ISTA 352

Lecture 12

Where does the light end up (part II)?

Review

Example from last time

Suppose that the axis of a camera are in the directions of

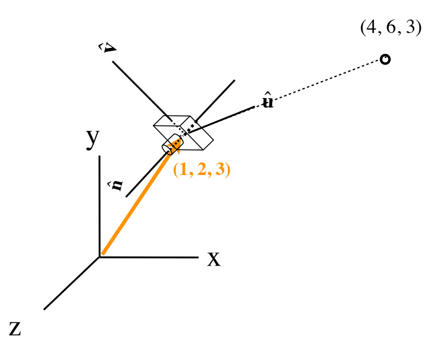
(3, 4, 0)   (-4, 3, 0)   (0, 0 5)

Further suppose that the center of the camera is at (1, 2, 3)

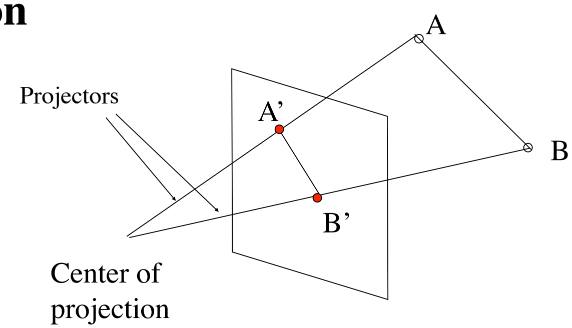
These triples are points used to describe vectors. By convention, vectors are column vectors, although it really only matters when they need to play with matrices.

Construct a matrix in homogenous coordinates that rewrites points given in world coordinates into points in camera coordinates.

(Express your matrix is a product of two others. No need to actually multiply them together).

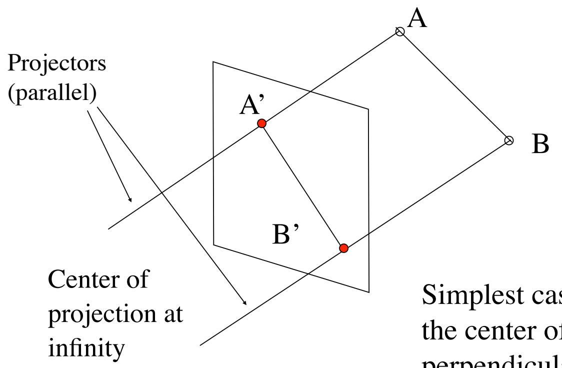


Projection



- Points in 3D world are mapped onto the image plane.
  - The point on the plane is the intersection of the camera plane with the line from the 3D point to the center of projection.
  - A maps to A’ and B maps to B’
  - A’ maps to A’ (doing it again has no effect).

Parallel Projection



Simplest case has the direction to the center of projection perpendicular to the image plane

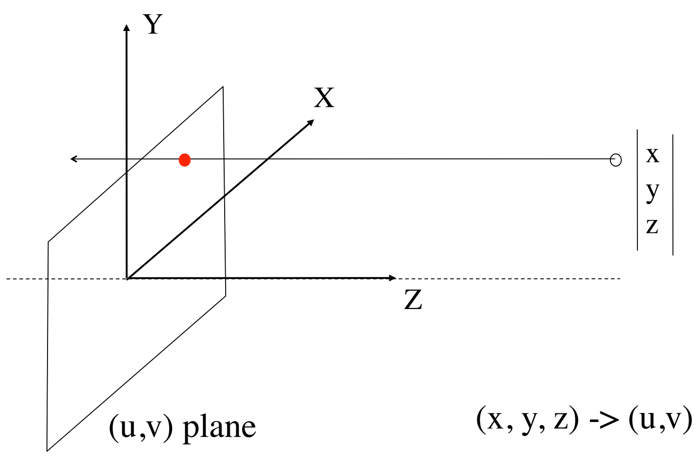
(orthographic projection)

Parallel Projection

Parallel lines remain parallel, some 3D measurements can be made using 2D picture

If projection plane is perpendicular to projectors the projection is orthographic

Orthographic example (onto z=0)



(x, y, z) -> (u,v)

The equation of projection (orthographic, onto z=0)

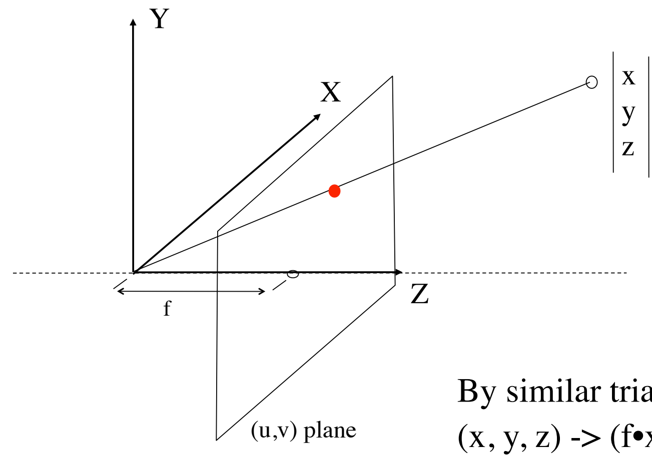
- In homogeneous coordinates

$$(x, y, z, 1) \Rightarrow (x, y, 1)$$

The projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective example (onto z=f)



By similar triangles,
(x, y, z) -> (f•x/z, f •y/z, f)

The equation of projection

- In homogeneous coordinates

$$\underbrace{(x,\ y,\ z,\ 1)}_{\text{3D world point (h.c.)}} \Rightarrow \underbrace{(f\frac{x}{z},\ f\frac{y}{z},\ 1)}_{\text{2D image point (h.c.)}}$$

- But, equivalently

$$\underbrace{(x,\ y,\ z,\ 1)}_{\text{3D world point (h.c.)}} \Rightarrow \underbrace{(x,\ y,\ \frac{z}{f})}_{\text{2D image point (h.c.)}}$$

The equation of projection

- From the previous slide

$$\underbrace{(x,\ y,\ z,\ 1)}_{\text{3D world point (h.c.)}} \Rightarrow \underbrace{(x,\ y,\ \frac{z}{f})}_{\text{2D image point (h.c.)}}$$

- Homogeneous coordinates implement the non-linear part (divide by z)
- Homogeneous coordinates store the change in size due to distance
- Using regular coordinates does **not** yield a linear transformation

The projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix}$$

In computer vision we often use this 3x4 form. In graphics we use a 4x4 with 3rd row (0,0,1,0).

Camera matrix, M

Actual pixel coords are (u,v) = (U/W, V/W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \underbrace{\begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix}}_{M} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Projection. By convention we use **f=1** and put the scale of the W component into the intrinsic parameter matrix.

First part makes it so that we are in standard camera coords where we know how to project.