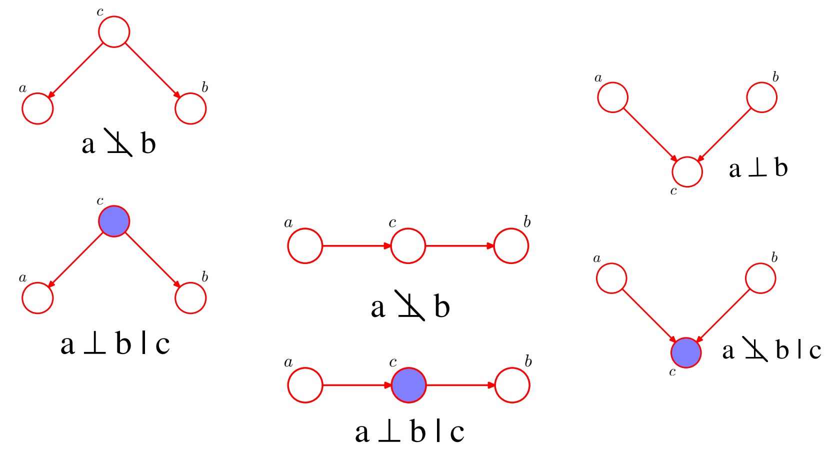
Announcements

- “midterm one” will be posted soon
  - This “midterm” is no different in format from the assignments so far

Three random variables summary



Three random variables summary

In cases one and two, *a* and *b* were not independent until the observation of *c* “blocked” the (connection) path from *a* to *b*.

(From Koller and Friedman, a path that is **not** blocked is “active”)

In case three, if *c* is not observed, the path is blocked. Observing *c* made the connection (path) active.

Three random variables summary

Put more generally, paths are blocked by

1) A tail-tail node in the conditioning set
2) A head-tail node in the conditioning set
3) A head-head node **not** in the conditioning set, AND that has no descendants in the conditioning set.

d-Separation (Pearl, 88)

"d" stands for "directed"

Generalizes the examples we have been studying.

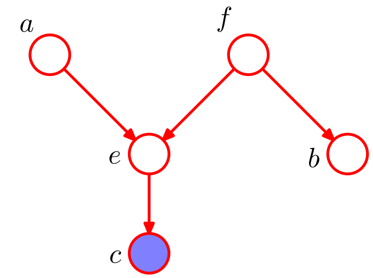
Consider non-overlapping subsets A, B, C of nodes of a graph.

Consider all paths from nodes in A to nodes in B.

A path is blocked if either:
a) The arrows meet either tail-to-tail or head-to-tail at a node in C.
b) The arrows meet head-to-head at some node that is not in C, nor are any of its descendants in C.

If all paths are blocked, then A and B are independent given C.

d-Separation (example one)

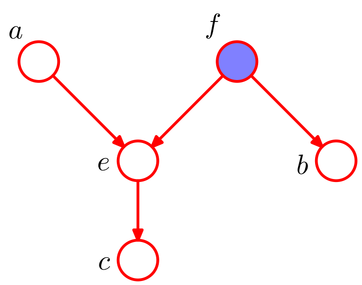


$$A \not\perp B \mid C$$

The path is not blocked by e because, although it is head-to-head, it has a descendant, c, in the conditioning set.

The path is not blocked by f because it is tail-to-tail, and f is not in C.

d-Separation (example two)



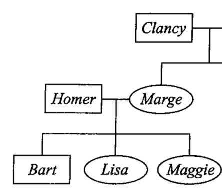
$$A \perp B \mid F$$

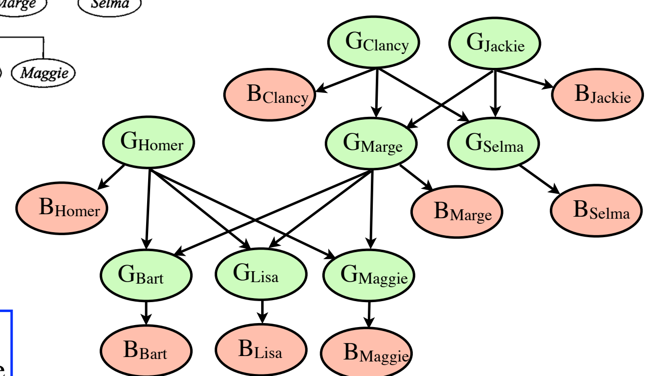
The path is blocked by e because it is head-to-head, and neither it, nor any of its descendants are in the conditioning set.

The path is also blocked by f because it is tail-to-tail, and f is in F.

Grounded example of a Bayesian Network

From Koller and Friedman





G is genotype
B is blood type

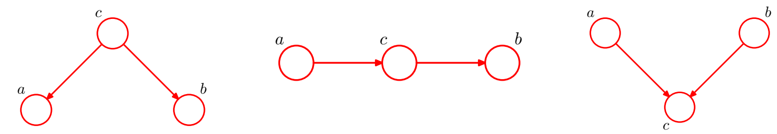
Bayesian network semantics

- Represents a factorization of p()
  - Random variables are nodes
  - Factors are CPD (conditional probability distributions) for child given parent (just p(`NODE`) if no parents).

Equivalent semantic specification (Proof is in K&F, ch. 3)

- For each X_i : $X_i \perp \text{NonDescendents}(X_i) \mid \text{Parents}(X_i)$
  - Notice no mention of factorization
  - Notice no mention of observed (shaded nodes)
  - Call such independence assertions for a graph, G, *I(G)*
  - Call such independence assertions for a distribution, P, *I(P)*

Check against our three random variables story



A few notes on notation and independence

We sometimes write $(A \perp B \mid \varnothing)$ for $A \perp B$

Also, we write $(A \perp B, C \mid X)$ for $(A \perp B \mid X)$ and $(A \perp C \mid X)$

Recall that $(A \perp B \mid C)$ means that $P(A \mid B, C) = P(A \mid C)$

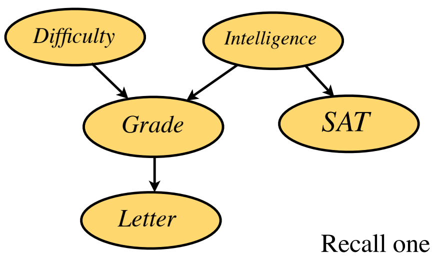
This generalizes to:

$$(A \perp B \mid \ldots, C, \ldots) \Rightarrow P(A \mid \ldots, B, C, \ldots) = P(A \mid \ldots, C, \ldots)$$

Going from independence to a factorization

From Koller and Friedman

For $P(I, D, G, L, S)$, what does $I(Graph)$ tells us?



Recall one version of DAG semantics is

$X_i \perp \text{NonDescendents}(X_i) \mid \text{Parents}(X_i)$

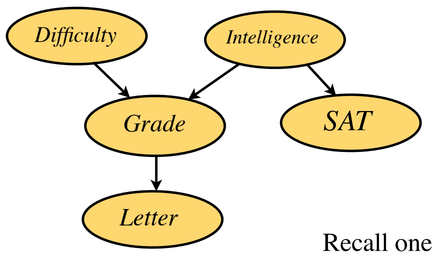
Example going from I-map to a factorization

From Koller and Friedman

For $P(I,D,G,L,S)$, $I(Graph)$ tells us

$(D \perp I|\varnothing) \quad (I \perp D|\varnothing) \quad (L \perp I,D,S|G) \quad (G \perp S|I,D) \quad (S \perp D,G,L|I)$

(Note that this is not necessarily all relationships that we can extract)



Recall one version of DAG semantics is

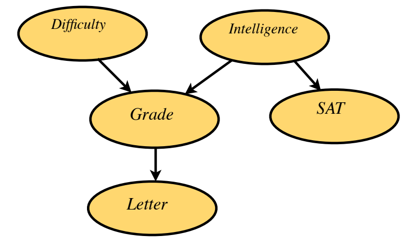
$\mathrm{X}_i \perp \mathrm{NonDescendents}(X_i) \,|\, \mathrm{Parents}(X_i)$

We can write the joint distribution as conditioning on non-descendents if we maintain a sensible "lexigraphical order" where parents occur before children.

$P(I,D,G,L,S) = P(I)P(D|I)P(G|I,D)P(L|I,D,G)P(S|I,D,G,L)$

This means that for each factor, all variables conditioned on are either the parents, or non-descendents.

This means that for each factor, we may have rule that gets rid of some non-descendents.



Going from independence to a factorization

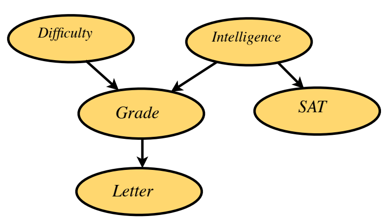
$P(I,D,G,L,S) = P(I)P(D|I)P(G|I,D)P(L|I,D,G)P(S|I,D,G,L)$

$(D \perp I|\varnothing) \Rightarrow P(D|I) = P(D)$

$(L \perp I,D,S|G) \Rightarrow P(L|I,D,G) = P(L|G)$

$(S \perp D,G,L|I) \Rightarrow P(S|I,D,G,L) = P(S|I)$

So, $P(I,D,G,L,S) = P(I)P(D)P(G|I,D)P(L|G)P(S|I)$



Conditional independence in distributions and graphs

Let $I(P)$ be the set of independence assertions of the form $(X \perp Y|Z)$ that are true for a distribution P.

Let $I(G)$ be the set of independence assertions represented by a DAG, G.

G is an I-map for P if I(G) $\subseteq$ $I(P)$

In other words, all independance represented in G are true.
(There could be some more in P that G does not reveal).

Summary on the equivalence of the two interpretations of directed graphical models

Factorization semantics
Factors are p(node | parents)

Abstract semantics
$$X_i \perp \text{NonDescendents}(X_i) \mid \text{Parents}(X_i)$$

These are equivalent
Proof of one direction by the one example just completed.

Interesting questions

- Does every probability distribution have a corresponding Bayesian network?

  Chain rule says yes

- Given the independence structure of a probability distribution, and a graph that captures them all (I(G)=I(P), is the corresponding graph unique (ignoring isomorphisms)?

  Case study of three nodes says no

- Do our graphs faithfully capture the independence structure of our distributions?

  TBA