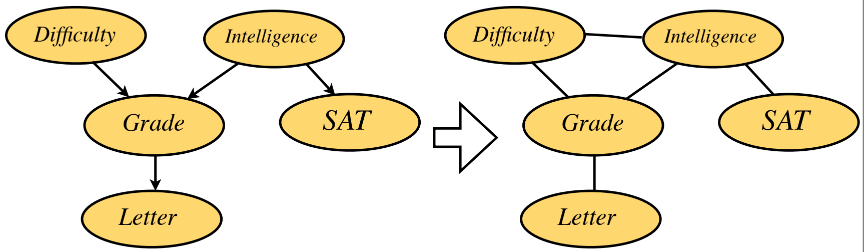
Example of converting directed to undirected



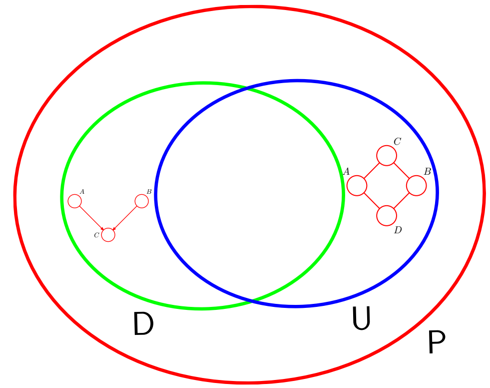
$$P(I,D,G,L,S)=P(I)P(D)P(G|I,D)P(L|G)P(S|I)$$

$$P=\psi(D,G,I)\psi(S,I)\psi(L,G)$$

$$\psi(D,G,I)=P(I)P(D)P(G|I,D) \quad \psi(S,I)=P(S|I) \quad \psi(L,G)=P(L|G)$$

$$\psi(D,G,I)=P(D)P(G|I,D) \quad \psi(S,I)=P(I)P(S|I) \quad \psi(L,G)=P(L|G)$$

Directed and undirected perfect maps



D is subset of distributions in P that are perfectly represented by directed graphs; similarly U for undirected graphs.

Inference on graphs

- Given a graph and its conditionals or potentials compute

  $p(\theta|e)$ (particular θ and e, marginalizing out other variables)

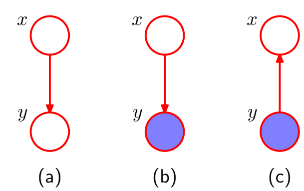
  $p(X)$ (particular event, marginalizing out other variables)

  argmax $p(\theta|e)$ (particular θ and e; marginalizing other variables)

  argmax $p(\theta,\theta_N,e_N|e)$ (all variables, will nuisance / unobserved)

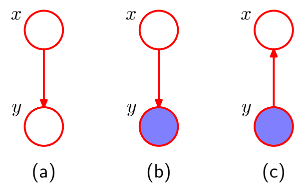
Inference on graphs

- Simplest example (Bayes rule)
  - (a) model
  - (b) illustrates observed
  - (c) inference **reverses** the arrow



Inference on graphs

- Simplest example (Bayes' rule)
  - (a) model
  - (b) illustrates observed
  - (c) inference **reverses** the arrow



- Computationally

$$p(x|y) = \frac{p(y|x)\,p(x)}{\sum_{x'} p(y|x')\,p(x')}$$

Marginals on a chain

Recall

$$p(\mathbf{x}) = p(x_1)\,p(x_2|x_1)\,p(x_3|x_2) \;\cdots\; p(x_{N-1}|x_{N-2})\,p(x_N|x_{N-1})$$

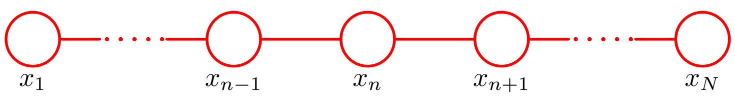
Converted to

$$p(\mathbf{x}) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \;\;\ldots\;\; \psi_{N-2,N-1}(x_{N-2},x_{N-1})\psi_{N-1,N}(x_{N-1},x_N)$$

Assume N discrete variables, with K values each.

Compute the marginal of a node in the middle, $p(x_n)$

Marginals on a chain

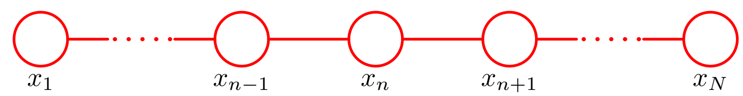


$$p(\mathbf{x}) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \;\;\ldots\;\; \psi_{N-2,N-1}(x_{N-2},x_{N-1})\psi_{N-1,N}(x_{N-1},x_N)$$

Direct calculation of $p(x_n)$

$p(x_n) = \;?$

Marginals on a chain



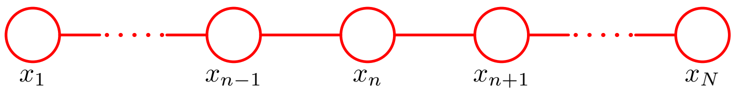
$$p(\mathbf{x}) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \;\;\ldots\;\; \psi_{N-2,N-1}(x_{N-2},x_{N-1})\psi_{N-1,N}(x_{N-1},x_N)$$

Direct calculation of $p(x_n)$

$$p(x_n) = \sum_{x_1}\sum_{x_2} \ldots \sum_{x_{n-1}}\sum_{x_{n+1}} \ldots \sum_{x_{N-1}}\sum_{x_N} p(\mathbf{x})$$

Skip x_n

Marginals on a chain



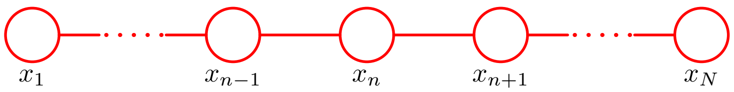
$$p(\mathbf{x}) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \;\ldots\; \psi_{N-2,N-1}(x_{N-2},x_{N-1})\psi_{N-1,N}(x_{N-1},x_N)$$

Direct calculation of $p(x_n)$

$$p(x_n) = \sum_{x_1}\sum_{x_2}\ldots\sum_{x_{n-1}}\sum_{x_{n+1}}\ldots\sum_{x_{N-1}}\sum_{x_N} p(\mathbf{x})$$

What is the computational complexity?

Marginals on a chain



$$p(\mathbf{x}) = \psi_{1,2}(x_1,x_2)\psi_{2,3}(x_2,x_3) \;\ldots\; \psi_{N-2,N-1}(x_{N-2},x_{N-1})\psi_{N-1,N}(x_{N-1},x_N)$$

Direct calculation of $p(x_n)$

$$p(x_n) = \sum_{x_1}\sum_{x_2}\ldots\sum_{x_{n-1}}\sum_{x_{n+1}}\ldots\sum_{x_{N-1}}\sum_{x_N} p(\mathbf{x})$$

Computational complexity is $O(K^N)$. Way too slow!