ISTA 352

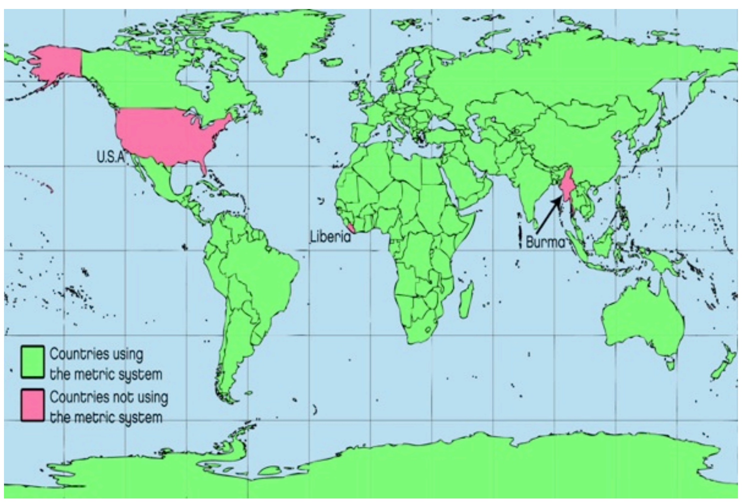
Lecture 21

Maps and mappings (III)

Review

Mapping a curved surface to a flat one

- We assume the mapping should be locally continuous
  - A small change on one surface corresponds to a small change on the other
- Three properties that would be helpful
  - A) Angles are preserved (conformal)
    - Preserving global shape is not well defined
  - B) Relative areas are preserved
  - C) Relative distances are preserved
- Unfortunately, for a sphere you can have no more than one of A or B, and C is not possible

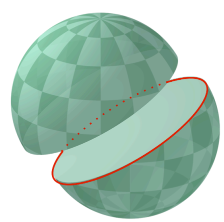


(metric map)

Review

Mapping a sphere to a flat surface

- A geodesic is the shortest path between two points on a surface
- On a sphere geodesics are on great circles
  - We get circles on the sphere with a cutting plane
  - For great circles, the cutting plane is through the center of the sphere

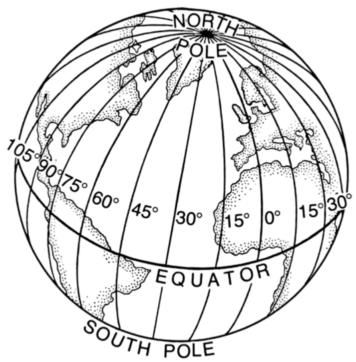


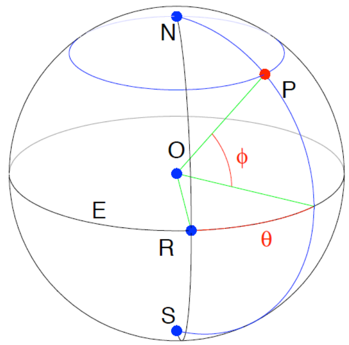
A great circle cuts the sphere into equal halves

Review

Mapping a sphere to a flat surface

- Meridians (e.g., lines of longitude) are great circles going through the poles

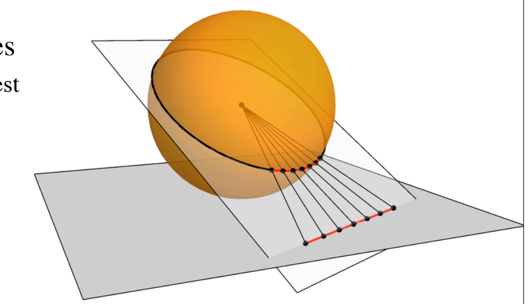




Review

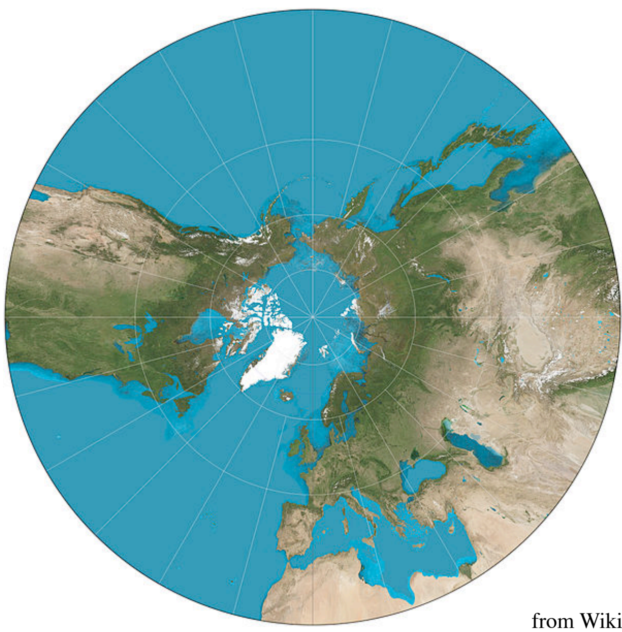
Planar gnomonic projection

- Central projection method
  - Projection point is the center of the sphere
- Chose a point (often north pole; south drawn) for a tangent plane
- Then project points on the surface by extending a line from the center to the plane
- Great circles map to lines
  - Convenient for the shortest shipping or flight path



from WikiMedia commons

Review

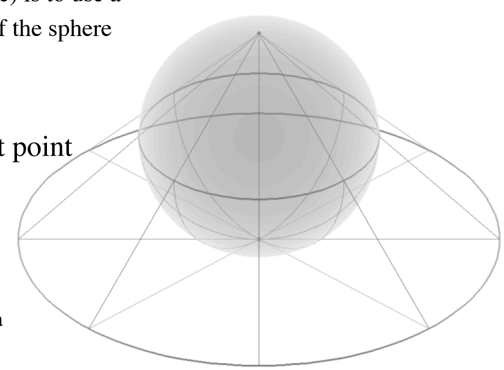


from WikiMedia commons

Review

Stereographic projection

- Projection point is on the surface of the sphere
- Usual mapping version projects onto a plane tangent to the projection point antipode.
  - Another version (next slide) is to use a plane through the center of the sphere
- Mapping is conformal
  - i.e., angles are preserved
- Lines through the tangent point map to great circles



WikiMedia commons

Review

WikiMedia commons

A few early maps

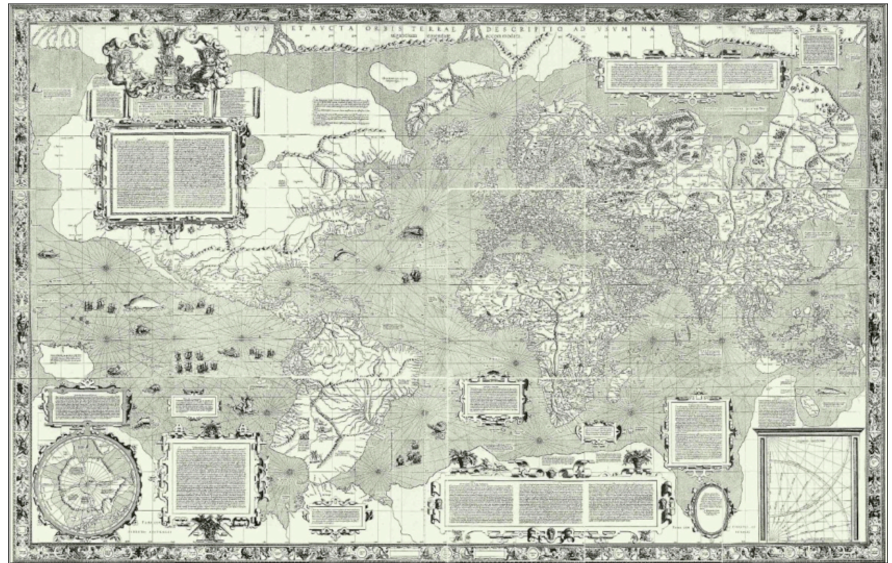
- Note that the western world developed the idea of the Earth as a sphere from about 6th to 3rd century BC

Ptolemy's world map (1467 reconstruction from information in 2nd century book)

1482 (pre-mercator)

1492

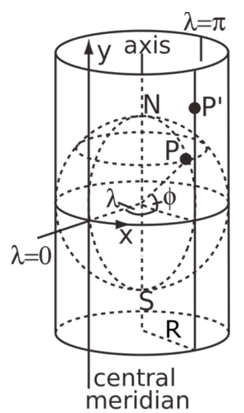
(oldest surviving globe)



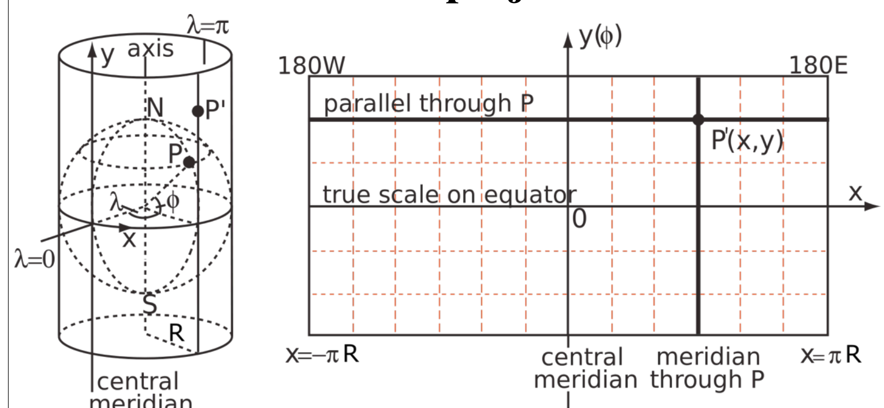
Mercator world map, 1569

Mercator projection

- Projects the world onto a cylinder, which can be unrolled to become a flat map
- If the cylinder axis goes through the poles (shown), lines of longitude are parallel to the vertical axis
- This mapping is also conformal

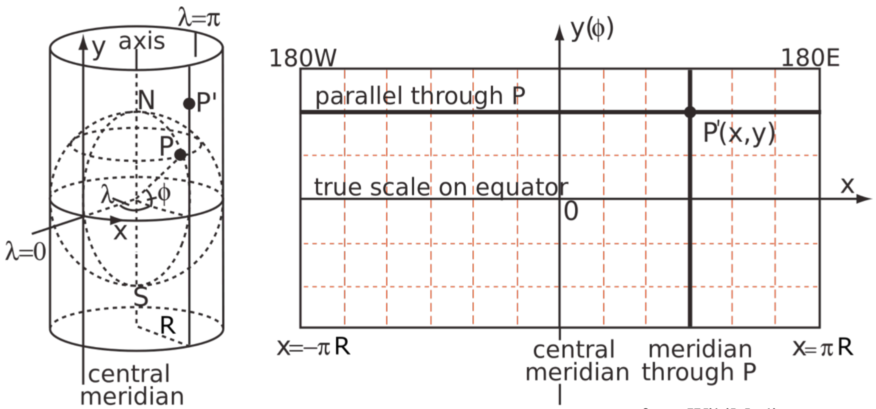


Mercator projection



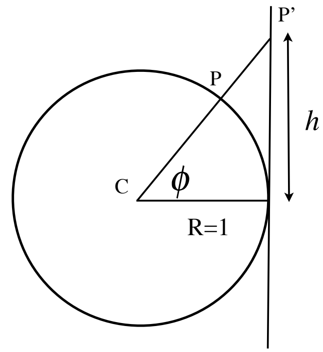
from WikiMedia commons

Mercator projection



from WikiMedia commons

In a basic cylindrical projection, P, and P' are all on a line and the distance above the equator line is easy to work out using trigonometry. For Mercator, P' has a fancy formula (next page)



The Mercator formula is fancier

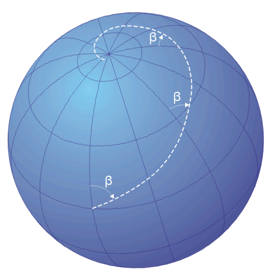
$$h = \ln\left(\sec(\phi) + \tan(\phi)\right)$$

(assuming that the radius is 1 unit)

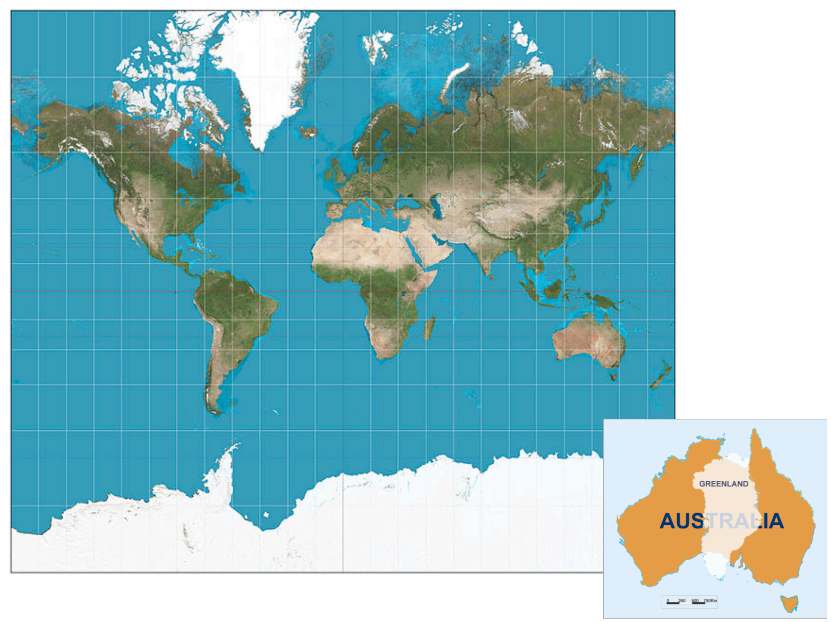
Basic cylindrical projection

Mercator projection

- Lines of constant course (rhumb lines) are straight
- The mapping is accurate near the tangent line (equator in this example).
- The representation of distance expands towards the poles
  - Significant distortion of size
  - Greenland is not that big!

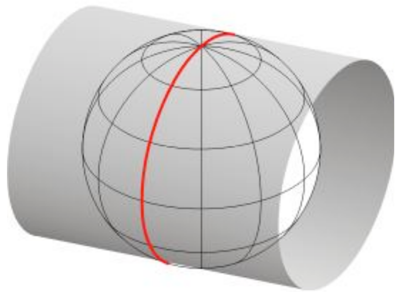


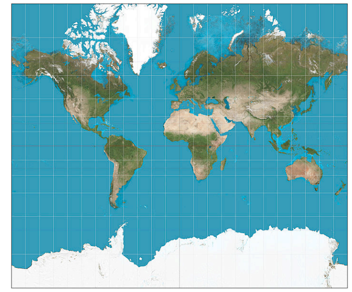
A rhumb line



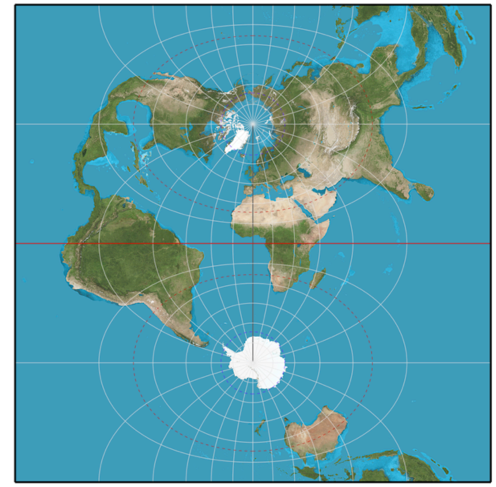
Transverse Mercator projection

- Mercator projection where the cylinder axis goes through the plane of the equator
- Standard approach is to step around the Earth in 6 degree increments (e.g., Universal Transverse Mercator (UTM)).
- Maps made from the slices are reasonably accurate
  - (Mercator is accurate near the tangent circle)





Regular

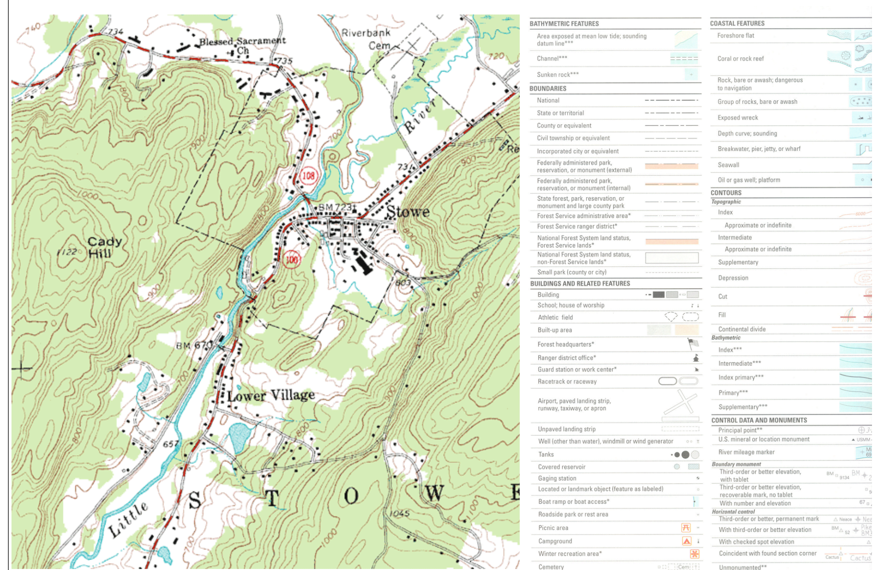


Transverse

Topographic maps

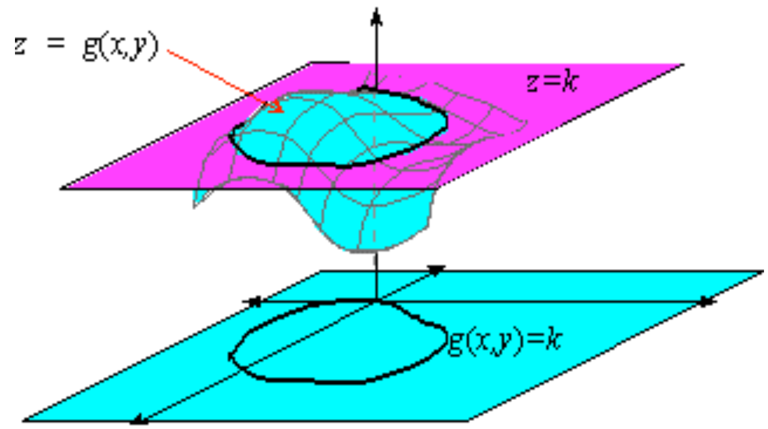
- Maps are useful because the overlay semantic information onto spatial representations

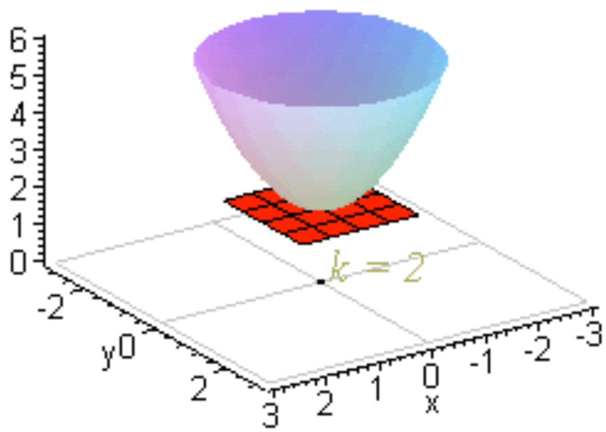
- A topographic map is is a detailed and accurate graphic representation of cultural and natural features on the ground
(Canadian Centre for Topographic Information, via WikiPedia)

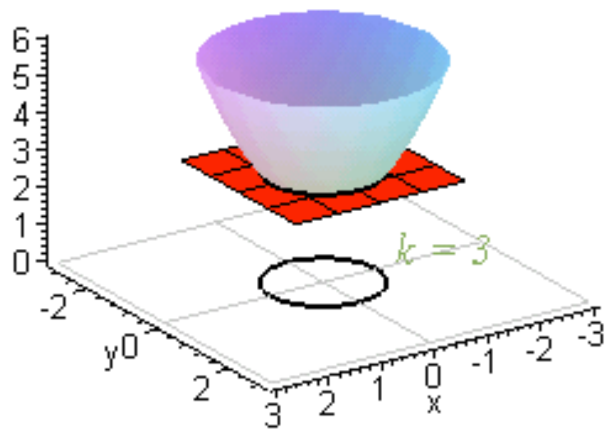


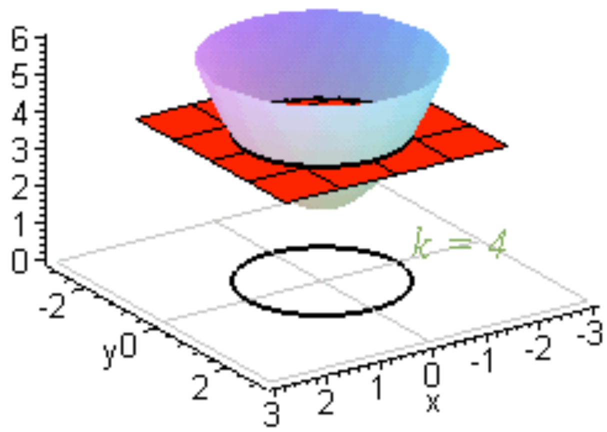
Topographic maps

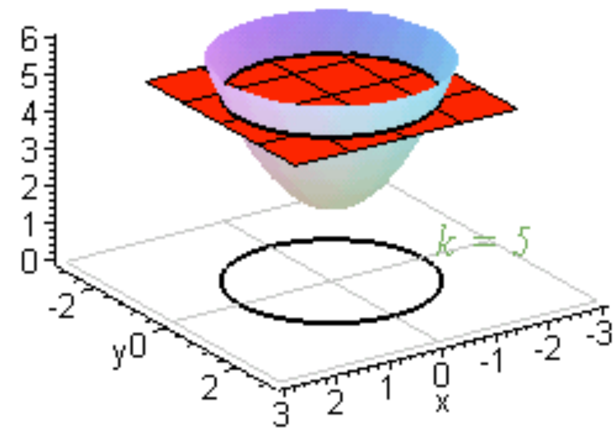
- Maps are useful because the overlay semantic information onto spatial representations
- A topographic map is is a detailed and accurate graphic representation of cultural and natural features on the ground (Canadian Centre for Topographic Information, via WikiPedia)
- One example is to elevation information via contours
  - Contour lines depict paths of constant elevation (typically at fixed intervals such as 40meters).
  - Mathematically, these are level sets of the elevation function of position.











Level sets for f(x,y)

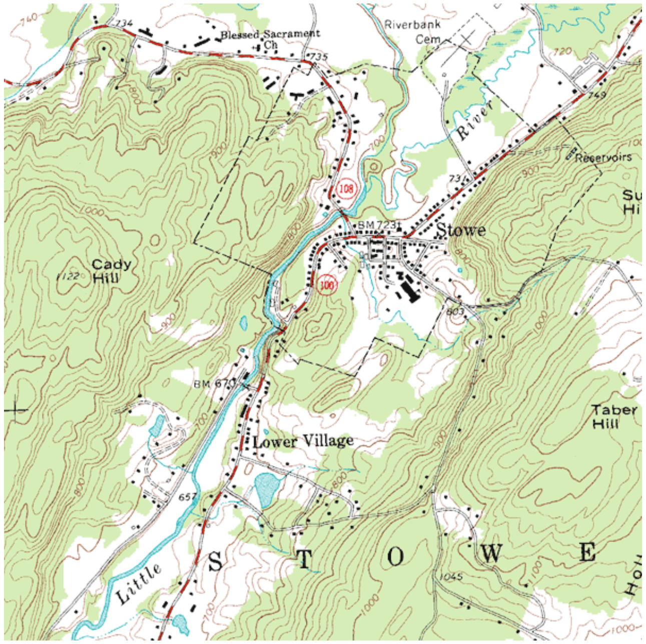
- Given continuous surface, these curves are continuous and typically closed, with the following special cases
  - They can be straight lines (circles with infinite radius)
  - They can be single points
  - They can be a region (if the surface has constant elevation)

Gradient of f(x,y)

- Ignoring special cases, at a point (x,y), f(x,y) has direction of maximal increase
  - Direction of maximal decrees is the opposite
- Mathematically we compute this direction in (x,y) by

$$\nabla f(x,y) = \left(\frac{\delta f}{\delta x}, \frac{\delta f}{\delta y} \right)$$

- Important fact is that the gradient is perpendicular to the level sets

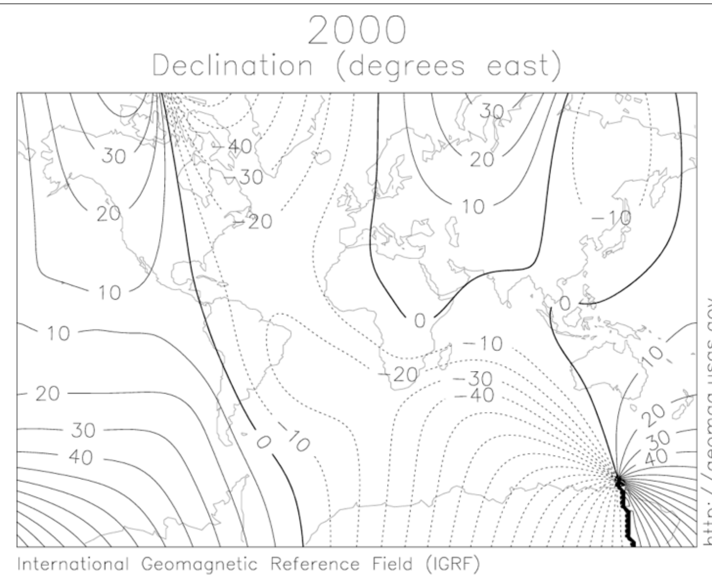


General use of level sets for mapping

- Elevation contours are a particular case of an important representation data defined spatially
  - Isolines (2D)   (iso means equal)
  - Isosurfaces (3D)
    - If f=f(x,y,z) is a function of three variables, the level sets are surfaces
- The isolines or isosurfaces are often called isoXXX, where XXX indicates the values of f() (in latin), as in
  - isobars (equal pressure)
  - isotherms (equal temperature)

Red line is a 10^{o}C isoline of the mean temperature in July

(defines Arctic region)



Isolines for magnetic declination