ISTA 410/510 Midterm II (take home)

For contribution to the final grade, due dates, current late policy, and instructions for handing the assignment in, see the assignment web page.

Please create a PDF document with your answers and/or the results of any programs that you write. You should also hand in your programs.

Because this is a midterm, you should not post questions to the maillist. Rather, send requests for clarification to the instructor.

Unless otherwise specified, each sub-question is worth one point. (One is worth two, another is worth four). There is a total of 16 points. Grad students should attempt to hand in the answers for all of them. Undergrads are responsible for 12 points. There is no extra points beyond 100%. Undergrads should simply exclude handing in answers for a set of questions worth four points in total.

1. Consider the trail network in the Figure. Suppose that a traveler who finds themselves at any junction, has equal probability of choosing all the directions, including the one they just came from.

   Suppose that whenever the traveller passes through a junction, they telephone their significant other who writes down the time of the phone call. However, the traveller does not know which junction they are at. We can attempt to determine their route with the following information. At each junction there is a sensor at that responds with 80% if the traveller passes through. Further, at that time, all other sensors responds randomly to animals, leaves blowing in the wind and so on with probability 20%. The sensors respond independently, and so none, one, or all sensors could respond. The observations are thus the responses of all the sensors. Also suppose we know that the traveler started at node "a".

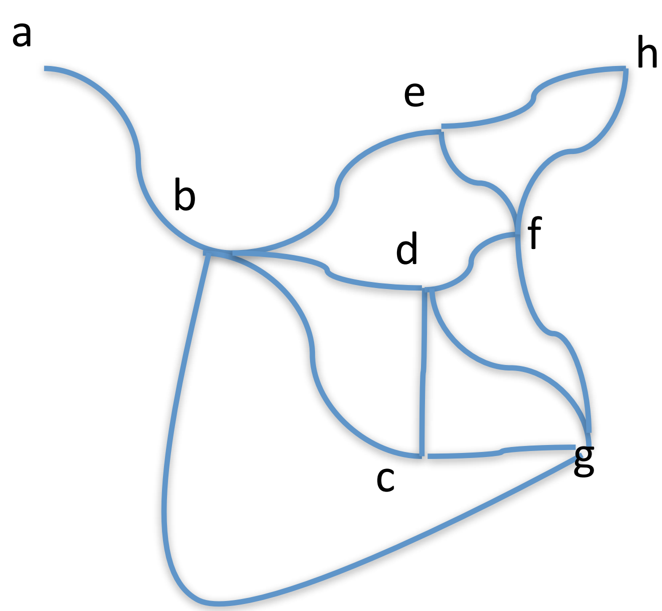
   (a) Write down the matrix of transition probabilities of going from one of the nodes to another.

   (b) Is our traveler's node sequence a Markov process? Explain.

   (c) Write down the probability model for the observations given the model algebraically, with determining the numeric quantities also specified. IE, if your model has a quantity T_{ij}, you could say something like "where T_{ij} is found in the matrix T specified in 1(a)". However you do it, your formulation should be clear enough that someone could compute the right number without having to read this document, or take this ISTA 410.

   (d) Suppose the traveler took the route: R1=[a,b,c,d,b,c,d,b,e,f,h]. What is the probability of sensor responses of

   (i) {a}, {b}, {c}, {d}, {b}, {c}, {d}, {b}, {e}, {f}, {h}
   (ii) {a,b}, {b,c}, {c,d}, {d,f}, {b,d}, {c,f}, {d,h}, {b,e}, {e,h}, {f,e}, {f,h}
   (iii) {}, {b}, {}, {f}, {}, {f}, {}, {h}, {}, {e}, {}



(e) Consider a second route, R2=[a,b,d,f,d,f,h,e,h,e,f]. What is the ratio of the probability of R1, to that of R2, given sensor responses (i), (ii), (iii)?

(f) (four points) Write a computer program to compute the most probable route given which sensors responded for a path through 11 nodes (ideally, N nodes). Run your program on the sequences (i), (ii), and (iii) and report if the answer is what you expected based on (d)?

2.

a) Create an example (hopefully relatively simple) that shows that the most probable sequence of HMM states is not necessarily the sequence of most probable states.

b) Write down a probability model for the following. A magnetic object of mass M is moving in a known force field F(x,y,z) with latent (unknown) position Z(t)=(x,y,z) and velocity V(t). We measure the position X(t) as normally distributed about Z(t). We have V(t+1)=V(t)+(k1)F(Z) and Z(t+1)=Z(t)+(k2)V(t) where k1 and k2 are constants. You can assume priors over initial states. Hint: The latent variables are like states in an HMM and the transition between states are perhaps most easily thought about in terms of matrix operations.

c) (two points) Create a state model with transitions shown by arrows that breaks the following activities into state sequences for each of the actors. The set of models, which depend on each other are *coupled*, but you can consider that for the model for one actor, the states of the other actors are given. 1) Two people meet each other at a specific location L (not necessarily arriving at the same time), have a conversation, then leave L once it is over: 2) One of the persons, "A", gives a package to "B", and "B" leaves with the package: 3) A third person arrives, grabs the package while the transfer in (2) is taking place, and runs off with the package. "A" and "B" head towards the police station at location P.

d) Explain how (or why it is not possible) the following can be modeled as an HMM: Observed sequences of steps taken by jet plane mechanics doing a particular complex repair following instructions in a manual. The mechanic takes decisions (i.e., jump to step 12) based on measurements, but we cannot observe that. Also the mechanic sometimes makes an error and has to go back to a previous step.

e) Explain how (or why it is not possible) the following can be modeled as an HMM: Observed sequences of coin flips from a fair coin.

f) Explain how (or why it is not possible) the following can be modeled as an HMM: A simple metronome which malfunctions with age. When it is working, the state of the pendulum being on the left side is followed by the pendulum being on the right side. There is a small probability that the pendulum fails and stays in the same state for a transition. The probability that the pendulum fails this increases asymptotically to one with increasing age of the device.