ISTA 352

Lecture 11

Where does the light end up?

Administrivia

- Homework II due in one week
- Kyle would like all files your program needs (I said different in class (sorry!)---no marks off for HW1).
- You should give the problems a good effort on your own, but after struggling for a reasonable amount of time seek help.
  - If you give up too soon you are not learning
  - If you are beating your head against a wall, you are not using your time effectively

Highlights from the survey

- Learning Matlab quickly can be hard
  - Treating Matlab in class or tutorial might help
- Amount of math is OK (some variance here!)
  - But linear algebra tutorials were a bit fast
  - More examples would help
- Overall pace is OK (again some variance)
- Lectures so far are “typical” (with large variance)

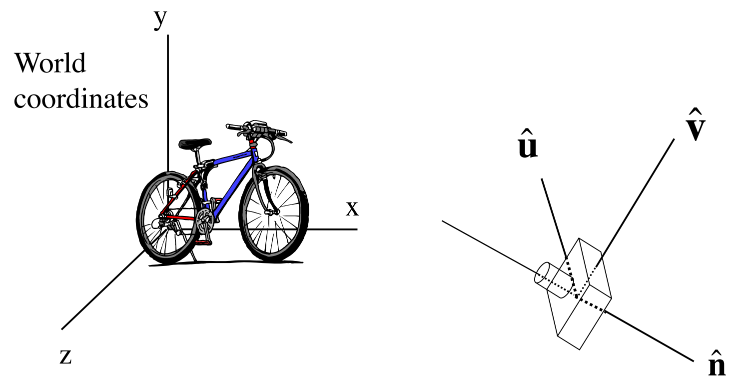
Highlights from the survey

- It is not yet clear what will be on the quiz
  - I will provide some guidance and study questions next week
  - My focus on quizzes will be on conceptual understanding
  - Assignment material is expected to be understood better
  - Matlab programming will NOT be on quizes
    - At most you might see (or use) Matlab notation (e..g, A.*B), but it will NOT be the subject of a question

Geometric Camera Model

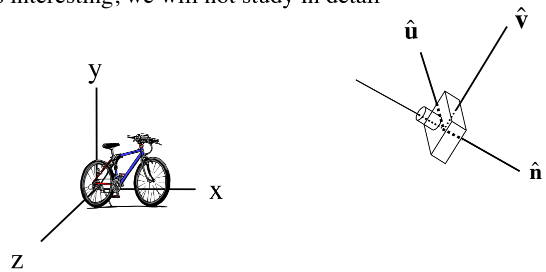
- Let **P**=(X,Y,Z) be a point in space.
- Let (u,v) be image coordinates.
- A geometric camera model, G, tells us where P goes in the image.
- (u,v) = G(**P**)

World and camera coordinates



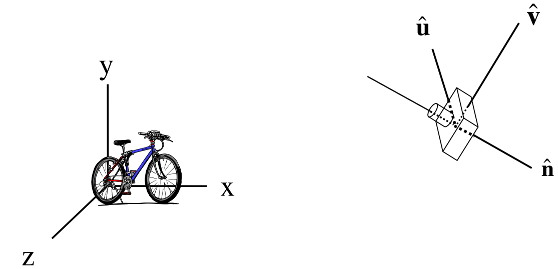
Geometric camera model steps

- (1) Rewrite world coordinates as camera centric coordinates
- (2) Project the world onto the camera plane
  - Easy because of step 1.
- (3) Map resulting canonical image in pixels
  - For example, pixels may not be square, origin may not be in the center
  - Less interesting, we will not study in detail



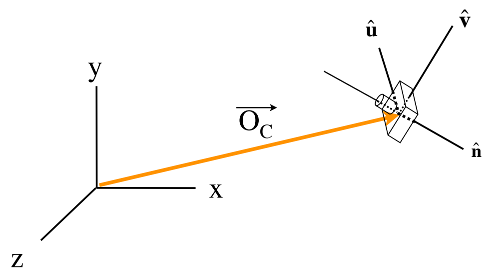
First step of geometric camera model

- Rewrite world coordinates as camera centric coordinates
  - Note that the origins are not the same and the axis are not aligned
  - Note that our rotation matrices are about an axis.
  - Hence we need to translate the world coordinates, and then rotate them.



Transform object expressed in world coords to camera coords

Step 1A. Rewrite world coordinates so that the camera at O_c=(cx, cy, cz) in original coordinates is at the new origin.



Transform object expressed in world coords to camera coords

Step 1A. Rewrite world coordinates so that the camera at O_c=(cx, cy, cz) in original coordinates is at the new origin.

Translation vector is simply negative O_C.

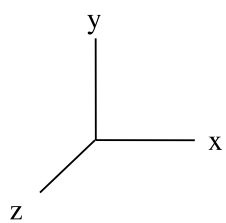
(We want world coordinates to **change** so that the camera location **becomes** the origin).

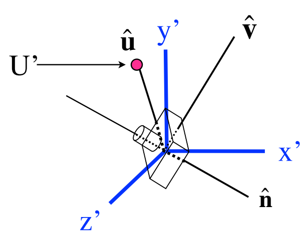
$$T = \begin{vmatrix} 1 & & & -cx \\ & 1 & & -cy \\ & & 1 & -cz \\ & & & 1 \end{vmatrix}$$

Transform object expressed in world coords to camera coords

Step 1B. Rewrite coordinates obtained from step 1 so that the camera axis **becomes** the standard axis—e.g, **û** (now expressed in terms of the translated axis (x',y',z')) becomes (1,0,0).

Similarly, **v**ˆ becomes (0,1,0) and **n**ˆ becomes (0,0,1).





The point, U', shown in red, defines the û vector, described in terms of the translated axis (x',y',z'). We now want to change the basis so that U' becomes (1,0,0).

$$\begin{vmatrix} & ? & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \hat{\mathbf{u}} \\ \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} & ? & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \hat{\mathbf{v}} \\ \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 0 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} & ? & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \hat{\mathbf{n}} \\ \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 1 \end{vmatrix}$$

Transform object expressed in world coords to camera coords

By inspection (see also the tutorial notes)

$$\begin{vmatrix} \hat{\mathbf{u}}^T & 0 \\ \hat{\mathbf{v}}^T & 0 \\ \hat{\mathbf{n}}^T & 0 \\ 0\ 0\ 0 & 1 \end{vmatrix} \begin{vmatrix} \hat{\mathbf{u}} \\ \\ \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

(And similarly, $\hat{\mathbf{v}}$-->Y-axis unit vector, $\hat{\mathbf{n}}$-->Z-axis unit vector)

So in general, our rotation matrix is

$$\begin{vmatrix} \hat{\mathbf{u}}^T & 0 \\ \hat{\mathbf{v}}^T & 0 \\ \hat{\mathbf{n}}^T & 0 \\ 0\ 0\ 0 & 1 \end{vmatrix}$$

Transform object expressed in world coords to camera coords

To get the answer so far (rewrite world coordinates in camera coordinates, we simply multiply the two matrices together:

$$\begin{vmatrix} \hat{\mathbf{u}}^T & 0 \\ \hat{\mathbf{v}}^T & 0 \\ \hat{\mathbf{n}}^T & 0 \\ 0\ 0\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & & & -cx \\ & 1 & & -cy \\ & & 1 & -cz \\ & & & 1 \end{vmatrix}$$

Example

Could be a quiz question

Suppose that the axis of a camera are in the directions of

(3, 4, 0) (-4, 3, 0) (0, 0 5)

Further suppose that the center of the camera is at (1, 2, 3)

A point of confusion from the class exercise was whether these are row vectors since they are written that way. They are actually points used to describe vectors. By convention, vectors are column vectors, although it really only matters when they need to play with matrices.

Construct a matrix in homogenous coordinates that rewrites points given in world coordinates into points in camera coordinates.

(Express your matrix is a product of two others. No need to actually multiply them together).

Solution

We first deal with translation. This means we subtract the camera origin (in world coordinates) from the points.

Intuitive check --- the camera origin should now be (0, 0, 0)

From the problem statement the camera is at (1, 2, 3)

The matrix is:

$$\begin{vmatrix} 1 & & & -1 \\ & 1 & & -2 \\ & & 1 & -3 \\ & & & 1 \end{vmatrix}$$

Solution

Next we deal with rotation. We need orthonormal basis vectors.

First we notice that they are pairwise orthogonal.

$$(3, 4, 0) \bullet (-4, 3, 0) = 0$$
$$(-4, 3, 0) \bullet (0, 0, 5) = 0$$
$$(3, 4, 0) \bullet (0, 0, 5) = 0$$

Next we make them unit length by scaling them by their length.

Solution

Next we make them unit length by scaling them by their length.

$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

$$\frac{(3,4,0)}{|(3,4,0)|} = \frac{(3,4,0)}{\sqrt{3^2+4^2+0^2}} = \frac{(3,4,0)}{\sqrt{9+16}} = \frac{(3,4,0)}{\sqrt{25}} = \frac{(3,4,0)}{5} = (0.6, 0.8, 0)$$

$$\frac{(-4,3,0)}{|(-4,3,0)|} = \frac{(-4,3,0)}{\sqrt{(-4)^2+3^2+0^2}} = (-0.8, 0.6, 0)$$

$$\frac{(0,0,5)}{|(0,0,5)|} = \frac{(0,0,5)}{5} = (0, 0, 1)$$

Solution

Our rotation matrix is

$$\begin{vmatrix} \hat{\mathbf{u}}^T & 0 \\ \hat{\mathbf{v}}^T & 0 \\ \hat{\mathbf{n}}^T & 0 \\ 0\ 0\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 0.6 & 0.8 & 0 & 0 \\ -0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Final answer

$$\begin{vmatrix} 0.6 & 0.8 & 0 & 0 \\ -0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & & & -1 \\ & 1 & & -2 \\ & & 1 & -3 \\ & & & 1 \end{vmatrix}$$