Probabilistic Fitting

• Generative probabilistic model
  – Tells a story about how stochastic data comes to be
  – Darts fall around the center of the board, but where exactly?
  – Consider a model with parameters, \( \theta \)
  – Consider an observation, \( x_i \)
  – We denote the probability of seeing \( x_i \) under the model by:

\[
p(x_i | \theta)
\]

Read “given” or “conditioned on”
Restricts to the case of \( \theta \)
Defined by \( p(A | B) = \frac{p(A,B)}{p(B)} \)

Probabilistic Fitting

• Multiple observations
  – Suppose we have multiple observations, in a vector \( x \)
  – What is the probability of \( x \)?

• If observations are independent then probability is the product of the individual observations
  – Essentially a definition, but is consistent with intuition
  – The observations are conditionally independent given the model

• So, the probability of \( x \) is then:

\[
p(x | \theta) = \prod p(x_i | \theta)
\]

Probabilistic Fitting

• So, given the model, we have the probability of observing the data

\[
p(x | \theta) = \prod p(x_i | \theta)
\]

• But what we really want is the probability of the model (parameters) given the data!

• Bayes rule comes to the rescue!