ISTA 352

Lecture 20

Maps and mappings (II)

Review

What is a map?

- According to Wikipedia

  "A **map** is a visual representation of an area—a symbolic depiction highlighting relationships between elements of that space such as objects, regions, and themes"

- Types of maps
  - Geographic maps
    - Perhaps with overlaid data
  - Star maps
  - Topological maps
    - What is connected to what matters, the details of how it happens is less important

Review

Perspective image of a flat surface

- This transformation is a homography (linear transformation in homogenous coordinates).

$$(x,y) \Rightarrow (u,v) = (U/W, V/W) \quad \text{where}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Application --- matching slides to video frames.

Mapping a curved surface to a flat one

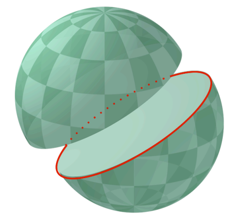
- We assume the mapping should be locally continuous
  - A small change on one surface corresponds to a small change on the other
- Three properties that would be helpful
  - A) Angles are preserved (conformal)
    - Preserving global shape is not well defined
  - B) Relative areas are preserved
  - C) Relative distances are preserved
- Unfortunately, for a sphere you can have no more than one of A or B, and C is not possible

Mapping a sphere to a flat surface

- Classic problem motivated by mapping the Earth
  - Note that the Earth is not exactly a sphere
    - Modern mapping uses a reference ellipsoid
- No perfect solution that preserves all that is desired
  - We can preserve angles
  - We can preserve relative area
  - Doing both, or relative distances, is not feasible.
- Standard solutions are compromises

Mapping a sphere to a flat surface

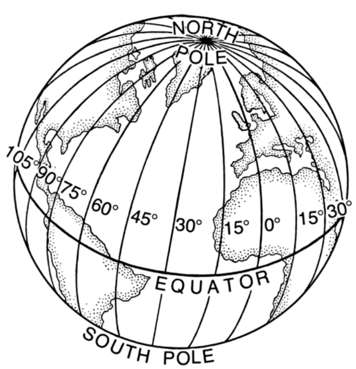
- A geodesic is the shortest path between two points on a surface
- On a sphere geodesics are on great circles
  - We get circles on the sphere with a cutting plane
  - For great circles, the cutting plane is through the center of the sphere

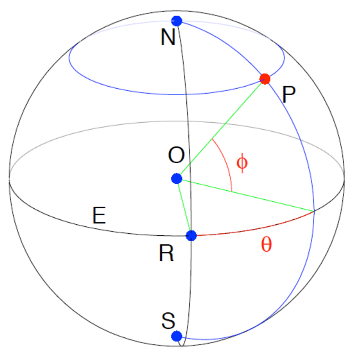


A great circle cuts the sphere into equal halves

Mapping a sphere to a flat surface

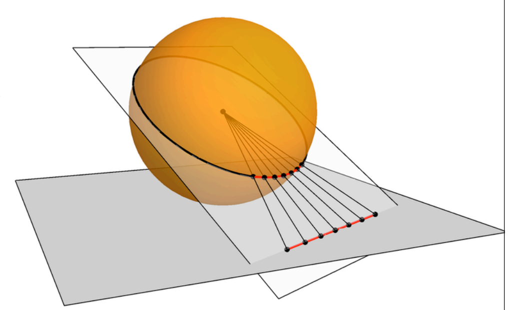
- Meridians (e.g., lines of longitude) are great circles going through the poles





Planar gnomonic projection

- Central projection method
  - Projection point is the center of the sphere
- Chose a point (often north pole; south drawn) for a tangent plane
- Then project points on the surface by extending a line from the center to the plane
- Great circles map to lines
  - Convenient for the shortest shipping or flight path



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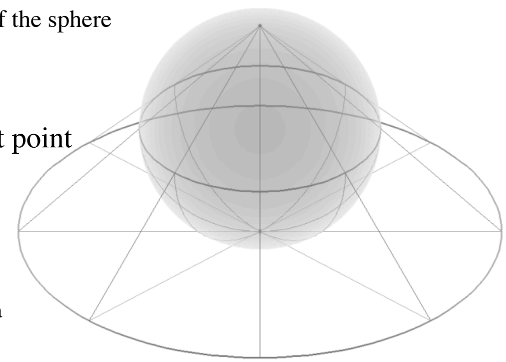
Stereographic projection

- Projection point is on the surface of the sphere
- Usual mapping version projects onto a plane tangent to the projection point antipode.
  - Another version (next slide) is to use a plane through the center of the sphere
- Mapping is conformal
  - i.e., angles are preserved
- Lines through the tangent point map to great circles



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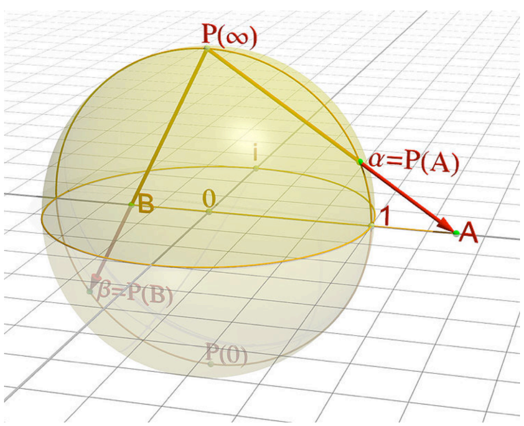
Stereographic projection

- Another version is to use a plane through the center of the sphere



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