentation as clustering

- Cluster together (pixels, tokens, etc.) that belong together
- We assume that we can compute how close tokens are, or how close a token is to cluster.

Why is clustering hard?

Main reason

- The number of possible clusterings is exponential in the number of data points

Other issues

- The number of clusters is usually **not** known
- A good distance function between points may not be known
- A good model explaining the existence of clusters is usually not available.
- High dimensionality

Data Representation

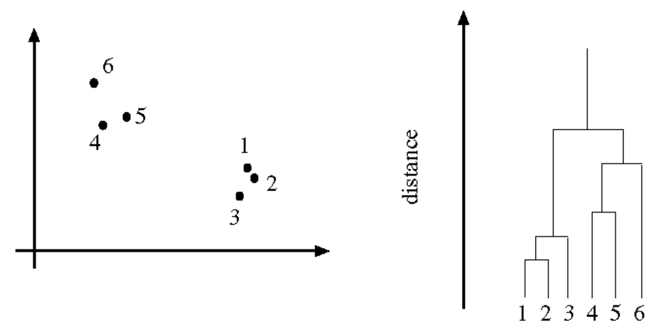
- Most common is an N dimensional "feature" vector.
- Most common distance is Euclidian distance.
- Be careful with scaling and units!
- Probabilistic modals finesse multiple modalities
- Problems with correlated variables can be mitigated using transformations and data reduction methods such as PCA, ICA.

Clustering approaches

- Agglomerative clustering
  - initialize: every item is a cluster
  - attach item that is "closest" to a cluster to that cluster
  - repeat
- Divisive clustering
  - split cluster along best boundary
  - repeat
- Probabilistic clustering
  - Define a probabilistic grouping model

Simple clustering approaches

- Point-Cluster or Cluster-Cluster distance
  - single-link clustering (minimum distance from point to points in clusters or among pairs of points, one from each cluster)
  - complete-link clustering (maximum)
  - group-average clustering (average)
  - (terms are not important, but concepts are worth thinking about)
- Dendrograms
  - classic picture of output as clustering process continues



Syllabus Notes

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K-Means

- Choose a fixed number of clusters ("K")
- Choose cluster centers (**means**) and point-cluster allocations (membership) to minimize the error

$$\sum_{i \in \text{clusters}} \left\{ \sum_{j \in \text{elements of i'th cluster}} \left\| x_j - \mu_i \right\|^2 \right\}$$

- **x**'s could be any set of features for which we can compute a distance (careful with scaling)

K-Means

- Want to minimize

$$\sum_{i \in \text{clusters}} \left\{ \sum_{j \in \text{elements of i'th cluster}} \left\| x_j - \mu_i \right\|^2 \right\}$$

- **Cannot** do this optimization by search, because there are too many possible allocations.
- Standard difficulty which we handle with an iterative process (chicken and egg)

K-Means algorithm (intuition)

- If we know the cluster centers, the best cluster for each point is easy to compute
  - Just compute the distance to each to find the closest
- If we know the best cluster for each point, the cluster centers are also easy to compute
  - Just average the points in each cluster
- Algorithm
  - 1) Guess one of the two.
  - 2) Alternatively re-compute the values for each

K-means flow chart

Choose K

Guess membership — OR — Guess the means

Assume membership is **fixed**. Take averages to get cluster centers (means)

Assume means are **fixed**. Find cluster with closest mean for each point

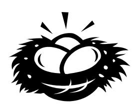


Image | Clusters on intensity | Clusters on color

K-means clustering using intensity alone and color alone
(Assuming 5 segments, i.e. k=5)

Image

Clusters on color

K-means using color alone, 11 segments

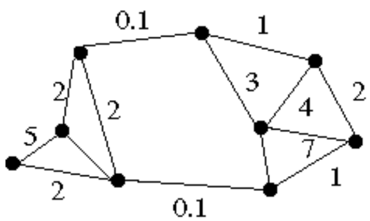
K-means using color alone, 11 segments.

Notes on K-Means

- K-means is “hard” clustering-each point is completely in exactly one cluster
- What you get is a function of starting “guess”
- The error goes down with every iteration
  - This means you get a local minimum
- Unfortunately, the dimension of the space is usually large, and high-dimensional space have lots of local maximum (standard problem!)
  - Dimensionality here is K*dim(**x**)
- Finding the global minimum for a real problem is very optimistic!

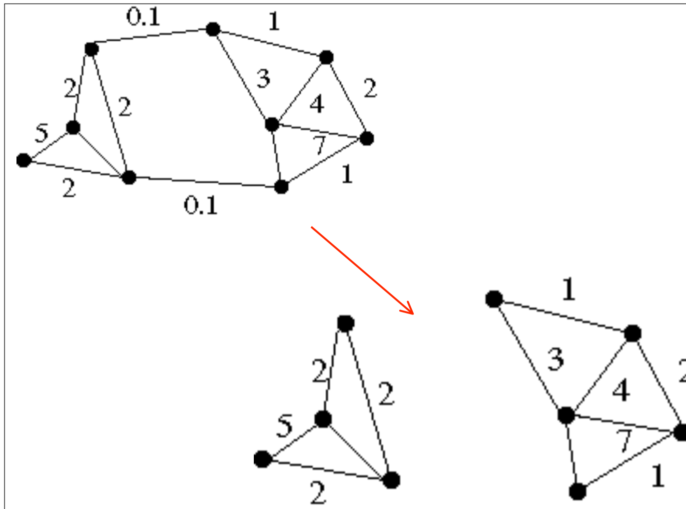
Graph theoretic clustering

- Represent distance between tokens using a weighted graph.
  - affinity matrix
- Cut up this graph to get subgraphs with strong interior links (and weak links between the subgraphs).



Graph for 9 tokens

Image representation of weight matrix



Measuring Affinity

Intensity

$$aff(x,y)=\exp\left\{-\left(\frac{1}{2\sigma_i^2}\right)\left(\|I(x)-I(y)\|^2\right)\right\}$$

Distance

$$aff(x,y)=\exp\left\{-\left(\frac{1}{2\sigma_d^2}\right)\left(\|x-y\|^2\right)\right\}$$

Texture

$$aff(x,y)=\exp\left\{-\left(\frac{1}{2\sigma_t^2}\right)\left(\|c(x)-c(y)\|^2\right)\right\}$$

~~exam~~

Eigenvectors and cuts

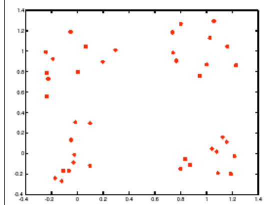
- For some cluster, i, consider a vector $\mathbf{a}_i$ giving the association between each element and that cluster
- We want elements within this cluster to, on the whole, have strong affinity with one another
- This suggests maximizing $a^T A a$
- But need the constraint $a^T a = 1$

Eigenvectors and cuts

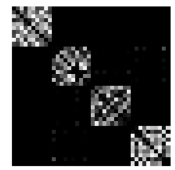
ex am

- We want to maximize $a^T A a$ subject to $a^T a = 1$
- This is an eigenvalue problem - choose the eigenvector of A with largest eigenvalue
- This gives the cluster with greatest internal affinity
  - Ideally, most elements of the eigenvalue are near zero, and the others tell us which tokens are in the cluster

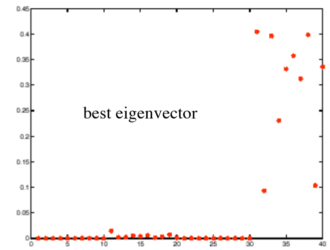
Example eigenvector



points



matrix



Normalized cuts

- Previous criterion evaluates **within** cluster similarity, but does not promote large differences **between** clusters across cluster difference
- N-cuts proposes maximizing the within cluster similarity **compared** to the across cluster difference
- Write graph as V, one cluster as A and the other as B. (V=AUB).
- Maximize

$$\left(\frac{assoc(A,A)}{assoc(A,V)}\right)+\left(\frac{assoc(B,B)}{assoc(B,V)}\right)$$

- (Solution follows to keep notes self-contained).

Optional

Normalized cuts

- Write a vector y whose elements are 1 if item is in A, -b if it's in B
- Write the matrix of the graph as W, and the matrix which has the row sums of W on its diagonal as D, 1 is the vector with all ones.
- With some algebra, the criterion becomes $\min_y \left(\frac{y^T(D-W)y}{y^T D y}\right)$
- And we have a constraint $y^T D 1 = 0$
- This is hard to do, because y's values are quantized

Optional

Normalized cuts

- Instead, solve the generalized eigenvalue problem

$$\max_y \left(y^T (D - W) y \right) \text{ subject to } \left(y^T D y = 1 \right)$$

- which gives

$$(D - W) y = \lambda D y$$

- Now look for a quantization threshold that maximizes the criterion --- i.e all components of y above that threshold go to one, all below go to -b

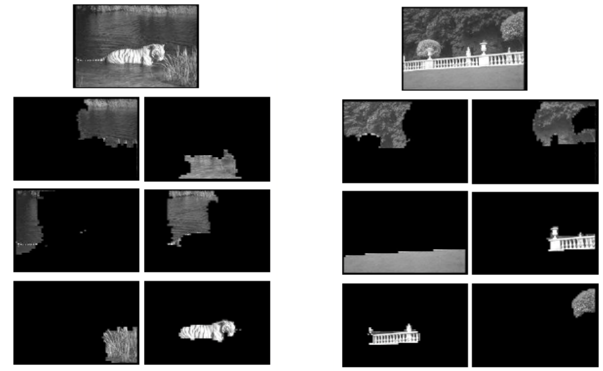


Figure from “Image and video segmentation: the normalised cut framework”, by Shi and Malik, copyright IEEE, 1998