Today

- Assignment 3 now up
- Vision project opportunities
- Review of eigenvectors and intuition about H.L.S.
- Fitting lines
- Back to cameras
- Generalizing what we have learned
- Feedback

Wisdom from tea dipper handle

Math aside, #4

Homogenous linear least squares

Thus the problem inspired by our camera calibration problem is

Solve $U\mathbf{x}=\mathbf{0}$ subject to $|\mathbf{x}|=1$

Again, **there is no exact solution**.

Least squares solution is the value of $\mathbf{x}$ so that the magnitude of $U\mathbf{x}$ is as close zero as possible.

(still §3.1.1)

Math aside, #4

Homogenous linear least squares

The least squares problem is thus

Minimize $\|U\mathbf{x}\|$ subject to $|\mathbf{x}|=1$

This is solved by **magic** (see supplementary slides)

Important

Specifically, the minimum is reached when $\mathbf{x}$ is set to the eigenvector corresponding to the minimum eigenvalue of U^TU.

Math aside, #4

Homogenous linear least squares

Pragmatic solution of

Solve $U\mathbf{x} = \mathbf{0}$ subject to $|\mathbf{x}| = 1$

In Matlab, form $Y = U^{T} U$

Then use eig() to get the eigenvalues and eigenvectors of Y.

These are provided in order.

Extract the eigenvector that you need.

("proof" --- try 1,000,000 random solutions to see if you can do better)

Homogenous linear least squares

Example 3.1 in book

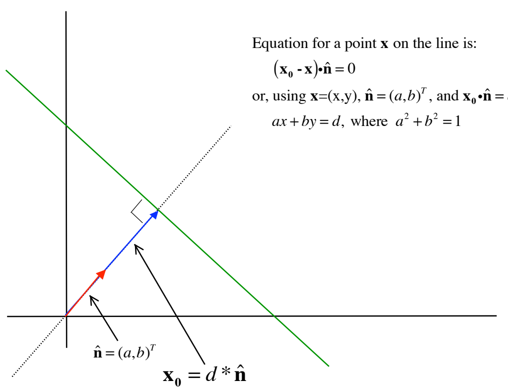
(fitting a line to points, a better way for many applications)

Key initial point: The perpendicular distance from a point x_i, to a line ax+by=d, where a^2+b^2=1 is given by:

$d_i = | ax_i + by_i - d |$

(See next two slides for geometry)

Line Fitting



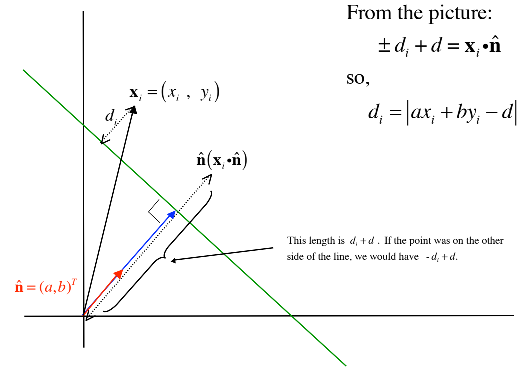
Equation for a point **x** on the line is:

$(\mathbf{x_0} - \mathbf{x}) \bullet \hat{\mathbf{n}} = 0$

or, using **x**=(x,y), $\hat{\mathbf{n}} = (a,b)^T$, and $\mathbf{x_0} \bullet \hat{\mathbf{n}} = d$,

$ax + by = d$, where $a^2 + b^2 = 1$

Line Fitting



From the picture:

$\pm d_i + d = \mathbf{x}_i \bullet \hat{\mathbf{n}}$

so,

$d_i = |ax_i + by_i - d|$

Line Fitting (continued)

$$E = \sum d_i^2 = \sum \left(d - a x_i - b y_i \right)^2$$

$$\frac{\partial E}{\partial d} = 2 \sum \left(d - a x_i - b y_i \right) = 0$$

So, $d = a\overline{x} + b\overline{y}$

Line Fitting (continued)

$E = \sum d_i^2 = \sum \left(d - a x_i - b y_i \right)^2$ and $d = a\overline{x} + b\overline{y}$ (previous slide)

$$E = \sum \left(a\overline{x} - a x_i + b\overline{y} - b y_i \right)^2$$

$$= \sum \left(\left(\overline{x} - x_i, \overline{y} - y_i \right) \bullet (a,b) \right)^2$$

$$= \left| U\mathbf{n} \right|^2, \text{ where } U = \begin{pmatrix} \overline{x} - x_1 & \overline{y} - y_1 \\ \dots & \dots \\ \overline{x} - x_n & \overline{y} - y_n \end{pmatrix}$$

So, we solve $U\mathbf{n}$=0 in the least squares sense, with $a^2 + b^2 = 1$

Back to cameras (§3.2.1)

Goal one: Find the matrix M linking world coordinates to image coordinates from image of calibration object.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = MP$$

Recall, that since the above is in terms of homogeneous coordinates we have to work in terms of the observed image coordinates, u=U/W and v=V/W

Recall that we form column vectors from the rows of M and stack the columns on top of one another to get the vector of unknowns, **m**.

Recall that we derived the following equation for **m**, to be solved subject to |**m**|=1 in the least squares sense.

$$\begin{pmatrix} \mathbf{P}_1^T & & -u_1\mathbf{P}_1^T \\ & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \dots & \dots & \dots \\ \mathbf{P}_i^T & & -u_i\mathbf{P}_i^T \\ & \mathbf{P}_i^T & -v_i\mathbf{P}_i^T \\ \dots & \dots & \dots \\ \mathbf{P}_n^T & & -u_n\mathbf{P}_n^T \\ & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix} = \mathbf{0}$$

So, now we can simply apply the eigenvalue method in the previous slides to solve for **m**.

Intrinsic/extrinsic parameters

Recall goal two: Given M, recover the intrinsic parameters.

See §3.2.2 for the development of some formulas. Grad students will use a simplified version of them in assignment three (relatively straight forward, but a bit complex)

Camera summary

- Model for the brightness due to light reaching a region (say, CCD element)

$$(R,G,B) = \int_{380}^{780}$$

$$d\lambda$$

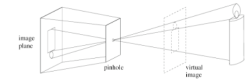
$$\upsilon^{(k)} = F^{(k)}(\rho^{(k)}) = F^{(k)}\left(\int L(\lambda)R^{(k)}(\lambda)d\lambda\right)$$

or

$$\rho^{(k)} = \mathbf{L} \bullet \mathbf{R}^{(\mathbf{k})}$$

- Spectral camera calibration task is to determine R (and F) from data.

- Model the image location corresponding to a point in the world by projection (developed using pinhole camera model).



- Represent using matrix multiplication using homogenous coordinates

- (u,v,w) = **M** * (X, Y, Z, 1)

- Geometric camera calibration task is to determine **M** from data.

Supplementary material

Real cameras

(Supplementary material on these topics posted on-line)

- Real cameras need lenses
  - Focus now depends on distance (unlike pinhole cameras)
  - Various aberrations and distortions
    - E.g. Chromatic aberration
- Brightness falls off towards edges
  - Fall off due to projection onto flat surface
  - Vignetting
- Scattering at optical surfaces (flare)
- Capture process has many sources of noise

Generalizing what we have learned