ISTA 410/510 Midterm I (take home)

For contribution to the final grade, due dates, current late policy, and instructions for handing the assignment in, see the assignment web page.

Please create a PDF document with your answers and/or the results of any programs that you write. You should also hand in your programs. (This midterm does **not** require any programing, but if you choose to write one for some part, you should hand it in).

Because this is a midterm, you should not post content questions to the mail list. Rather, send requests for clarification to the instructor. (Simple clarifying questions can be posted to the mail list).

There are 20 sub-questions (i.e., 1(a), 2(b)). Grad students should attempt to hand in the answers for all 20. Undergrads are responsible for 14 of them. They are all worth the same, and there is no extra points beyond 100%. Undergrads should just hand in answers for up to 14 different questions.

1. Supposed that your undergraduate **grades** influence the **letters** that professors write for you (higher grade, better letter). Another thing that influences those letters is your **interactions** with those professors (better interaction, better letter). Now suppose that the **rank** of the graduate program you get into is influenced by your **GRE**, your grades, and your letter (better letter increases probability that rank will be higher). Finally, supposed that whether or not you get an academic **job** depends on the graduate program rank (better rank increases your chances), and the **state** where the job is located (political comment goes here).

(a) Draw a directed graphical model for this problem.

(b) Write down an expression for the probability distribution that corresponds to your model.

(c) Argue that getting the job is independent of your grades, *conditioned on your grad program*. (Yup, eventually they are forgotten about!).

(d) Are GRE scores independent of grades in your model? Provide a formal argument based on rules learned in class.

(e) Are GRE scores independent of the quality of the letter in your model? Provide an argument based on rules learned in class.

(f) Are GRE scores independent of the quality of the letter *given the grad program*? Provide an argument based on rules learned in class

(g) To make it easier to think about, suppose that your model is applied is someone you know. In particular it is easier to imagine that you do not know their grades. Now suppose that getting the job *also* depends on their interaction with their professors (better interaction, higher chance). In other words, the interaction with the professor might have revealed some qualities have which may also help them in the interview. Is getting the job *still* independent of their grades *given their grad program*? Provide an argument based on rules learned in class.

If you argue "no", then explain it in English as well. In particular address whether learning that your friend had really high grades increases or decreases the amount of money you are willing to bet that they get the job.

(h) Using the extended model from (g), suppose you now have also come across the letter (the professor forgot to shred the draft). Conditioned on your knowledge of both the *grad program* that your friend is in, and the *letter*, are their grades independent of them getting a job? Provide an argument based on rules learned in class.

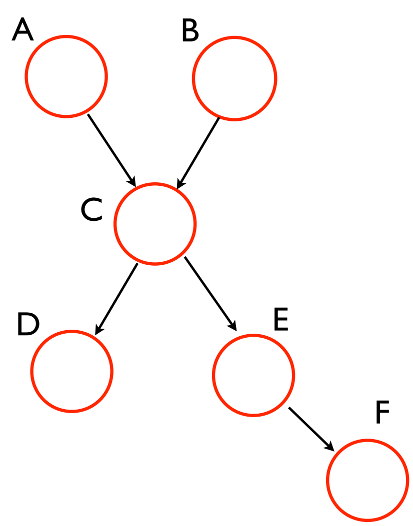
If you argue "no", then explain it in English as well. In particular address whether learning that your friend had really high grades increases or decreases the amount of money you are willing to bet that they get the job.

2.

a) Convert your initial model in (1) to an undirected graph. Explain your steps. Does doing so loose any independence information?

b) Provide potential functions for the undirected graph just made based on the directed graphical model.

3. Consider this graph:



a) Using the abstract definition of what a directed graphical model means in terms of conditional independencies, write down all the conditional independencies that it expresses.

b) Use this to derive an expression for the probability distribution expressed.

c) Write down the probability distribution available by our first definition of a directed graphical model that specified a particular factorization of the distribution. Is is the same? Do you expect it to be?

4. The following matrix tells us who are friends in a cohort of 10 students who conveniently have a different first initial, and all initials are in the first ten letters (e.g., Alice, Bob, Carol, Doug, ...). The cells are "1" if the persons with the initials listed in the row label and the column label are friends. The matrix is symmetric.

	A	B	C	D	E	F	G	H	I	J
A		1	1							
B	1		1	1						
C	1	1		1						
D		1	1		1					
E				1		1			1	
F					1		1	1		
G						1				
H						1				
I					1					1
J									1	

a) Draw an undirected graph, one node per person, with links between friends. We will associate with each person a random variable representing whether their preferred computer is a mac or not. We also have a MicroSoft spy camera that detects whether each person is using a mac or not, but it is not very reliable. Represent the detector output on your graph as well.

b) Identify the cliques in the graph.

c) Provide an expression using potential functions for the probability distribution for the assignment of all the values over the network (e.g., a distribution over any particular assignment of who prefers macs and what they were detected using).

d) Construct an energy function for this model that has the following properties: 1) Everyone has a fixed energy (bias) preference for macs. 2) If one of your friends has a mac, then you are more likely to have one; 3) Three mutual friends having a mac is less likely because mac users have to be different, and so one of them will switch to a PC running linux, rather than be in a group of three mac users; 4) The detector output for which computer you were spotted using is statistically correlated with your actual preferred computer.

e) Is mac preference by A independent of mac preference of J? Provide an argument based on rules learned in class.

f) Is mac preference by A independent of mac preference by D given mac preference by C? Provide an argument based on rules learned in class.

g) Is mac preference by A independent of mac preference by E given mac preference by D? Provide an argument based on rules learned in class.