What is a map?

• What are some kinds of maps?

• What is the difference between and image and a map?

• What are some good properties of maps (perhaps a function of the kind of map?)

What is a map

• When we studied what might qualify as image “data”, we discussed that spatial arrangement matters

• Maps are typically (representable as) images so space matters, but we add semantic information

• Typically much of that information is about space and relations
  – 10 miles to Tucson
  – This is where the Parthenon is
  – There is a river (coastline, border) following this curve
  – This subway line gets you to the station
    • Does it matter what the path is?
    – This neuron connects to that one
What is a map

• According to Wikipedia
  
  “A map is a visual representation of an area—a symbolic depiction highlighting relationships between elements of that space such as objects, regions, and themes”

• Types of maps
  – Geographic maps
    • Perhaps with overlaid data
  – Star maps
  – Topological maps
    • What is connected to what matters, the details of how it happens is less important

So you want it on a flat piece of paper?

• Mappings of 2D surfaces to 2D surfaces

• Mathematically, something is n-D if you need n numbers to specify (index) it

• What is n for ...
  – A line?
  – A flat piece of paper?
  – A section of the surface of the earth (in Kansas)?
  – A section of the surface of the earth (with Mountains)?
  – A color?

So you want it on a flat piece of paper?

• Things that can be preserved while mapping 2D surfaces to 2D surfaces
  – Shape (e.g., triangles map to similar triangles)
  – Angles (and local shape)
    • Conformal mapping
  – Lines
  – Ratios of line segment lengths
  – Relative areas
  – Local ordering of points
  – Information (i.e., is the mapping invertible)

So you want it on a flat piece of paper?

• Mappings of 2D surfaces to 2D surfaces
  – Simple stretch of a flat surface
    • What does this preserve?
  – What about the surface of the Earth onto flat piece of paper?
    • Details later
  – What about a picture of the surface taken with a camera?
    • Note that unlike 3D->2D projection, this does not necessarily lose information
    • What is preserved?
Perspective image of a flat surface

Angles are not preserved, but lines go to lines, and order is preserved. The transformation is invertible.

"Key-holing"

Perspective image of a flat surface

- This transformation is a homography (linear transformation in homogenous coordinates).

\[
(x, y) \Rightarrow (u, v) = (U/W, V/W)
\]

where

\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} = \begin{bmatrix}
h_{11} & h_{12} & h_{13} \\
h_{21} & h_{22} & h_{23} \\
h_{31} & h_{32} & h_{33}
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

- Application --- matching slides to video frames.