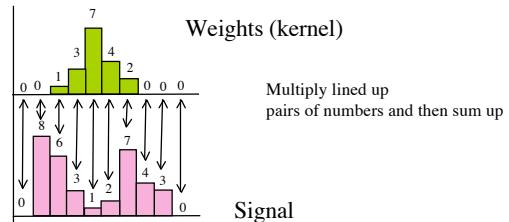


Part Three

- Single pixels do not carry that much information
- The spatial structure of the image carries most of the interpretation
- The next part of the course looks at local indicators of spatial structure
 - smoothing to deal with noise
 - understanding image scale
 - edges
 - texture
 - scale and rotation invariant informative keypoints

Linear Filters (§7)

- General process:
 - Form new image whose pixels are a **weighted sum** of original pixel values, using the same set of weights at each point.
- Much like a 2D version of the sensor response computation (sensitivities are like weights), but now compute a similar weighted sum for each point



Linear Filters (§7)

- Example: smoothing by averaging
 - form the average of pixels in a neighbourhood (weights are equal)
- Example: smoothing with a Gaussian
 - form a weighted average of pixels in a neighbourhood (weights follow a Gaussian function)
- Example: finding a derivative
 - negative weights on one side, positive ones on the other

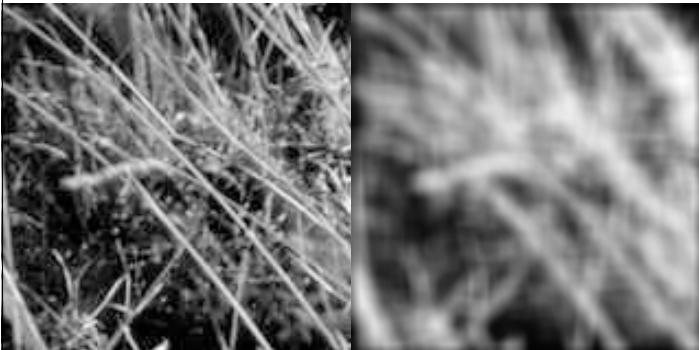
Linear Filter Example

- Compute a new image which is an average of 3 by 3 blocks

- H (the kernel)
$$H(i^*, j^*) = \begin{cases} \frac{1}{9} & i^*, j^* \in \{-1, 1\} \\ 0 & \text{otherwise} \end{cases}$$

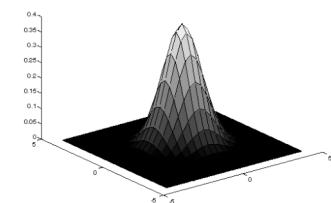
- Result is
$$R_{ij} = \sum_{i'=i-1}^{i+1} \sum_{j'=j-1}^{j+1} \frac{1}{9} F(i', j')$$

Example: Smoothing by Averaging



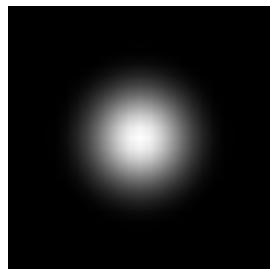
Smoothing with a Gaussian

- Smoothing with an average actually doesn't really make sense because points close to the center should count more.
- Also, it does not compare at all well with a defocused lens
 - Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob; but the averaging process would give a little square.



- A Gaussian gives a good model of a fuzzy blob

An Isotropic Gaussian



- The picture shows a smoothing kernel proportional to

$$\exp\left(-\left(\frac{x^2 + y^2}{2\sigma^2}\right)\right)$$

(a reasonable model of a circularly symmetric fuzzy blob)

Smoothing with a Gaussian



Block Averaging

Gaussian

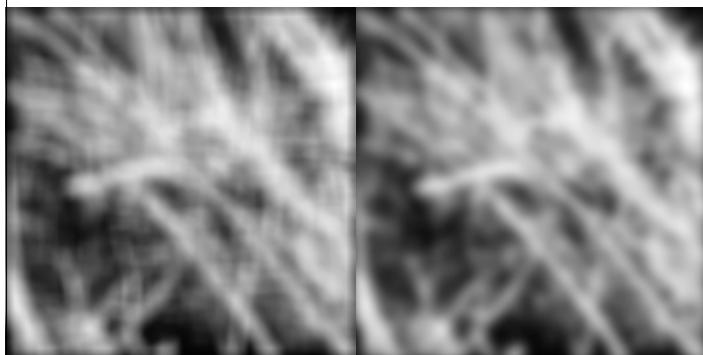
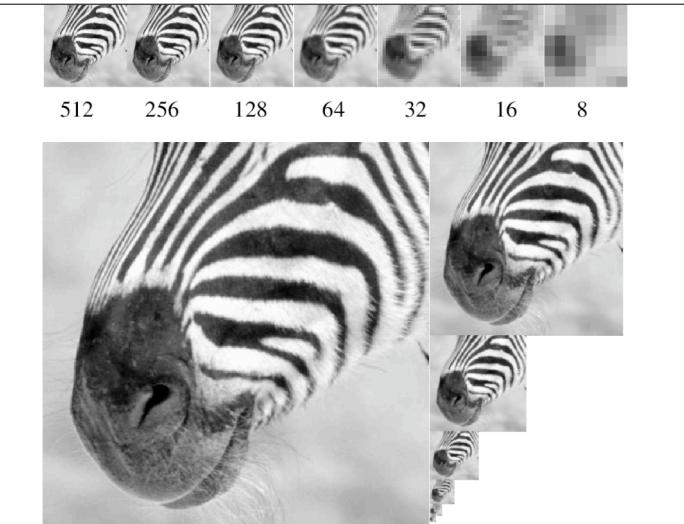
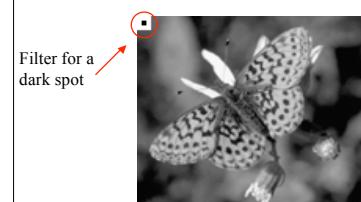


Image Scale

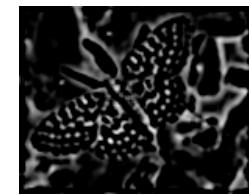
- The difference between a tree in the distance, and its leaves up close, is one of image scale
- An arbitrary image will have multiple arbitrary scales
- Typically we analyze images at various scales
- A good way to think of rescaling an image is to smooth with a Gaussian and sub sample the results.



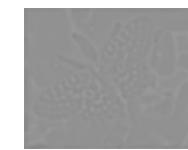
Filters are templates



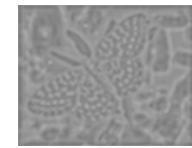
Filter for a
dark spot



Positive responses
(negatives values are set to zero)



To visualize negative values,
make mean value gray .



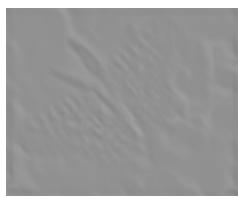
Increase contrast by scaling.



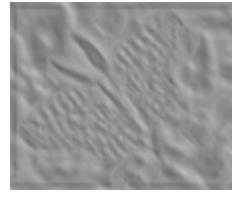
Original image. Bar filter in upper left corner.



Positive responses. Negatives are clipped at zero.



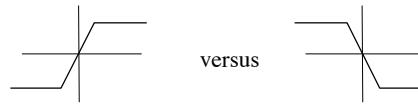
Zero is now gray.



Scaled to push actual max/min towards display max/min.

Linear Filters (§7)

- Properties
 - Output is a **linear** function of the input
 - Can represent as a matrix (but usually do not)
- Terminology
 - Array of weights is referred to as the kernel (H)
 - (Sometimes referred to as "mask" or "template")
- Be aware of two forms
 - (Cross) Correlation (more natural, often what we visualize)
 - Convolution (more commonly referred to, has some useful mathematical properties)
 - Convolution is correlation by a flipped kernel (if kernel is symmetric, then no difference)



Convolution vs. Correlation

- Perhaps easiest to think about in just one dimension (time)
- Correlation
 - New signal by moving desired mask/template around.
 - New values follow the mask
- Convolution
 - Filter is the response to a unit bar (or delta function)
 - Convolution gives response to the entire signal
 - Your signal comes to the filter
- Can switch from one to the other by flipping the filter
 - $h_{corr}(x) = h_{conv}(-x)$
 - $h_{corr}(x, y) = h_{conv}(-x, -y)$
- No difference if filter is symmetric