ISTA 352

Lecture 18

Maps and mappings

Administrivia

- Homework 3A due date extended (still soon!)
  - Now due late Tuesday, October 09

What is a map?

- What are some kinds of maps?
- What is the difference between and image and a map?
- What are some good properties of maps (perhaps a function of the kind of map?)

What is a map

- When we studied what might qualify as image “data”, we discussed that spatial arrangement matters
- Maps are typically (representable as) images so space matters, but we add semantic information
- Typically much of that information is about space and relations
  - 10 miles to Tucson
  - This is where the Parthenon is
  - There is a river (coastline, border) following this curve
  - This subway line gets you to the station
    - Does it matter what the path is?
  - This neuron connects to that one

What is a map

- According to Wikipedia

> "A **map** is a visual representation of an area—a symbolic depiction highlighting relationships between elements of that space such as objects, regions, and themes"

- Types of maps
  - Geographic maps
    - Perhaps with overlaid data
  - Star maps
  - Topological maps
    - What is connected to what matters, the details of how it happens is less important

So you want it on a flat piece of paper?

- Mappings of 2D surfaces to 2D surfaces
- Mathematically, something is **n**-D if you need **n** numbers to specify (index) it
- What is **n** for ...
  - A line?
  - A flat piece of paper?
  - A section of the surface of the earth (in Kansas)?
  - A section of the surface of the earth (with Mountains)?
  - A color?

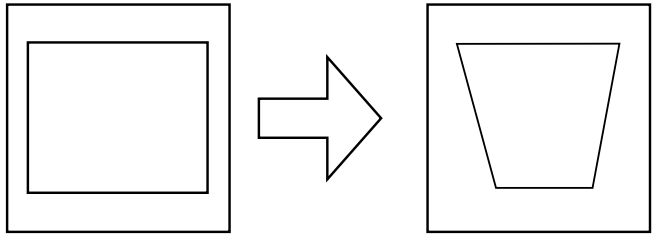
So you want it on a flat piece of paper?

- Things that can be preserved while mapping 2D surfaces to 2D surfaces
  - Shape (e.g., triangles map to similar triangles)
  - Angles (and local shape)
    - Conformal mapping
  - Lines
  - Ratios of line segment lengths
  - Relative areas
  - Local ordering of points
  - Information (i.e., is the mapping invertible)

So you want it on a flat piece of paper?

- Mappings of 2D surfaces to 2D surfaces
  - Simple stretch of a flat surface
    - What does this preserve?
  - What about the surface of the Earth onto flat piece of paper?
    - Details later
  - What about a picture of the surface taken with a camera?
    - Note that unlike 3D->2D projection, this does not necessarily loose information
    - What is preserved?

Perspective image of a flat surface



"Key-holing"

Angles are not preserved,
but lines go to lines,
and order is preserved.
The transformation is invertible

Perspective image of a flat surface

- This transformation is a homography (linear transformation in homogenous coordinates).

$$(x,y) \Rightarrow (u,v) = (U/W, V/W) \quad \text{where}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Application --- matching slides to video frames.