Example of converting directed to undirected

Directed and undirected perfect maps

D is subset of distributions in P that are perfectly represented by directed graphs; similarly U for undirected graphs.

Inference on graphs

- Given a graph and its conditionals or potentials compute
  \[ p(\theta|e) \quad \text{(particular } \theta \text{ and } e, \text{ marginalizing out other variables)} \]
  \[ p(\mathcal{X}) \quad \text{(particular event, marginalizing out other variables)} \]
  \[ \arg\max p(\theta|e) \quad \text{(particular } \theta \text{ and } e, \text{ marginalizing other variables)} \]
  \[ \arg\max p(\theta,\theta_\Delta,\epsilon|e) \quad \text{(all variables, will nuisance / unobserved)} \]

Simplest example (Bayes rule)
- (a) model
- (b) illustrates observed
- (c) inference reverses the arrow
Inference on graphs

- Simplest example (Bayes’ rule)
  - (a) model
  - (b) illustrates observed
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- Computationally

\[
p(x|y) = \frac{p(y|x)p(x)}{\sum_{x'} p(y|x')p(x')}
\]

Marginals on a chain

Recall

\[
p(x) = p(x_1)p(x_2|x_1)p(x_3|x_2) \cdots p(x_{N-1}|x_{N-2})p(x_N|x_{N-1})
\]

Converted to

\[
p(x) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \cdots \psi_{N-2,N-1}(x_{N-2},x_{N-3})\psi_{N-1,N}(x_{N-1},x_N)
\]

Assume N discrete variables, with K values each.

Compute the marginal of a node in the middle, \( p(x_n) \)

Marginals on a chain

Direct calculation of \( p(x_n) \)

\[
p(x_n) = ?
\]

Marginals on a chain

Direct calculation of \( p(x_n) \)

\[
p(x_n) = \sum_{x_1} \sum_{x_2} \cdots \sum_{x_{n-1}} \sum_{x_{n+1}} \cdots \sum_{x_N} p(x)
\]

Skip \( x_n \)
Marginals on a chain

\[ p(x) = \psi_{1,2}(x_1, x_2)\psi_{2,3}(x_2, x_3) \ldots \psi_{N-2,N-1}(x_{N-2}, x_{N-1})\psi_{N-1,N}(x_{N-1}, x_N) \]

Direct calculation of \( p(x_n) \)

\[ p(x_n) = \sum_{x_1} \sum_{x_2} \ldots \sum_{x_{n-1}} \sum_{x_{n+1}} \ldots \sum_{x_N} p(x) \]

What is the computational complexity?

Computational complexity is \( O(K^N) \). Way too slow!